FTC Classwork/Homework **Name** ME

Due: ______________

Directions: Show work to solve all problems, then choose the correct answer. Calculator problems have a C next to them.

C 1. The function $f(x) = \frac{x}{e^x}$ has the derivative $f'(x) = \frac{1-x}{e^x}$. Use the *Fundamental Theorem of Calculus* to find the exact value of the definite integral $\int_0^1 \frac{1-x}{e^x}\,dx$. $= f(1) - f(0)$

$= \frac{1}{e} - \frac{0}{e^0} = \frac{1}{e}$

A. $\frac{1}{3e}$ B. $\frac{1}{2e}$ C. $\frac{1}{e}$ D. e E. None of these

D 2. If F and f are continuous functions such that $F'(x) = f(x)$ for all x, then $\int_a^b f(x)\,dx =$ $\int_a^b F'(x)\,dx$

$= F(b) - F(a)$

A. $F'(b) - F'(a)$ B. $F'(a) - F'(b)$ C. $F(a) - F(b)$ D. $F(b) - F(a)$ E. None of these

C 3. A particle is moving along the x-axis so that at time t its velocity is given by $v(t) = 3t^2 - 2t$. At the instant when $t = 0$, the particle's location is $x = 2$. The position of the particle at $t = 3$ is

$S(0) = 2$ $S(3) = ?$

$\int_0^3 v(t)\,dt = S(3) - S(0)$

A. 12 B. 16 C. 20 D. 24 E. None of these

$S(0) + \int_0^3 v(t)\,dt = S(3)$

$2 + \left[t^3 - t^2\right]_0^3 = 2 + 27 - 9$

$= 20$

D 4. If $\int_{-1}^{k} (3x^2)\,dx = 9$, then $k =$

A. -1 B. 0 C. 1 D. 2 E. 3

C

A

5. If f is the function defined by $f(x) = \sqrt[3]{x^2 + 4x}$ and g is an antiderivative of f such that $g(5) = 7$, then $g(1) \approx$

$\int_{-1}^{k} (x^2)\,dx = \frac{x^3}{3}\Big|_{-1}^{k} = 9$ $\quad \frac{k^3}{3} - \frac{(-1)^3}{3} = 9$ $\quad k^3 = 8 \quad k = 2$

$\int_1^5 f(x)\,dx = g(5) - g(1)$ $\quad \int_1^5 f(x)\,dx = 7 - g(1)$

A. -3.882 B. -3.557 C. 1.710 D. 3.557 E. 3.882

$7 - \int_1^5 f(x)\,dx = g(1) \Leftarrow -7 + \int_1^5 f(x)\,dx = -g(1)$

C

E

6. The rate at which ice is melting in a pond is given by $\frac{dV}{dt} = \sqrt{1 + e^t}$, where V is the volume of ice in cubic feet and t is the time in minutes. Suppose that 1.642 ft^3 of ice melted in the first minute, $V(1) = 1.642$. Find $V(3)$, the total amount of ice has melted after 3 minutes.

$\int_1^3 V'(t)\,dt = V(3) - V(1)$

A. 7.60 ft^3 B. 7.62 ft^3 C. 7.64 ft^3 D. 7.66 ft^3 E. 7.68 ft^3

$V(1) + \int_1^3 V'(t)\,dt = V(3)$

$1.642 + \int_1^3 V'(t)\,dt$

e 7. $\frac{d}{dx}\left(\int_0^{x^2} \sin\left(t^3\right)dt\right) =$ $\sin x^6 \cdot 2x$

$= 2x \sin x^6$

(a) $-\cos(x^6)$ (b) $\sin(x^3)$ (c) $\sin(x^6)$ (d) $2x\sin(x^3)$ (e) $2x\sin(x^6)$

C

d

8. Let g be the function given by:

$$g(x) = \int_0^x \sin\left(t^2\right)dt$$

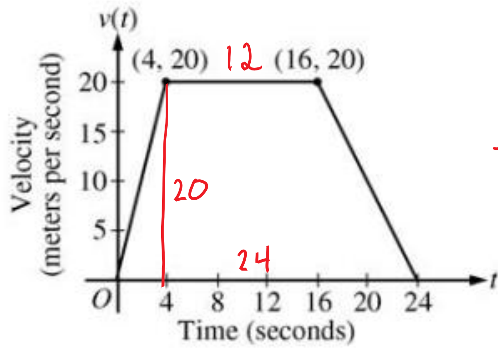
$g'(x) = \sin x^2$

for $-1 \le x \le 3$. On which of the following intervals is g decreasing?

(a) $-1 \le x \le 0$ (b) $0 \le x \le 1.772$ (c) $1.253 \le x \le 2.171$ (d) $1.772 \le x \le 2.507$ (e) $2.802 \le x \le 3$

Free Response: Show work and justify where required.

9.



$\frac{1}{2}(20)(12+24)$

$10(36)$

360

A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

(a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.

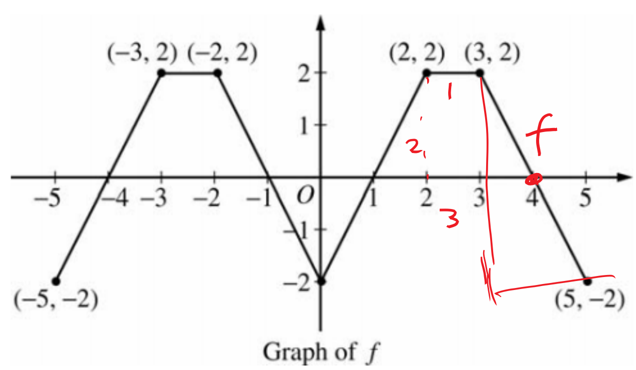
The car traveled 360 meters over the time interval 0 to 24 seconds.

10.

The graph of the function f shown above consists of six line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(4)$, $g'(4)$, and $g''(4)$.

(b) Does g have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.



$g(4) = \int_0^4 f(t)\,dt$

$= -\frac{1}{2}(1)(2) + \frac{1}{2}(2)(3+1) = -1 + 4 = 3$

$g'(x) = f(x)$

$g'(x) = f(x)$ $\quad g'(4) = 0$ $\quad g''(4) = \frac{-4}{2} = -2$

g(x) has a relative min @ x=1 b/c

$g'(x) = f(x)$ goes from − to +.