AP Calculus AB
FTC Review 1

Name___________________________

RECAP:

Fundamental Theorem of Calculus Part II:

$\int_a^b f(x)dx = F(b) - F(a)$

Where F is the antiderivative of f

a and b are both constants and $a \leq b$

i.e $\int_0^{\frac{\pi}{4}} sec^2x\, dx = tan\frac{\pi}{4} - tan0 = 1 - 0 = 1$

Fundamental Theorem of Calculus Part I:

Let $y = \int_a^u f(t)dt,\ Where\ u\ is\ a\ function\ of\ x$

Then $y = F(u) - F(a)$

$\frac{dy}{dx} = y' = \frac{d}{dx}\big(F(u) - F(a)\big) = F'(u) \cdot u' = f(u) \cdot u'$

i.e. $\frac{d}{dx}\int_3^{5x^2} \sqrt{\tan t}\, dt = 10x\sqrt{tan(5x^2)}$

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1. Evaluate:

A. $\int_3^4 \frac{3}{x^2} dx =$ $\frac{1}{4}$

$\int_3^4 3x^{-2} dx = -3x^{-1}\Big|_3^4$

$= \frac{-3}{4} - \left(\frac{-3}{3}\right) = \frac{-3}{4} + 1$

B. $\int_0^{\frac{\pi}{3}} secxtanx\, dx =$ 1

$\sec x\Big|_0^{\pi/3} = \sec\frac{\pi}{3} - \sec 0$

$= 2 - 1$

2. For each problem, find y'.

A. $y(x) = \int_{-\frac{\pi}{6}}^{x} 2\sec^2 t\,dt$

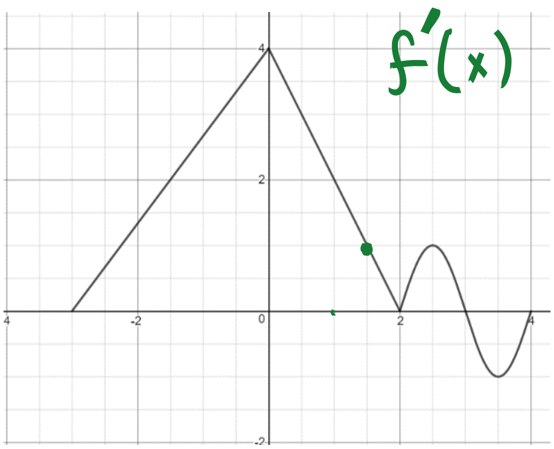
$y' = 2\sec^2 x$

B. $y(x) = \int_{-\frac{\pi}{6}}^{3x} 2\sec t \tan t\,dt$

$y' = 3 \cdot 2\sec(3x)\tan(3x)$

$= 6\sec(3x)\tan(3x)$

3. The following is a graph of $f'(x)$. Let $g(x) = \int_{17}^{3x} f'(t)\,dt$. Find $g'(0.5)$.



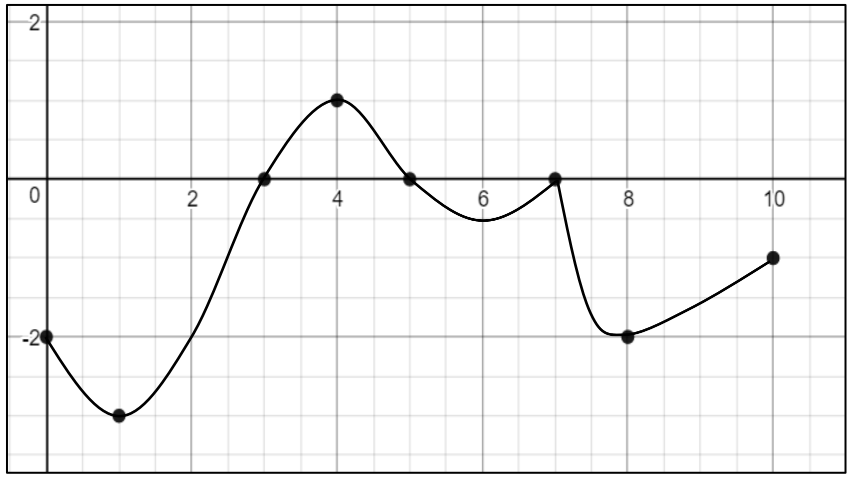
$g'(0.5)$

$g'(x) = f(3x) \cdot 3$

$g'(\frac{1}{2}) = f(\frac{3}{2}) \cdot 3$

$= 1 \cdot 3 = 3$

4. Let h be the continuous function defined by $h(x)=\int_0^x f(t)\,dt$. The graph of $f(x)$ is given below.



a) Find $f(1)$ and $f(4)$.

$f(1) = -3$

$f(4) = 1$

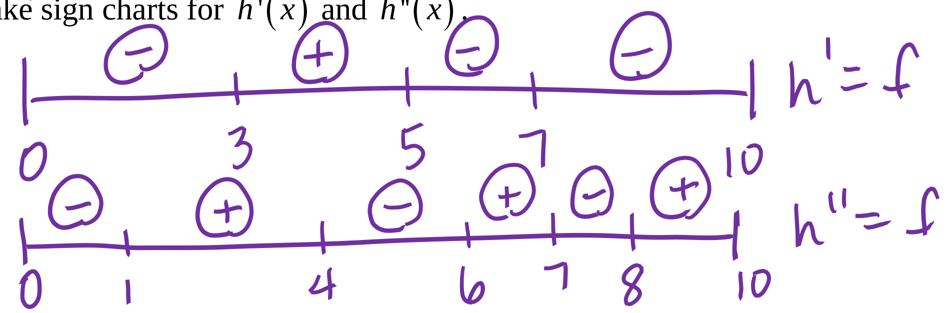
b) Is $h(2)$ positive or negative?

$h(2) < 0$

c) Is $h(5)$ positive or negative?

$h(5) < 0$

d) Make sign charts for $h'(x)$ and $h''(x)$.



e) Find any x values where $h(x)$ has a relative maximum. Justify your answer.

$x = 5$ because $h' = f$ changes from + to −.

$x = 0$ because this is a left endpt with $h' = f < 0$ on $(0,3)$.

f) Find the interval(s) where $h(x)$ is decreasing. Justify your answer.

$(0,3) \cup (5,7) \cup (7,10)$ because $h' = f < 0$.

g) Find all x values of inflection points of $h(x)$.

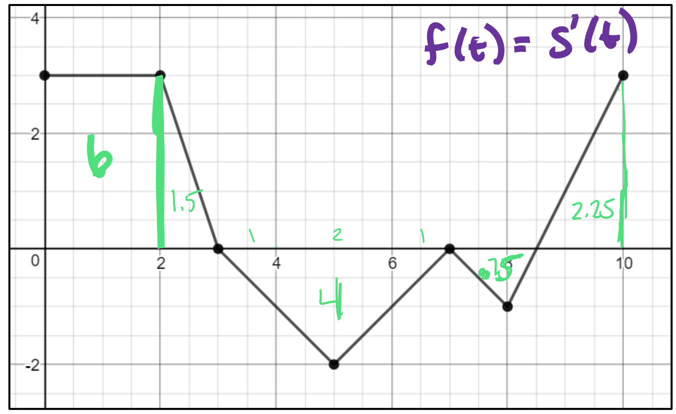
$x = 1, 4, 6, 7, 8$ because $h'' = f'$ changes sign.

h) Find the interval(s) where $h(x)$ is concave up. Justify your answer.

$(1\ 4) \cup (6\ 7) \cup (8\ 10)$ because $h'' = f' > 0$.

5. Let $s(t) = \int_0^t f(x)dx$ be the position (feet) of a particle moving along a coordinate axis.

The graph of $f(t)$ is given where t = time in seconds.



a. Find $s'(t)$ and state what it represents in context of the problem.

$f(t)$ which is the velocity (ft/sec) of the particle @ time t

b. When is the particle moving to the right, left, and stopped? Justify your answer.

Right $0 \le t < 3$ and $8.25 < t \le 10$ b/c $s'(t) = f(t) > 0$ on the intervals

Left $3 < t < 7$ and $7 < t < 8.25$ b/c $s'(t) = f(t) < 0$ on the interval

Stopped $t = 3, 7, 8.25$ b/c $s'(t) = f(t) = 0$

c. when will the graph of $s(t)$ have a relative maximum on the interval [0,10]? Justify your answer.

$s'(t) = f(t)$ [0 + 3 − 7 − 8.25 + 10]

$s(t)$ will have a rel. max @ $t = 3$ b/c $s'(t) = f(t)$ goes from + to − @ $t = 3$

$s(t)$ will have a rel max @ $t = 10$ b/c $t = 10$ is an endpt and $s'(t) = f(t) > 0$ to the left of $t = 10$

d. What is the particle's position at t = 7 seconds?

$s(7) = \int_0^7 f(x)\,dx = 7.5 - 4 = 3.5$ feet

e. When is the particle moving toward the origin?

$(3, 8.25)$

f. Is the particle speeding up, slowing down, or neither at t = 4 seconds. Justify your answer.

Speeding up b/c $s'(t) = f(t) < 0$ and $s''(t) = f'(t) < 0$