Recall:

Let’s revisit the last problem from the homework:

The wing lengths of houseflies are normally distributed with a mean of 4.6 millimeters and a standard deviation of 0.4 millimeter.

a. What percent of flies have wing lengths between 4.6 and 5.4?

b. What percent of flies have wing lengths less than 4.1? Now what?

How to find a z score:

\[ z = \frac{x - \mu}{\sigma} \]

we know that 5.4 is 2 standard deviations away from the mean.

\[ z = \frac{5.4 - 4.6}{0.4} = 2 \]

z-score for 4.1

\[ z = \frac{4.1 - 4.6}{0.4} = -1.25 \]

or 1.25 standard deviations to the left of \( \mu \).

How to use a z table:

Under 2 we find -1.2
look at .05 second decimal place 0.05 and find .1056
so, 10.56% of flies have a wingspan less than 4.1
Making sense of the Z score:

A z-score allows us to find the area under a normal curve to the left of that z-score.

Since the total area under a normal curve is one, area under a normal curve to the right of the z score can easily be found.

If we want to find an area under a normal curve BETWEEN two z scores:

Let's practice using the z-table. Find the following probabilities (areas under the normal curve).

a. \(P(z < 0)\)  
\[0.50\]

b. \(P(z > 0)\)  
\[0.50\]

c. \(P(z < 1.42)\)  
\[0.9222\]

d. \(P(z < -2.34)\)  
\[0.0096\]

e. \(P(z > 2.60)\)  
\[0.0053\]

d. \(P(-1.56 < z < 1.3)\)  
\[0.9687\]

e. \(P(|z| < 2.2)\)  
\[0.9713\]

f. \(P(|z| > 2.2)\)  
\[0.0287\]
**Ex. 1.** A study finds that the weights of infants at birth are normally distributed with a mean of 3270 grams and a standard deviation of 600 grams. An infant is randomly chosen.

a. What is the probability that the infant weighs less than 4170 grams or less?

\[
Z = \frac{4170 - 3270}{600} = 1.5
\]

\[P(Z \leq 1.5) = 0.9332\]

b. What is the probability that the infant weighs 3990 grams or more?

\[
Z = \frac{3990 - 3270}{600} = 1.2
\]

\[P(Z > 1.2) = 1 - P(Z \leq 1.2) = 1 - 0.8849 = 0.1151\]

c. What is the probability that the infant weighs between 3000 and 3576 grams?

\[
\begin{align*}
Z_{3000} &= \frac{3000 - 3270}{600} = -0.45 \\
Z_{3576} &= \frac{3576 - 3270}{600} = 0.51 \\
P(-0.45 \leq Z \leq 0.51) &= P(Z \leq 0.51) - P(Z \leq -0.45) \\
&= 0.6950 - 0.3044 \\
&= 0.3906
\end{align*}
\]

**Ex. 2** Scientists conducted aerial surveys of a seal sanctuary and recorded the number of seals they observed during each survey. The number of seals observed was normally distributed with a mean of 73 and a standard deviation of 14.1.

a. Find the probability that at most 50 seals were observed during a randomly chosen survey.

\[
Z = \frac{50 - 73}{14.1} \approx -1.63
\]

\[P(Z \leq -1.63) = 0.0516\]

b. Approximately how many seals were observed if the observed number of seals is 1.5 standard deviations below the mean.

\[
Z = -1.5
\]

\[-1.5 = \frac{x - 73}{14.1}
\]

\[x \approx 51.83\] About 52 seals

(c) Find the number of seals observed if approximately 86% of all the observations resulted in fewer seals.

\[
Z = 1.08
\]

\[1.08 = \frac{x - 73}{14.1}\]

\[x \approx 87.89\] Approx. 88 seals

(d) Find the number of seals observed that fall within 6% of the mean.

\[
Z = \pm 1.5
\]

\[\frac{x - 73}{14.1} = \pm 1.5\]

For 56% or 44% of values it will have symmetric \(Z\)-scores.

\[56\% \text{ or } 44\% \text{ of } \frac{x - 73}{14.1}\]

\[Z = \pm 1.5\]

Between 71 and 75 seals

\[X \mp 15\text{ seals}\]
**Day 3 Homework**

1. In the figure, the shaded region represents 47.5% of the area under a normal curve. What are the mean and standard deviation of the normal distribution?

2. Use the z-table to help you answer the questions below. Find the following probabilities:
   a. \( P(z < 2.4) \)
   b. \( P(z < -1.51) \)
   c. \( (-1.23 < z < 1.5) \)
   d. \( P(|z| < 1.7) \)
   e. \( P(|z| > 1.7) \)

3. Find the value of \( z \) from the standard normal distribution that satisfies each of the following conditions. (Use the value that comes the closest!)
   a. 
   b. 
   c. 

4. A busy time to visit a bank is during its Friday evening rush hours. For these hours, the waiting times at the drive through window are normally distributed with a mean of 8 minutes and a standard deviation of 2 minutes. You have no more than 11 minutes to do your banking and still make it to your meeting on time. What is the probability that you will be late for the meeting?
5. The guayule plant, which grows in southwestern United States and in Mexico, is one of several plants that can be used as a source of rubber. In a large group of guayule plants, the heights of the plants are normally distributed with a mean of 12 inches and a standard deviation of 2 inches.

a. What percent of the plants are at most 13 inches?

\[ z = \frac{13 - 12}{2} = \frac{1}{2} \quad z = 0.5 \quad 0.6915 \quad 69.15\% \]

b. What percent of the plants are taller than 16 inches?

\[ 1 - P(z < 2) = 1 - \Phi(2) = 1 - 0.9772 = 0.0228 \quad 2.28\% \]

c. What percent of the plants are between 13 inches and 16 inches?

\[ z_1 = \frac{13 - 12}{2} = \frac{1}{2} \quad z_2 = \frac{16 - 12}{2} = 2 \quad 28.57\% \]

6. Elephants have the longest pregnancy of all mammals. One species of elephant has a mean gestation period of 525 days and a standard deviation of 32 days. Their pregnancy length follows an approximately normal distribution.

a. What percent of elephant pregnancies last less than 461 days?

\[ P(z < -2) = \Phi(-2) = 0.0228 \]

b. The longest 20.9% of all elephant pregnancies last at least how many days?

Find the z for 79.1

\[ z = 0.81 \]

\[ x \approx \frac{525}{32} \approx 551 \text{ days} \]

c. The middle 68% of elephant pregnancies last between how many days?

\[ 525 + 32 = 557 \]
\[ 525 - 32 = 493 \]

Between 493 and 557 days.