6.5 (Tuesday 1/28)

Monday, January 27, 2020 12:33 PM

Section 6.5 Notes **Another Way of Estimating Area**

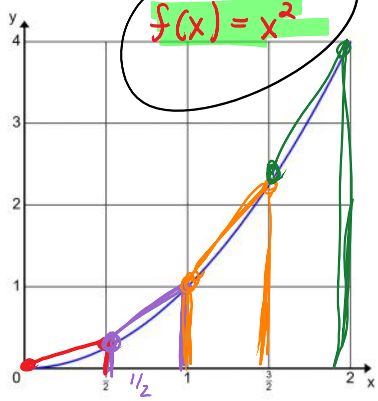
Besides doing LRAM, RRAM or MRAM, what could be another more accurate way of estimating area under a curve that doesn't used rectangles?

Let's look at an example of the graph of $y = x^2$ over the interval [0,2] with four subintervals.

$f(x) = x^2$

Area of Trap $= \frac{1}{2}h(b_1 + b_2)$

$\frac{1}{2}\Delta x(f(x_1) + f(x_2))$



$h = \Delta x$

$\frac{2-0}{4} = \frac{1}{2} = \Delta x$

$\frac{1}{2} \cdot \frac{1}{2}(f(0) + f(\tfrac{1}{2})) + \frac{1}{2} \cdot \frac{1}{2}(f(\tfrac{1}{2}) + f(1)) + \frac{1}{2} \cdot \frac{1}{2}(f(1) + f(\tfrac{3}{2})) + \frac{1}{2} \cdot \frac{1}{2}(f(\tfrac{3}{2}) + f(2))$

$\frac{1}{2} \cdot \frac{1}{2}(f(0) + f(\tfrac{1}{2}) + f(\tfrac{1}{2}) + f(1) + f(1) + f(\tfrac{3}{2}) + f(\tfrac{3}{2}) + f(2))$

$\frac{1}{2} \cdot \frac{1}{2}(f(0) + 2f(\tfrac{1}{2}) + 2f(1) + 2f(\tfrac{3}{2}) + f(2))$

$\frac{1}{4}(0^2 + 2(\tfrac{1}{2})^2 + 2(1)^2 + 2(\tfrac{3}{2})^2 + 2^2)$

$= 11/4$

overestimate b/c f(x) ccu

estimate $\int_0^2 x^2\,dx \approx 11/4$

This approximation method is called the trapezoidal Approximation Method.

Let's compare to the actual area!

Actual $= \int_0^2 x^2\,dx = \frac{x^3}{3}\Big|_0^2 = 8/3$

In this case, it was an overestimate, but will it always be an overestimate? When do you think it'll be an underestimate?

overestimate $\rightarrow$ ccu $\rightarrow$ $f'' > 0$

underestimate $\rightarrow$ ccd $\rightarrow$ $f'' < 0$

General Rule to approximate an integral using Trapezoid Rule:

$$T_n = \frac{1}{2}\Delta x\left(f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_n)\right)$$

*****Note: This only works if** equal intervals **!!!*****

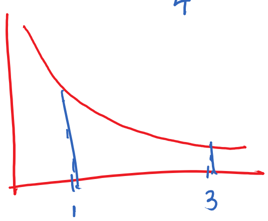
Ex: Use the trapezoid rule to approximate the value of the integral over four subintervals, decide if it's an overestimate or an underestimate, then use find the exact value of the integral to check your answer.

a) $\int_1^3 \frac{1}{x}dx$

$$\frac{3-1}{4} = \frac{2}{4} = \frac{1}{2} = \Delta x$$

$$\frac{1}{2}\cdot\overset{\Delta x}{\frac{1}{2}}\left(f(1) + 2f(3/2) + 2f(2) + 2f(5/2) + f(3)\right)$$

$$\frac{1}{4}\left(\frac{1}{1} + 2\left(\frac{2}{3}\right) + 2\left(\frac{1}{2}\right) + 2\left(\frac{2}{5}\right) + \frac{1}{3}\right)$$



$f(x) = \frac{1}{x}$

$f' = \frac{-1}{x^2}$ $\quad f'' = \frac{2}{x^3}$

overestimate b/c $f'' > 0$ $[1, 3]$

b) $\int_0^2 x^3 dx$

Ex: The temperature of room 209 changes throughout the day. In order to make a case to the Principal that her room is on average too cold, Mrs. Saller takes a reading of the temperature in the room at different times on one day. Let t = 0 be 7:00AM and the table of her readings is given below:

Time	7:00AM	9:00	10:00	1:00PM	2:00PM	4:00PM
Temp (°F)	53	84	73	68	59	53

What was the average temperature over the 9 hours? Use trapezoid rule to get an estimate.

Ex: The table below shows the time-to-velocity data for a train. How far had the train travelled in the 12 minutes? Use trapezoid rule to estimate how far it traveled. (What do we need to be careful of in this problem?)

Δt 2   3   3   4

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	120	150

$$\int_0^{12} v(t)\,dt \approx \frac{1}{2}(2)(0+100) + \frac{1}{2}(3)(100+40) + \frac{1}{2}(3)(40+120) + \frac{1}{2}(4)(120+150)$$