# 9.3 Day 2 Intervals of Convergence

Monday, March 2, 2020 8:58 AM

When finding intervals of converge, use **ratio test** to find the basic interval of abs. convergence, but then you need to **test endpoints** to see if the converge abs, converge conditionally, or diverge.

Ex: $\sum_{n=1}^{\infty} \frac{x^n}{n}$ • Find the radius & interval of convergence.

$$\lim_{n\to\infty}\left|\frac{x^{n+1}}{n+1}\cdot\frac{n}{x^n}\right| \Rightarrow \lim_{n\to\infty}\frac{x\cdot n}{n+1} = |x|$$

$$|x|<1$$

$$-1<x<1 \Rightarrow \text{abs. converge}$$

• $x=1$ $\sum_{n=1}^{\infty}\frac{1}{n} \Rightarrow$ Diverges by the p-series test

• $x=-1$ $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$

1. $\frac{1}{n}\geq 0$ ✓
2. $\frac{1}{n}\geq\frac{1}{n+1}$ ✓
3. $\lim_{n\to\infty}\frac{1}{n}=0$ ✓

converges cond by alt series test

$$\boxed{\begin{array}{l}\text{Converges } [-1, 1) \\ \text{conv. abs } (-1, 1) \\ \text{conv. cond } x = -1 \\ R = 1\end{array}}$$

Ex: $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x+3)^n}{3^n}$

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1} (x+3)^{n+1}}{3^{n+1}} \cdot \frac{3^n}{(-1)^n (x+3)^n} \right|$$

$$\lim_{n\to\infty} \left| \frac{(-1)^1 (x+3)}{3} \right|$$

$$\left| \frac{x+3}{3} \right| < 1$$

$$-1 < \frac{x+3}{3} < 1$$

$$-3 < x+3 < 3$$

$$-6 < x < 0 \Rightarrow \text{abs. converge}$$

- $x = 0$ $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (3)^n}{3^n} = \sum_{n=0}^{\infty} (-1)^n \Rightarrow$ Diverge since $|r| \geq 1$

- $x = -6$ $\displaystyle\sum_{n=0}^{\infty} \frac{\cancel{(-1)^n} (-3)^n}{3^n}$ $|r| \geq 1$

Converge $(-6, 0)$

conv. Abs $(-6, 0)$

conv. cond. Never

$R = 3$

you try.... $\sum_{n=1}^{\infty} \frac{(x-3)^{n-1}}{4^{n-1}}$

$$\lim_{n\to\infty} \left| \frac{(x-3)^n}{4^n} \cdot \frac{4^{n-1}}{(x-3)^{n-1}} \right| = \lim_{n\to\infty} \left| \frac{(4)^{-1}}{(x-3)^{-1}} \right| = \lim_{n\to\infty} \left| \frac{x-3}{4} \right| < 1$$

$$-1 < \frac{x-3}{4} < 1$$

$$-4 < x-3 < 4$$

$-1 < x < 7$ conv. abs

$x = 7$ $\sum \frac{4^{n-1}}{4^{n-1}} = \sum 1^{n-1}$ Diverges since $|r| \geq 1$

$x = -1$ $\sum \frac{(-4)^{n-1}}{4^{n-1}} = \sum (-1)^{n-1}$

- converges $(-1, 7)$
- conv. abs $(-1, 7)$
- conv cond. never

$R = 4$