9.7 Day 2

Saturday, January 11, 2020 1:43 PM

Pg 644 #17, 26, 45, 46, 55, 56, 59, 67

17. $f(x) = \sin x \quad n=5$ Maclaurin $c=0$

$f(0) = 0$

$f' = \cos x \quad f'(0) = 1$

$f'' = -\sin x \quad f''(0) = 0$

$f''' = -\cos x \quad f'''(0) = -1$

$f^{(4)} = \sin x \quad f^{(4)}(0) = 0$

$f^{(5)} = \cos x \quad f^{(5)}(0) = 1$

$$P_5(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

26. $f(x) = \frac{1}{x^2} \quad n=4 \quad c=2$

$= x^{-2}$

$f(2) = 1/4$

$f' = -2x^{-3} \quad f'(2) = \frac{-2}{8} = \frac{-1}{4}$

$f'' = 6x^{-4} \quad f''(2) = \frac{6}{16} = \frac{3}{8}$

$f''' = -24x^{-5} \quad f'''(2) = \frac{-24}{32} = -\frac{3}{4}$

$f^{(4)} = 120x^{-6} \quad f^{(4)}(2) = \frac{120}{64} = 15/8$

$$P_4(x) = \frac{1}{4} - \frac{1}{4}(x-2) + \frac{3}{8}\frac{(x-2)^2}{2!} - \frac{3}{4}\frac{(x-2)^3}{3!} + \frac{15}{8}\frac{(x-2)^4}{4!}$$

45. $\cos(0.3) \approx 1 - \frac{(0.3)^2}{2!} + \frac{(0.3)^4}{4!}$

sin x & cos x can't get bigger than 1

$0 \le z \le 0.3$

$|R_n(0.3)| \le \frac{(0.3)^5}{5!} \approx 2.025 \times 10^{-5}$ Error Bound

Exact 1.011×10^{-6}

46. $e \approx 1 + 1 + \frac{1^2}{2!} + \frac{1^3}{3!} + \frac{1^4}{4!} + \frac{1^5}{5!}$ error $\approx e \cdot \frac{1^6}{6!}$

$0 \le z \le 1$

$|R_n(e)| \le \frac{e(1^6)}{6!} \approx 0.003775$ error Bound

Exact 0.00162

55. $f(x) = e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$

$x < 0$

$f^{(4)}(x) = e^x$

$\left| f^{(4)}(z)\, x^4 \right| , \; 0.001$

$f^{(4)}(x) = e^x$

$$|\text{error}| \le \left| \frac{f^{(4)}(z)\, x^4}{4!} \right| \le 0.001$$

$$\left| \frac{e^{x} x^4}{24} \right| \le 0.001 \qquad \boxed{-0.439 \le x \le 0}$$

56. $f(x) = \sin x \approx x - \frac{x^3}{3!}$

$$\left| \frac{1\, x^4}{4!} \right| \le 0.001$$

$f' = \cos x$ → Don't need because

$f'' = -\sin x$

$f''' = -\cos x$ max of $f^{(n)}(x) = 1$

$f^{(4)} = \sin x$

$f^{(5)} = \cos x$

20 60 120

$$-.394 \le x \le .394$$

59. P_n estimates estimate $f(x)$ better as $x \to c$ an $P(c) = f(c)$

67. $f(x) = \sin\left(\frac{\pi x}{4}\right)$ $\qquad P_2(x) = 1 - \left(\frac{\pi^2}{32}\right)(x-2)^2$

a. $Q_2(x) = -1 + \left(\frac{\pi^2}{32}\right)(x+2)^2$

b. $R_2(x) = -1 + \frac{\pi^2}{32}(x-6)^2$

c. No, the slopes $\neq 0$ at that point so the polys are different @ $x = 2$ & 4.

$\therefore$