

1. $\sum_{n=0}^{\infty} nx^n = 0 + x + 2x^2 + 3x^3 + \dots \quad C=0$

4. $\sum_{n=0}^{\infty} \frac{(-1)^n (x-\pi)^{2n}}{(2n)!} = 1 - \frac{x-\pi}{2} + \frac{(x-\pi)^2}{4!} + \dots \quad C=\pi$

5. $\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n+1} = 1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \dots \quad C=0$

$$\lim_{n \rightarrow \infty} \left| \frac{x^{n+1}}{(n+1)+1} \cdot \frac{(n+1)}{x^n} \right| = \lim_{n \rightarrow \infty} \left| \frac{x \cdot x \cdot (n+1)}{(n+2) \cdot x^n} \right| = \lim_{n \rightarrow \infty} |x| \quad |x| < 1$$

$R=1$

6. $\sum_{n=0}^{\infty} (3x)^n = 1 + 3x + 9x^2 + 27x^3 + \dots \quad C=0$

$$\lim_{n \rightarrow \infty} \left| \frac{(3x)^{n+1}}{(3x)^n} \right| = \lim_{n \rightarrow \infty} |3x| \quad |3x| < 1 \quad -1 < 3x < 1$$

$$-\frac{1}{3} < x < \frac{1}{3} \quad R = \frac{1}{3}$$

7. $\sum_{n=1}^{\infty} \frac{(4x)^n}{n^2} = 4x + \frac{16x^2}{4} + \frac{64x^3}{9} + \dots \quad C=0$

$$\lim_{n \rightarrow \infty} \left| \frac{(4x)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(4x)^n} \right| = \lim_{n \rightarrow \infty} |4x| \quad |4x| < 1 \quad -\frac{1}{4} < \frac{4x}{4} < \frac{1}{4}$$

$R = \frac{1}{4}$

10. $\sum_{n=0}^{\infty} \frac{(2n)! x^{2n}}{n!} \quad \lim_{n \rightarrow \infty} \left| \frac{(2(n+1))! x^{2(n+1)}}{(n+1)!} \cdot \frac{n!}{(2n)! x^{2n}} \right| = \lim_{n \rightarrow \infty} \left| \frac{(2n+2)! \cdot n!}{(n+1)! \cdot (2n)!} \cdot \frac{x \cdot x \cdot x^2}{x^{2n}} \right|$

$$\lim_{n \rightarrow \infty} \left| \frac{(2n+2) \cdot (2n+1) \cdot (2n)!}{(2n)!} \cdot \frac{n!}{(n+1)!} \cdot x^2 \right| = \infty > 1 \quad R=0$$

41. $\sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=1}^{\infty} \frac{x^{n-1}}{(n-1)!}$ 44. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n-1}}{2n-1}$

55a $|x| < 2$ many possibilities

one is a geometric $\sum_{n=1}^{\infty} \left(\frac{x}{2}\right)^n \quad \left|\frac{x}{2}\right| < 1 \quad -1 < \frac{x}{2} < 1$

$$-2 < x < 2$$