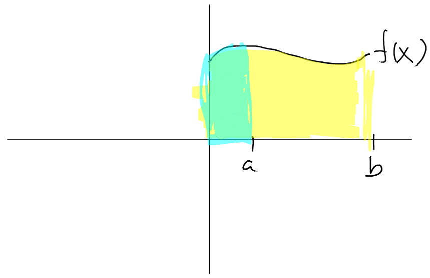
Fundamental Theorem of Calculus Part II

FTC Part II

If f is a continuous function @ every point on $[a,b]$ and F is any antiderivative of f on $[a,b]$

then:

$$\underbrace{\int_a^b f(x)\,dx = F(b) - F(a)}$$



☆ $F(a) + \int_a^b f(x)\,dx = F(b)$ ☆☆

Evaluate $\int_{-3}^{-1} (x^3 + x + 6)\,dx = \frac{x^4}{4} + \frac{x^2}{2} + 6x \Big|_{-3}^{-1} = \left(\frac{1}{4} + \frac{1}{2} - 6\right) - \left(\frac{81}{4} + \frac{9}{2} - 18\right)$

$\frac{1}{4} + \frac{1}{2} - 6 - \frac{81}{4} - \frac{9}{2} + 18$

$-\frac{80}{4} - \frac{8}{2} + 12$

$-20 - 4 + 12 = \boxed{-12}$

you try... Evaluate:

Think about $\frac{d}{dx} 2^x = 2^x \ln 2$

a. $\int_{-2}^{0} \frac{e^x}{2}\,dx$

$\frac{1}{2}\int_{-2}^{0} e^x\,dx = \frac{1}{2}e^x \Big|_{-2}^{0}$

$\frac{1}{2}e^0 - \left(\frac{1}{2}e^{-2}\right)$

$\boxed{\frac{1}{2} - \frac{1}{2e^2}}$

b. $\int_{3}^{1} \frac{1-x^2}{x}\,dx$

$-\int_1^3 \frac{1-x^2}{x}\,dx$

$-\int_1^3 \left(\frac{1}{x} - \frac{x^2}{x}\right)dx$

$-\int_1^3 \left(\frac{1}{x} - x\right)dx$

$-\left(\ln x - \frac{x^2}{2}\right)\Big|_1^3 = \left(-\ln x + \frac{x^2}{2}\right)\Big|_1^3 = \left(-\ln 3 + \frac{9}{2}\right) - \left(-\ln 1 + \frac{1}{2}\right)$

$-\ln 3 + 9/2 + 0 - \frac{1}{2}$

$-\ln x + \frac{x^2}{2}\Big|_1^3 = -\ln 3 + \frac{9}{2} - \left(-\ln 1 + \frac{1}{2}\right)$

$\boxed{= -\ln 3 + 4}$

c. $\int_0^2 2^x\,dx$

$= \frac{2^x}{\ln 2}\Big|_0^2$

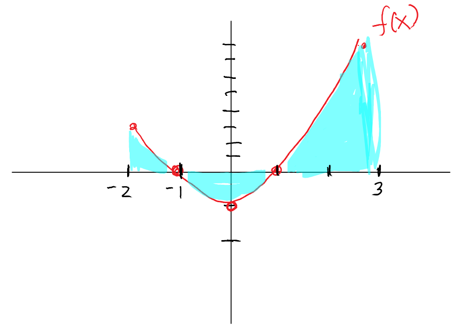
$= \frac{2^2}{\ln 2} - \frac{2^0}{\ln 2}$

$\frac{4}{\ln 2} - \frac{1}{\ln 2} = \frac{3}{\ln 2}$

Finding Area using Anti-derivatives

Finding Area using Anti-derivatives

ex: Find the area of the region between the curve $f(x) = x^2 - 1$ and the x-axis on $[-2, 3]$.



$\boxed{\text{w/o calculator}}$

$$\int_{-2}^{-1} (x^2-1)\,dx - \int_{-1}^{1} (x^2-1)\,dx + \int_{1}^{3} (x^2-1)\,dx$$

This positive

$$\left.\frac{x^3}{3} - x\right|_{-2}^{-1} - \left.\left(\frac{x^3}{3} - x\right)\right|_{-1}^{1} + \left.\left(\frac{x^3}{3} - x\right)\right|_{1}^{3} = \underline{9.\overline{3}}$$

with calculator

$$\int_{-2}^{3} |x^2-1|\,dx = \underline{9.\overline{3}}$$

graphically