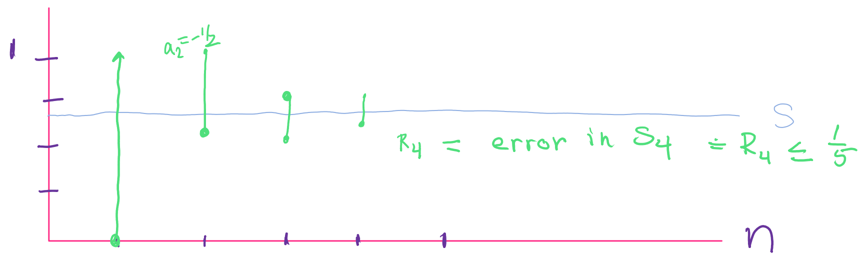
9.5 Day 2

Thursday, February 13, 2020 8:11 AM

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \underbrace{1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}}_{S_4 = \text{ estimate of } S} + \ldots = S = ???$$



error in $S_n \le a_{n+1}$

Alt. Series Remainder / Error

If an alternating series converges by alternating series test, then the abs. value of the remainder, R_n, involved in approximating the sum, S, by S_n is less than or equal to the 1^{st} neglected term.

$$|\text{error}| = \underbrace{|S - S_n|}_{\text{error}} = R_n \le \underbrace{|a_{n+1}|}_{1^{st}\text{ neglected term after the partial sum}}$$

Ex: Determine the error bound when S_4 is used to approx. the sum of the series: $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n n}{n^2-1}$

1. $\frac{n}{n^2-1} \ge 0$

$$|\text{Error}| = R_4 \le \left| \frac{(-1)^6 \cdot 6}{6^2 - 1} \right|$$

1. $\frac{n}{n^2-1} \geq 0$ ✓

2. $\frac{n}{n^2-1} \geq \frac{n+1}{(n+1)^2-1}$ ✓

3. $\lim_{n\to\infty} \frac{n}{n^2-1} = 0$ ✓

$|\text{Error}| = R_4 \leq \left|\frac{6}{6^2-1}\right|$

$\text{Error} \leq \frac{6}{35}$

Ex: How many terms of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ are needed to approx. the sum of the series w/ error ≤ 0.001

$|\text{error}| = R_n \leq a_{n+1}$

$\frac{1}{(n+1)^2} \leq .001$

$1000 \leq (n+1)^2$

$31.623 \text{ terms} \leq n+1$

$n \geq 30.623 \text{ terms}$

use $n \geq 31$ terms

Defn: for Absolute convergence

The alt. series is absolutely convergent when $\sum |a_n|$ converges

Defn of conditional convergence

The alt. series $\sum a_n$ is conditionally convergent when $\sum a_n$ converges but $\sum |a_n|$ diverges.

☆ It converges under the conditions of an alt. series

Ex: Determine if each series is absolutely or conditionally convergent.

A) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$

1. $\frac{1}{\ln(n)} \geq 0$ ✓

2. $\frac{1}{\ln(n)} \geq \frac{1}{\ln(n+1)}$ ✓

3. $\lim_{n \to \infty} \frac{1}{\ln n} = 0$ ✓

$\sum \left|\frac{(-1)^n}{\ln n}\right| = \sum \frac{1}{\ln n}$

$\frac{1}{\ln n} \geq \frac{1}{n}$ diverges by p-series test

Diverges by Direct comp. Test

$\therefore \displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ converges conditionally

you try...

$\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{5n^4}$

$\displaystyle\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{5n^4}\right| = \sum_{n=1}^{\infty} \frac{1}{5n^4}$ converges by p-series test

1. $\frac{1}{5n^4} \geq 0$ ✓

$\therefore \displaystyle\sum^{\infty} \frac{(-1)^n}{5n^4}$ Absolute convergence

1. $\frac{1}{5n^4} \geq 0$ ✓

2. $\frac{1}{5n^4} \geq \frac{1}{5(n+1)^4}$ ✓

3. $\lim_{n \to \infty} \frac{1}{5n^4} = 0$ ✓

$\therefore \sum_{n=1}^{\infty} \frac{(-1)^n}{5n^4}$ Absolute convergence

Absolute Convergence Theorem:

If $\sum |a_n|$ converges the $\sum a_n$ converges also