7.4 Day 2

Thursday, February 16, 2017 12:24 PM

Modeling Growth and Decay

Derive the general solution to $\frac{dy}{dt} = ky$, where k is a constant.

$B = e^C$

$\int \frac{1}{y}\,dy = \int k\,dt$

$\ln y = kt + C$

$e^{kt+C} = y$

$y = e^{kt} \cdot e^C$

$y = Be^{kt}$

Law of exponential change:

If y changes at a rate proportional to the amount present, then

$$y = y_0 e^{kt}$$

where y_0 is the initial value and k is a constant.

For what values of k will this function represent exponential growth?

positive

For what values of k will this function represent exponential decay?

negative

You may choose to have the base be another constant other than e based on the information given in the problem.

Ex 1 There are 40 bees in a hive on April 1st. Just 30 days later, there are 120 bees in the hive. Assume this growth will continue. Find:

a. an exponential model for the number of bees in the hive in terms of time in days.

$(0, 40)$ $y_0 = 40$ $(30, 120)$

$120 = 40 \cdot e^{30k}$

$3 = e^{30k}$ $\ln e^{30k}$

$\ln 3 = 30k \cdot \ln e$

$k = \ln 3 / 30 \approx .037$

$y = 40e^{.037t}$

b. When will the number of bees reach 1000? 10,000?

$1000 = 40e^{.037t}$

$\frac{1000}{40} = e^{.037t}$

$\frac{\ln(1000/40)}{.037} = \frac{.037t}{.037}$

$t \approx 87.898$ days

Ex 2 Carbon-14 has a half-life of 5700 years.

a. Find a decay function to model the amount of Carbon-14 left in terms of time in years.

$y_0 = 1$ $\quad (5700, \tfrac{1}{2})$ $\quad t \quad y$

$\frac{1}{2} = 1e^{5700k}$ $\quad \frac{\ln \frac{1}{2}}{5700} = \frac{5700k}{5700}$ $\quad k = \frac{\ln \frac{1}{2}}{5700}$

$y = e^{\frac{\ln \frac{1}{2}}{5700} t}$

b. When will 20% of the original amount of Carbon-14 be all that remains?

$y_0 = 1$ $\quad (t, .20)$ $\quad .20 = e^{\frac{\ln \frac{1}{2}}{5700} t}$

Ex 3 The processing of raw sugar has an "inversion" step that changes the sugar's molecular structure. Once the process has begun, the rate of change of the amount of raw sugar is proportional to the amount of raw sugar remaining. If 1000 kg of raw sugar reduces to 800 kg of raw sugar during the first 10 hours, how much raw sugar will remain after another 14 hours?

$y_0 = 1000$ $\quad (10, 800)$

Ex 4 You are hungry! You see that your mom is baking a dish of lasagna at 350°F and you take it out of the oven into a kitchen that is 73°F. After 6 minutes, the temperature of the lasagna is 298.9°F. What will its temperature be 11 minutes after you take it out of the oven? Round your answer to the nearest tenth of a degree.