Chapter P
Prerequisites

■ Section P.1 Real Numbers

Quick Review P.1

1. $\{1, 2, 3, 4, 5, 6\}$

2. $\{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$

3. $\{-3, -2, -1\}$

4. $\{1, 2, 3, 4\}$

5. **(a)** 1187.75 **(b)** −4.72

6. **(a)** 20.65 **(b)** 0.10

7. $(-2)^3 - 2(-2) + 1 = -3$; $(1.5)^3 - 2(1.5) + 1 = 1.375$

8. $(-3)^2 + (-3)(2) + 2^2 = 7$

9. 0, 1, 2, 3, 4, 5, 6

10. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

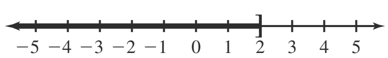
Section P.1 Exercises

1. −4.625 (terminates)

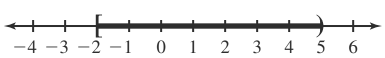
2. $0.\overline{15}$ (repeats)

3. $-2.1\overline{6}$ (repeats)

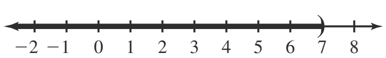
4. $0.\overline{135}$ (repeats)

5.


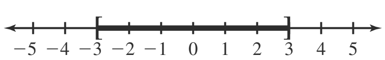
all real numbers less than or equal to 2 (to the left of and including 2)

6.


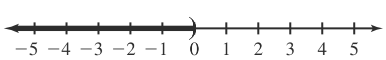
all real numbers between −2 and 5, including −2 and excluding 5

7.


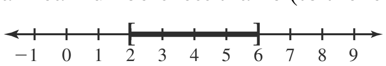
all real numbers less than 7 (to the left of 7)

8.


all real numbers between −3 and 3, including both −3 and 3

9.


all real numbers less than 0 (to the left of 0)

10.


all real numbers between 2 and 6, including both 2 and 6

11. $-1 \le x < 1$; all numbers between −1 and 1 including −1 and excluding 1

12. $-\infty < x \le 4$, or $x \le 4$; all numbers less than or equal to 4

13. $-\infty < x < 5$, or $x < 5$; all numbers less then 5

14. $-2 \le x < 2$; all numbers between −2 and 2, including −2 and excluding 2

15. $-1 < x < 2$; all numbers between −1 and 2, excluding both −1 and 2

16. $5 \le x < \infty$, or $x \ge 5$; all numbers greater than or equal to 5

17. $(-3, \infty)$; all numbers greater than −3

18. $(-7, -2)$; all numbers between −7 and −2, excluding both −7 and −2

19. $(-2, 1)$; all numbers between −2 and 1, excluding both −2 and 1

20. $[-1, \infty)$; all numbers greater than or equal to −1

21. $(-3, 4]$; all numbers between −3 and 4, excluding −3 and including 4

22. $(0, \infty)$; all numbers greater than 0

23. The real numbers greater than 4 and less than or equal to 9.

24. The real numbers greater than or equal to −1, or the real numbers which are at least −1.

25. The real numbers greater than or equal to −3, or the real numbers which are at least −3.

26. The real numbers between −5 and 7, or the real numbers greater than −5 and less than 7.

27. The real numbers greater than −1.

28. The real numbers between −3 and 0 (inclusive), or greater than or equal to −3 and less than or equal to 0.

29. $-3 < x \le 4$; endpoints −3 and 4; bounded; half-open

30. $-3 < x < -1$; endpoints −3 and −1; bounded; open

31. $x < 5$; endpoint 5; unbounded; open

32. $x \ge -6$; endpoint −6; unbounded; closed

33. His age must be greater than or equal to 29: $x \ge 29$ or $[29, \infty)$; $x =$ Bill's age

34. The costs are between 0 and 2 (inclusive): $0 \le x \le 2$ or $[0, 2]$; $x =$ cost of an item

35. The prices are between \$3.099 and \$4.399 (inclusive): $3.099 \le x \le 4.399$ or $[3.099, 4.399]$; $x =$ \$ per gallon of gasoline

36. The raises are between 0.02 and 0.065: $0.02 < x < 0.065$ or $(0.02, 0.065)$; $x =$ average percent of all salary raises

37. $a(x^2 + b) = a \cdot x^2 + a \cdot b = ax^2 + ab$

38. $(y - z^3)c = y \cdot c - z^3 \cdot c = yc - z^3c$

39. $ax^2 + dx^2 = a \cdot x^2 + d \cdot x^2 = (a + d)x^2$

40. $a^3z + a^3w = a^3 \cdot z + a^3 \cdot w = a^3(z + w)$

41. The opposite of $6 - \pi$, or $-(6 - \pi) = -6 + \pi = \pi - 6$

42. The opposite of −7, or $-(-7) = 7$

43. In -5^2, the base is 5.

44. In $(-2)^7$, the base is −2.

45. **(a)** Associative property of multiplication
(b) Commutative property of multiplication
(c) Addition inverse property
(d) Addition identity property
(e) Distributive property of multiplication over addition

46. **(a)** Multiplication inverse property
(b) Multiplication identity property, or distributive property of multiplication over addition, followed by the multiplication identity property. Note that we also use the multiplicative commutative property to say that $1 \cdot u = u \cdot 1 = u$.
(c) Distributive property of multiplication over subtraction
(d) Definition of subtraction; associative property of addition; definition of subtraction
(e) Associative property of multiplication; multiplicative inverse; multiplicative identity

47. $\frac{x^2}{y^2}$

48. $\frac{(3x^2)^2y^4}{3y^2} = \frac{3^2(x^2)^2y^4}{3y^2} = \frac{9x^4y^4}{3y^2} = 3x^4y^2$

49. $\left(\frac{4}{x^2}\right)^2 = \frac{4^2}{(x^2)^2} = \frac{16}{x^4}$

50. $\left(\frac{2}{xy}\right)^{-3} = \left(\frac{xy}{2}\right)^3 = \frac{x^3y^3}{2^3} = \frac{x^3y^3}{8}$

51. $\frac{(x^{-3}y^2)^{-4}}{(y^6x^{-4})^{-2}} = \frac{x^{12}y^{-8}}{y^{-12}x^8} = \frac{x^4}{y^{-4}} = x^4y^4$

52. $\left(\frac{4a^3b}{a^2b^3}\right)\left(\frac{3b^2}{2a^2b^4}\right) = \left(\frac{4a}{b^2}\right)\left(\frac{3}{2a^2b^2}\right) = \frac{12a}{2a^2b^4} = \frac{6}{ab^4}$

53. 5.18997×10^{11}

54. 6.5882×10^{10}

55. 1.6691×10^{10}

56. 6.1011×10^{11}

57. 4.839×10^8 mi

58. -1.6×10^{-19} C

59. 0.000 000 033 3

60. 673,000,000,000

61. 5,870,000,000,000 mi

62. 0.000 000 000 000 000 000 000 001 674 7 (23 zeros between the decimal point and the 1)

63. $\frac{(1.3)(2.4) \times 10^{-7+8}}{1.3 \times 10^9} = \frac{2.4 \times 10^1}{1 \times 10^9}$
$= 2.4 \times 10^{1-9} = 2.4 \times 10^{-8}$

64. $\frac{(3.7)(4.3) \times 10^{-7+6}}{2.5 \times 10^7} = \frac{15.91 \times 10^{-1}}{2.5 \times 10^7}$
$= \frac{15.91}{2.5} \times 10^{-1-7} = 6.364 \times 10^{-8}$

65. **(a)** When $n = 0$, the equation $a^m a^n = a^{m+n}$ becomes $a^m a^0 = a^{m+0}$. That is, $a^m a^0 = a^m$. Since $a \neq 0$, we can divide both sides of the equation by a^m. Hence $a^0 = 1$.
(b) When $n = -m$, the equation $a^m a^n = a^{m+n}$ becomes $a^m a^{-m} = a^{m+(-m)}$. That is $a^{m-m} = a^0$. We know from **(a)** that $a^0 = 1$. Since $a \neq 0$, we can divide both sides of the equation $a^m a^{-m} = 1$ by a^m. Hence

$$a^{-m} = \frac{1}{a^m}.$$

66. **(a)**

Step	Quotient	Remainder
1	0	1
2	0	10
3	5	15
4	8	14
5	8	4
6	2	6
7	3	9
8	5	5
9	2	16
10	9	7
11	4	2
12	1	3
13	1	13
14	7	11
15	6	8
16	4	12
17	7	1

(b) When the remainder is repeated, the quotients generated in the long division process will also repeat.
(c) When any remainder is first repeated, the next quotient will be the same number as the quotient resulting after the first occurrence of the remainder, since the decimal representation does not terminate.

67. False. If the real number is negative, the additive inverse is positive. For example, the additive inverse of -5 is 5.

68. False. If the positive real number is less than 1, the reciprocal is greater than 1. For example, the reciprocal of $\frac{1}{2}$ is 2.

69. $[-2, 1)$ corresponds to $-2 \leq x < 1$. The answer is E.

70. $(-2)^4 = (-2)(-2)(-2)(-2) = 16$. The answer is A.

71. In $-7^2 = -(7^2)$, the base is 7. The answer is B.

72. $\frac{x^6}{x^2} = \frac{\cancel{x^2} \cdot x^4}{\cancel{x^2}} = x^4$. The answer is D.

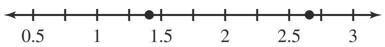
73. The whole numbers are 0, 1, 2, 3, …, so the whole numbers with magnitude less than 7 are 0, 1, 2, 3, 4, 5, 6.

74. The natural numbers are 1, 2, 3, 4, …, so the natural numbers with magnitude less than 7 are 1, 2, 3, 4, 5, 6.

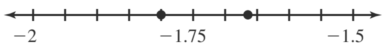
75. The integers are …, −2, −1, 0, 1, 2, …, so the integers with magnitude less than 7 are −6, −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, 6.

■ Section P.2 Cartesian Coordinate System

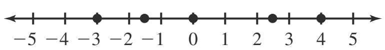
Quick Review P.2

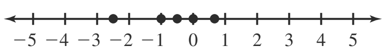
1. 

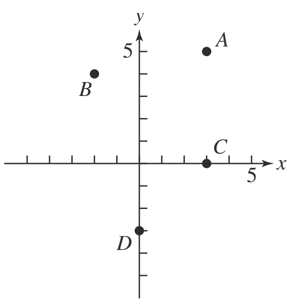
Distance: $|\sqrt{7} - \sqrt{2}| = \sqrt{7} - \sqrt{2} \approx 1.232$

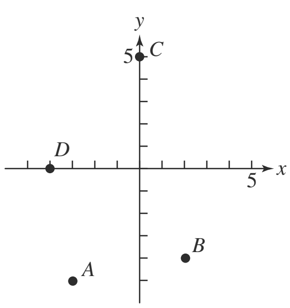
2. 

Distance: $\left|-\frac{5}{3} - \left(-\frac{9}{5}\right)\right| = \left|-\frac{25}{15} + \frac{27}{15}\right| = \left|\frac{2}{15}\right| = \frac{2}{15}$ or $0.1\overline{3}$

3. 

4. 

5. 

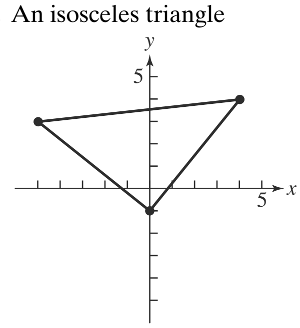
6. 

7. 5.5 **8.** 21.40

9. 10 **10.** 18.44

Section P.2 Exercises

1. $A(1, 0); B(2, 4); C(-3, -2); D(0, -2)$
2. $A(0, 3); B(-3, 1); C(-2, 0); D(4, -1)$
3. **(a)** First quadrant
   (b) On the y-axis, between quadrants I and II
   (c) Second quadrant
   (d) Third quadrant
4. **(a)** First quadrant
   (b) On the x-axis, between quadrants II and III
   (c) Third quadrant
   (d) Third quadrant
5. $3 + |-3| = 3 + 3 = 6$
6. $2 - |-2| = 2 - 2 = 0$
7. $|(-2)3| = |-6| = 6$
8. $\frac{-2}{|-2|} = \frac{-2}{2} = -1$
9. Since $\pi \approx 3.14 < 4$, $|\pi - 4| = 4 - \pi$.
10. Since $\sqrt{5} \approx 2.236$ and $\frac{5}{2} = 2.5$, $\left|\sqrt{5} - \frac{5}{2}\right| = \frac{5}{2} - \sqrt{5}$ or $2.5 - \sqrt{5}$.
11. $|10.6 - (-9.3)| = |10.6 + 9.3| = 19.9$
12. $|-17 - (-5)| = |-17 + 5| = |-12| = 12$
13. $\sqrt{(-3 - 5)^2 + [-1 - (-1)]^2} = \sqrt{(-8)^2 + 0^2} = \sqrt{64} = 8$
14. $\sqrt{(-4 - 1)^2 + (-3 - 1)^2} = \sqrt{(-5)^2 + (-4)^2} = \sqrt{25 + 16} = \sqrt{41} \approx 6.403$
15. $\sqrt{(0 - 3)^2 + (0 - 4)^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$
16. $\sqrt{(-1 - 2)^2 + [2 - (-3)]^2} = \sqrt{(-3)^2 + 5^2} = \sqrt{9 + 25} = \sqrt{34} \approx 5.830$
17. $\sqrt{(-2 - 5)^2 + (0 - 0)^2} = \sqrt{(-7)^2 + 0^2} = \sqrt{49} = 7$
18. $\sqrt{(0 - 0)^2 + [-8 - (-1)]^2} = \sqrt{0^2 + (-7)^2} = \sqrt{49} = 7$
19. An isosceles triangle



$\sqrt{(0 - (-5))^2 + (-1 - 3)^2} = \sqrt{5^2 + (-4)^2} = \sqrt{41}$

$\sqrt{(0 - (-4))^2 + (-1 - 4)^2} = \sqrt{(-4)^2 + (-5)^2} = \sqrt{41}$

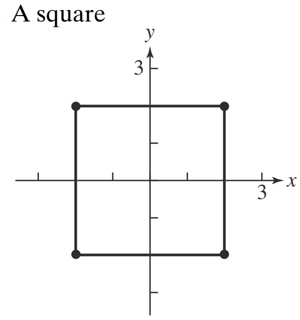
$\sqrt{(-5 - 4)^2 + (3 - 4)^2} = \sqrt{(-9)^2 + (-1)^2} = \sqrt{82}$

Perimeter $= 2\sqrt{41} + \sqrt{82} \approx 21.86$

Since $(\sqrt{41})^2 + (\sqrt{41})^2 = (\sqrt{82})^2$, this is a right triangle.

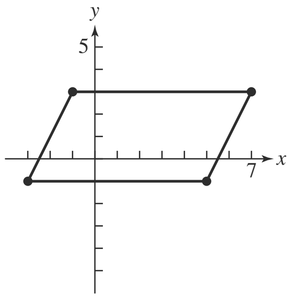
Area $= \frac{1}{2}(\sqrt{41})(\sqrt{41}) = 20.5$

20. A square



This is a square with sides of length 4. Perimeter $= 16$; Area $= 16$.

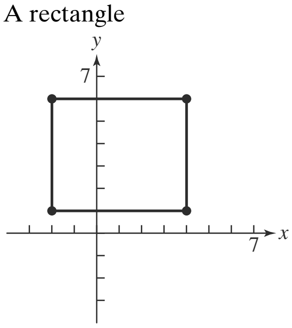
21. A parallelogram



$\sqrt{[3 - (-1)]^2 + [-1 - (-3)]^2} = \sqrt{4^2 + 2^2} = \sqrt{20}$

This is a parallelogram with base 8 units and height 4 units. Perimeter $= 2\sqrt{20} + 16 \approx 24.94$; Area $= 8 \cdot 4 = 32$

22. A rectangle



This is a rectangle with length 6 units and height 5 units. Perimeter $= 22$; Area $= 30$

23. $\dfrac{10.6 + (-9.3)}{2} = \dfrac{1.3}{2} = 0.65$

24. $\dfrac{-17 + (-5)}{2} = \dfrac{-22}{2} = -11$

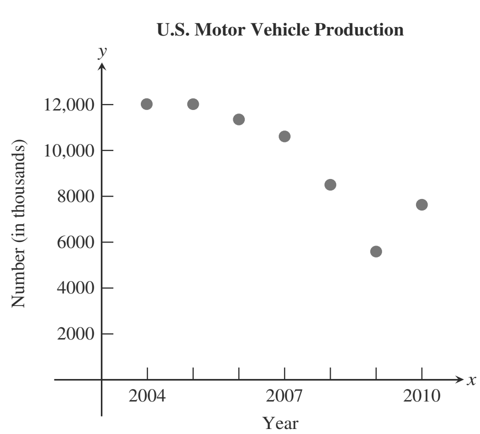
25. $\left(\dfrac{-1 + 5}{2}, \dfrac{3 + 9}{2}\right) = \left(\dfrac{4}{2}, \dfrac{12}{2}\right) = (2, 6)$

26. $\left(\dfrac{3 + 6}{2}, \dfrac{\sqrt{2} + 2}{2}\right) = \left(\dfrac{9}{2}, \dfrac{2 + \sqrt{2}}{2}\right)$

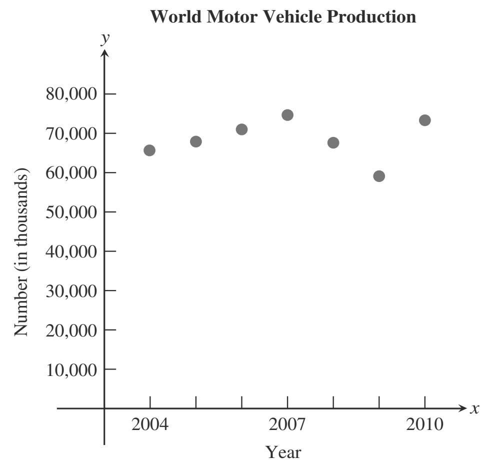
27. $\left(\dfrac{-\frac{7}{3} + \frac{5}{3}}{2}, \dfrac{\frac{3}{4} + \left(-\frac{9}{4}\right)}{2}\right) = \left(\dfrac{-\frac{2}{3}}{2}, \dfrac{-\frac{6}{4}}{2}\right) = \left(-\dfrac{1}{3}, -\dfrac{3}{4}\right)$

28. $\left(\dfrac{5 + (-1)}{2}, \dfrac{-2 + (-4)}{2}\right) = \left(\dfrac{4}{2}, \dfrac{-6}{2}\right) = (2, -3)$

29.



30.



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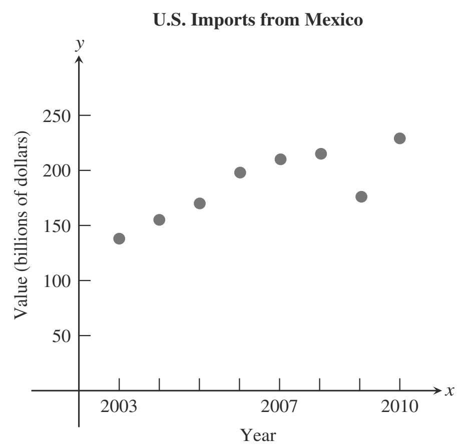
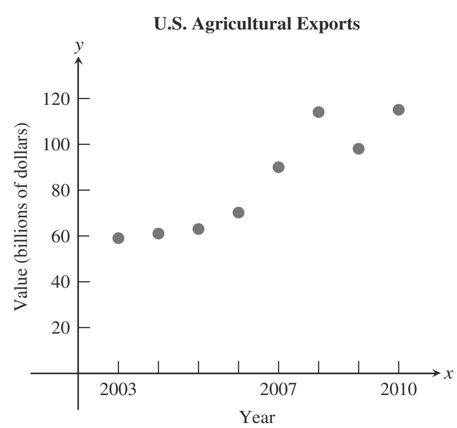
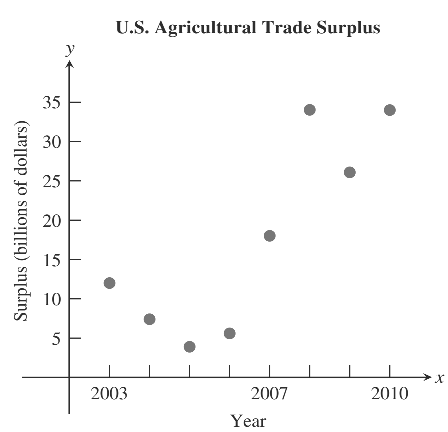


32.



33.



34.

35. (a) about \$1.40 **(b)** about \$2.80 **(c)** about \$2.80

36. (a) 2001: about \$146; 2006: about \$259

$$\frac{259 - 146}{146} \approx 0.77$$

An increase of about 77%

(b) 2009: about \$235; 2011: about \$365

$$\frac{365 - 235}{235} \approx 0.55$$

An increase of about 55%

37. The three side lengths (distances between pairs of points) are

$\sqrt{(4-1)^2 + (7-3)^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16}$
$= \sqrt{25} = 5$

$\sqrt{(8-4)^2 + (4-7)^2} = \sqrt{4^2 + (-3)^2} = \sqrt{16 + 9}$
$= \sqrt{25} = 5$

$\sqrt{(8-1)^2 + (4-3)^2} = \sqrt{7^2 + 1^2} = \sqrt{49 + 1}$
$= \sqrt{50} = 5\sqrt{2}.$

Since two sides of the triangle formed have the same length, the triangle is isosceles.

38. (a) Midpoint of diagonal from $(-7, -1)$ to $(3, -1)$ is

$$\left(\frac{-7+3}{2}, \frac{-1+(-1)}{2}\right) = (-2, -1)$$

Midpoint of diagonal from $(-2, 4)$ to $(-2, -6)$ is

$$\left(\frac{-2+(-2)}{2}, \frac{4+(-6)}{2}\right) = (-2, -1)$$

Both diagonals have midpoint $(-2, -1)$, so they bisect each other.

(b) Midpoint of diagonal from $(-2, -3)$ to $(6, 7)$ is

$$\left(\frac{-2+6}{2}, \frac{-3+7}{2}\right) = (2, 2)$$

Midpoint of diagonal from $(0, 1)$ to $(4, 3)$ is

$$\left(\frac{0+4}{2}, \frac{1+3}{2}\right) = (2, 2)$$

Both diagonals have midpoint $(2, 2)$, so they bisect each other.

39. (a) Vertical side: length $= 6 - (-2) = 8$; horizontal side: length $= 3 - (-2) = 5$; diagonal side:
length $= \sqrt{[6-(-2)]^2 + [3-(-2)]^2}$
$= \sqrt{8^2 + 5^2} = \sqrt{89}$

(b) $8^2 + 5^2 = 64 + 25 = 89 = (\sqrt{89})^2$, so the Pythagorean Theorem implies the triangle is a right triangle.

40. (a) $\sqrt{(4-0)^2 + (-4-0)^2} = \sqrt{4^2 + (-4)^2} = \sqrt{32}$

$\sqrt{(4-3)^2 + (-4-3)^2} = \sqrt{1^2 + (-7)^2} = \sqrt{50}$

$\sqrt{(3-0)^2 + (3-0)^2} = \sqrt{3^2 + 3^2} = \sqrt{18}$

(b) Since $(\sqrt{32})^2 + (\sqrt{18})^2 = (\sqrt{50})^2$, the triangle is a right triangle.

41. $(x-1)^2 + (y-2)^2 = 5^2$, or $(x-1)^2 + (y-2)^2 = 25$

42. $[x-(-3)]^2 + (y-2)^2 = 1^2$, or $(x+3)^2 + (y-2)^2 = 1$

43. $[x-(-1)]^2 + [y-(-4)]^2 = 3^2$, or
$(x+1)^2 + (y+4)^2 = 9$

44. $(x-0)^2 + (y-0)^2 = (\sqrt{3})^2$, or $x^2 + y^2 = 3$

45. $(x-3)^2 + (y-1)^2 = 6^2$, so the center is $(3, 1)$ and the radius is 6.

46. $[x-(-4)]^2 + (y-2)^2 = 11^2$, so the center is $(-4, 2)$ and the radius is 11.

47. $(x-0)^2 + (y-0)^2 = (\sqrt{5})^2$, so the center is $(0, 0)$ and the radius is $\sqrt{5}$.

48. $(x-2)^2 + [(y-(-6)]^2 = 5^2$, so the center is $(2, -6)$ and the radius is 5.

49. $|x - 4| = 3$

50. $|y - (-2)| \geq ,4$ or $|y + 2| \geq 4$

51. $|x - c| < d$

52. The distance between y and c is greater than d, so $|y - c| > d$.

53. $\frac{1+a}{2} = 4$ and $\frac{2+b}{2} = 4$

$1 + a = 8$ $\quad$ $2 + b = 8$

$a = 7$ $\quad$ $b = 6$

54. Show that two sides have the same length, but not all three sides have the same length:

$\sqrt{[3-(-1)]^2 + (2-0)^2} = \sqrt{4^2 + 2^2} = \sqrt{16 + 4}$
$= \sqrt{20} = 2\sqrt{5}$

$\sqrt{[5-(-1)]^2 + (4-2)^2} = \sqrt{6^2 + 2^2} = \sqrt{36 + 4}$
$= \sqrt{40} = 2\sqrt{10}$

$\sqrt{(5-3)^2 + (4-0)^2} = \sqrt{2^2 + 4^2} = \sqrt{4 + 16}$
$= \sqrt{20} = 2\sqrt{5}.$

55. The midpoint of the hypotenuse is $\left(\frac{5+0}{2}, \frac{0+7}{2}\right)$
$= \left(\frac{5}{2}, \frac{7}{2}\right) = (2.5, 3.5)$. The distances from this point to the vertices are:

$\sqrt{(2.5-0)^2 + (3.5-0)^2} = \sqrt{2.5^2 + 3.5^2}$
$= \sqrt{6.25 + 12.25} = \sqrt{18.5}$

$\sqrt{(2.5-5)^2 + (3.5-0)^2} = \sqrt{(-2.5)^2 + 3.5^2}$
$= \sqrt{6.25 + 12.25} = \sqrt{18.5}$

$\sqrt{(2.5-0)^2 + (3.5-7)^2} = \sqrt{2.5^2 + (-3.5)^2}$
$= \sqrt{6.25 + 12.25} = \sqrt{18.5}.$

56. $|x - 2| < 3$ means the distance from x to 2 must be less than 3. So x must be between -1 and 5. That is, $-1 < x < 5$.

57. $|x + 3| \geq 5$ means the distance from x to -3 must be 5 or more. So x can be 2 or more, or x can be -8 or less. That is, $x \leq -8$ or $x \geq 2$.

58. True. An absolute value is always greater than or equal to zero. If $a > 0$, then $|a| = a > 0$. If $a < 0$, then $|a| = -a > 0$. If $a = 0$, then $|a| = 0$.

59. True. $\frac{\text{length of AM}}{\text{length of AB}} = \frac{1}{2}$ because M is the midpoint of AB.

By similar triangles, $\frac{\text{length of AM}'}{\text{length of AC}} = \frac{\text{length of AM}}{\text{length of AB}} = \frac{1}{2}$, so M' is the midpoint of AC.

60. $1 < \sqrt{3}$, so $1 - \sqrt{3} < 0$ and $|1 - \sqrt{3}| = -(1 - \sqrt{3}) = \sqrt{3} - 1$. The answer is B.

61. For a segment with endpoints at $a = -3$ and $b = 2$, the midpoint lies at $\frac{a + b}{2} = \frac{-3 + 2}{2} = \frac{-1}{2} = -\frac{1}{2}$.

The answer is C.

62. $(x - 3)^2 + (y + 4)^2 = 2$ corresponds to $(x - h)^2 + (y - k)^2 = (\sqrt{2})^2$, with $h = 3$ and $k = -4$. So the center, (h, k), is $(3, -4)$. The answer is A.

63. In the third quadrant, both coordinates are negative. The answer is E.

64. **(a)** $2 + \frac{1}{3}(8 - 2) = 2 + \frac{1}{3}(6) = 2 + 2 = 4$;
$2 + \frac{2}{3}(6) = 2 + 4 = 6$

(b) $-3 + \frac{1}{3}(7 - (-3)) = -3 + \frac{1}{3}(10) = -3 + \frac{10}{3}$
$= \frac{1}{3}$; $-3 + \frac{2}{3}(7 - (-3)) = -3 + \frac{2}{3}(10)$
$= -3 + \frac{20}{3} = \frac{11}{3}$

(c) $a + \frac{1}{3}(b - a) = a + \frac{1}{3}b - \frac{1}{3}a = \frac{2}{3}a + \frac{1}{3}b$
$= \frac{1}{3}(2a + b) = \frac{2a + b}{3}$;
$a + \frac{2}{3}(b - a) = a + \frac{2}{3}b - \frac{2}{3}a = \frac{1}{3}a + \frac{2}{3}b$
$= \frac{1}{3}(a + 2b) = \frac{a + 2b}{3}$

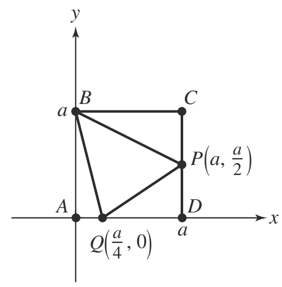
(d) $\left(\frac{2(1) + 7}{3}, \frac{2(2) + 11}{3}\right) = \left(\frac{9}{3}, \frac{15}{3}\right) = (3, 5)$;
$\left(\frac{1 + 2(7)}{3}, \frac{2 + 2(11)}{3}\right) = \left(\frac{15}{3}, \frac{24}{3}\right) = (5, 8)$

(e) $\left(\frac{2a + c}{3}, \frac{2b + d}{3}\right)$; $\left(\frac{a + 2c}{3}, \frac{b + 2d}{3}\right)$

65. If the legs have lengths a and b, and the hypotenuse is c units long, then without loss of generality, we can assume the vertices are $(0, 0)$, $(a, 0)$, and $(0, b)$. Then the midpoint of the hypotenuse is $\left(\frac{a + 0}{2}, \frac{b + 0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$. The distance to the other vertices is

$$\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2} = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \frac{c}{2} = \frac{1}{2}c.$$

66.



(a) Area of $\triangle BPQ$ = area of $\square ABCD$ − area of $\triangle BCP$ − area of $\triangle BAQ$ − area of $\triangle DPQ$
$= a^2 - \frac{1}{2}(a)\left(\frac{a}{2}\right) - \frac{1}{2}(a)\left(\frac{a}{4}\right) - \frac{1}{2}\left(\frac{a}{2}\right)\left(\frac{3}{4}a\right)$
$= a^2 - \frac{a^2}{4} - \frac{a^2}{8} - \frac{3a^2}{16}$
$= \frac{7a^2}{16}$

(b) Area of $\triangle BPQ = \frac{7}{16}\cdot$(area of $\square ABCD$), which is just under half the area of the square $ABCD$.

Note that the result is the same if $a < 0$, but the location of the points in the plane is different.

For #67–69, note that since $P(a, b)$ is in the first quadrant, then a and b are positive. Hence, $-a$ and $-b$ are negative.

67. $Q(a, -b)$ is in the fourth quadrant, and since P and Q both have first coordinate a, PQ is perpendicular to the x-axis.

68. $Q(-a, b)$ is in the second quadrant, and since P and Q both have second coordinate b, PQ is perpendicular to the y-axis.

69. $Q(-a, -b)$ is in the third quadrant, and the midpoint of PQ is $\left(\frac{a + (-a)}{2}, \frac{b + (-b)}{2}\right) = (0, 0)$.

70. Let the points on the number line be $(a, 0)$ and $(b, 0)$. The distance between them is $\sqrt{(a - b)^2 + (0 - 0)^2} = \sqrt{(a - b)^2} = |a - b|$.

■ Section P.3 Linear Equations and Inequalities

Quick Review P.3

1. $2x + 5x + 7 + y - 3x + 4y + 2$
$= (2x + 5x - 3x) + (y + 4y) + (7 + 2)$
$= 4x + 5y + 9$

2. $4 + 2x - 3z + 5y - x + 2y - z - 2$
$= (2x - x) + (5y + 2y) + (-3z - z) + (4 - 2)$
$= x + 7y - 4z + 2$

3. $3(2x - y) + 4(y - x) + x + y$
$= 6x - 3y + 4y - 4x + x + y = 3x + 2y$

4. $5(2x + y - 1) + 4(y - 3x + 2) + 1$
$= 10x + 5y - 5 + 4y - 12x + 8 + 1$
$= -2x + 9y + 4$

5. $\frac{2}{y} + \frac{3}{y} = \frac{5}{y}$

6. $\frac{1}{y - 1} + \frac{3}{y - 2} = \frac{y - 2}{(y - 1)(y - 2)} + \frac{3(y - 1)}{(y - 1)(y - 2)}$
$= \frac{y - 2 + 3y - 3}{(y - 1)(y - 2)} = \frac{4y - 5}{(y - 1)(y - 2)}$

7. $2 + \frac{1}{x} = \frac{2x}{x} + \frac{1}{x} = \frac{2x+1}{x}$

8. $\frac{1}{x} + \frac{1}{y} - x = \frac{y}{xy} + \frac{x}{xy} - \frac{x^2y}{xy} = \frac{y + x - x^2y}{xy}$

9. $\frac{x+4}{2} + \frac{3x-1}{5} = \frac{5(x+4)}{10} + \frac{2(3x-1)}{10}$
$= \frac{5x + 20 + 6x - 2}{10} = \frac{11x + 18}{10}$

10. $\frac{x}{3} + \frac{x}{4} = \frac{4x}{12} + \frac{3x}{12} = \frac{7x}{12}$

Section P.3 Exercises

1. **(a)** and **(c):** $2(-3)^2 + 5(-3) = 2(9) - 15$
$= 18 - 15 = 3$, and $2\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) = 2\left(\frac{1}{4}\right) + \frac{5}{2}$
$= \frac{1}{2} + \frac{5}{2} = \frac{6}{2} = 3$. Meanwhile, substituting $x = -\frac{1}{2}$ gives -2 rather than 3.

2. **(a):** $\frac{-1}{2} + \frac{1}{6} = -\frac{3}{6} + \frac{1}{6} = -\frac{2}{6} = -\frac{1}{3}$ and $\frac{-1}{3} = -\frac{1}{3}$.
Or: Multiply both sides by 6: $6\left(\frac{x}{2}\right) + 6\left(\frac{1}{6}\right) = 6\left(\frac{x}{3}\right)$, so $3x + 1 = 2x$. Subtract $2x$ from both sides: $x + 1 = 0$. Subtract 1 from both sides: $x = -1$.

3. **(b):** $\sqrt{1 - 0^2} + 2 = \sqrt{1} + 2 = 1 + 2 = 3$. Meanwhile, substituting $x = -2$ or $x = 2$ gives $\sqrt{1-4} + 2 = \sqrt{-3} + 2$, which is undefined.

4. **(c):** $(10 - 2)^{1/3} = 8^{1/3} = 2$. Meanwhile, substituting $x = -6$ gives -2 rather than 2; substituting $x = 8$ gives $6^{1/3} \approx 1.82$ rather than 2.

5. Yes: $-3x + 5 = 0$.

6. No. There is no variable x in the equation.

7. No. Subtracting x from both sides gives $3 = -5$, which is false and does not contain the variable x.

8. No. The highest power of x is 2, so the equation is quadratic and not linear.

9. No. The equation has a root in it, so it is not linear.

10. No. The equation has $\frac{1}{x} = x^{-1}$ in it, so it is not linear.

11. $3x = 24$
$x = 8$

12. $4x = -16$
$x = -4$

13. $3t = 12$
$t = 4$

14. $2t = 12$
$t = 6$

15. $2x - 3 = 4x - 5$
$2x = 4x - 2$
$-2x = -2$
$x = 1$

16. $4 - 2x = 3x - 6$
$-2x = 3x - 10$
$-5x = -10$
$x = 2$

17. $4 - 3y = 2y + 8$
$-3y = 2y + 4$
$-5y = 4$
$y = -\frac{4}{5} = -0.8$

18. $4y = 5y + 8$
$-y = 8$
$y = -8$

19. $2\left(\frac{1}{2}x\right) = 2\left(\frac{7}{8}\right)$
$x = \frac{7}{4} = 1.75$

20. $3\left(\frac{2}{3}x\right) = 3\left(\frac{4}{5}\right)$
$2x = \frac{12}{5}$
$x = \frac{12}{10}$
$x = \frac{6}{5} = 1.2$

21. $2\left(\frac{1}{2}x + \frac{1}{3}\right) = 2(1)$
$x + \frac{2}{3} = 2$
$x = \frac{4}{3}$

22. $3\left(\frac{1}{3}x + \frac{1}{4}\right) = 3(1)$
$x + \frac{3}{4} = 3$
$x = \frac{9}{4} = 2.25$

23. $6 - 8z - 10z - 15 = z - 17$
$-18z - 9 = z - 17$
$-18z = z - 8$
$-19z = -8$
$z = \frac{8}{19}$

24. $15z - 9 - 8z - 4 = 5z - 2$
$7z - 13 = 5z - 2$
$7z = 5z + 11$
$2z = 11$
$z = \frac{11}{2} = 5.5$

25. $4\left(\frac{2x-3}{4} + 5\right) = 4(3x)$
$2x - 3 + 20 = 12x$
$2x + 17 = 12x$
$17 = 10x$
$x = \frac{17}{10} = 1.7$

26. $3(2x - 4) = 3\left(\frac{4x-5}{3}\right)$
$6x - 12 = 4x - 5$
$6x = 4x + 7$
$2x = 7$
$x = \frac{7}{2} = 3.5$

27. $24\left(\frac{t+5}{8} - \frac{t-2}{2}\right) = 24\left(\frac{1}{3}\right)$
$3(t + 5) - 12(t - 2) = 8$
$3t + 15 - 12t + 24 = 8$
$-9t + 39 = 8$
$-9t = -31$
$t = \frac{31}{9}$

28. $12\left(\frac{t-1}{3} + \frac{t+5}{4}\right) = 12\left(\frac{1}{2}\right)$
$4(t - 1) + 3(t + 5) = 6$
$4t - 4 + 3t + 15 = 6$
$7t + 11 = 6$
$7t = -5$
$t = -\frac{5}{7}$

29. **(a)** The figure shows that $x = -2$ is a solution of the equation $2x^2 + x - 6 = 0$.

(b) The figure shows that $x = \frac{3}{2}$ is a solution of the equation $2x^2 + x - 6 = 0$.

30. **(a)** The figure shows that $x = 2$ is not a solution of the equation $7x + 5 = 4x - 7$.

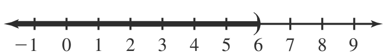
(b) The figure shows that $x = -4$ is a solution of the equation $7x + 5 = 4x - 7$.

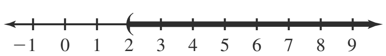
31. **(a)** $2(0) - 3 = 0 - 3 = -3 > 7$. Meanwhile, substituting $x = 5$ gives 7 (which is not less than 7); substituting $x = 6$ gives 9.

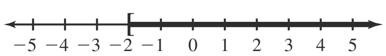
32. **(b)** and **(c):** $3(3) - 4 = 9 - 4 = 5 \geq 5$, and $3(4) - 4 = 12 - 4 = 8 \geq 5$.

33. **(b)** and **(c):** $4(2) - 1 = 8 - 1 = 7$ and $-1 < 7 \leq 11$, and also $4(3) - 1 = 12 - 1 = 11$ and $-1 < 11 \leq 11$. Meanwhile, substituting $x = 0$ gives -1 (which is not greater than -1).

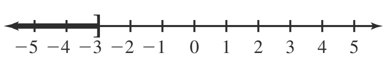
34. **(a)**, **(b)**, and **(c):** $1 - 2(-1) = 1 + 2 = 3$ and $-3 \leq 3 \leq 3$; $1 - 2(0) = 1 - 0 = 1$ and $-3 \leq 1 \leq 3$; $1 - 2(2) = 1 - 4 = -3$ and $-3 \leq -3 \leq 3$.

35.


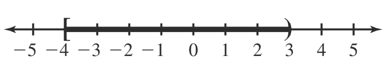
36.


37.


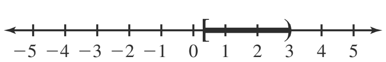
$$\begin{aligned} 2x - 1 &\leq 4x + 3 \\ 2x &\leq 4x + 4 \\ -2x &\leq 4 \\ x &\geq -2 \end{aligned}$$

38.


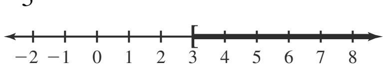
$$\begin{aligned} 3x - 1 &\geq 6x + 8 \\ 3x &\geq 6x + 9 \\ -3x &\geq 9 \\ x &\leq -3 \end{aligned}$$

39.


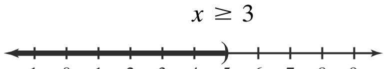
$$\begin{aligned} 2 \leq x + 6 < 9 \\ -4 \leq x < 3 \end{aligned}$$

40.


$$\begin{aligned} -1 \leq 3x - 2 < 7 \\ 1 \leq 3x < 9 \\ \frac{1}{3} \leq x < 3 \end{aligned}$$

41.


$$\begin{aligned} 10 - 6x + 6x - 3 &\leq 2x + 1 \\ 7 &\leq 2x + 1 \\ 6 &\leq 2x \\ 3 &\leq x \\ x &\geq 3 \end{aligned}$$

42.


$$\begin{aligned} 4 - 4x + 5 + 5x &> 3x - 1 \\ 9 + x &> 3x - 1 \\ 10 + x &> 3x \\ 10 &> 2x \\ 5 &> x \\ x &< 5 \end{aligned}$$

43. $$\begin{aligned} 4\left(\frac{5x + 7}{4}\right) &\leq 4(-3) \\ 5x + 7 &\leq -12 \\ 5x &\leq -19 \\ x &\leq -\frac{19}{5} \end{aligned}$$

44. $$\begin{aligned} 5\left(\frac{3x - 2}{5}\right) &> 5(-1) \\ 3x - 2 &> -5 \\ 3x &> -3 \\ x &> -1 \end{aligned}$$

45. $$\begin{aligned} 3(4) \geq 3\left(\frac{2y - 5}{3}\right) &\geq 3(-2) \\ 12 \geq 2y - 5 &\geq -6 \\ 17 \geq 2y &\geq -1 \\ \frac{17}{2} \geq y &\geq -\frac{1}{2} \\ -\frac{1}{2} \leq y &\leq \frac{17}{2} \end{aligned}$$

46. $$\begin{aligned} 4(1) > 4\left(\frac{3y - 1}{4}\right) &> 4(-1) \\ 4 > 3y - 1 &> -4 \\ 5 > 3y &> -3 \\ \frac{5}{3} > y &> -1 \\ -1 < y &< \frac{5}{3} \end{aligned}$$

47. $$\begin{aligned} 0 \leq 2z + 5 &< 8 \\ -5 \leq 2z &< 3 \\ -\frac{5}{2} \leq z &< \frac{3}{2} \end{aligned}$$

48. $$\begin{aligned} -6 < 5t - 1 &< 0 \\ -5 < 5t &< 1 \\ -1 < t &< \frac{1}{5} \end{aligned}$$

49. $$\begin{aligned} 12\left(\frac{x - 5}{4} + \frac{3 - 2x}{3}\right) &< 12(-2) \\ 3(x - 5) + 4(3 - 2x) &< -24 \\ 3x - 15 + 12 - 8x &< -24 \\ -5x - 3 &< -24 \\ -5x &< -21 \\ x &> \frac{21}{5} \end{aligned}$$

50. $$\begin{aligned} 6\left(\frac{3 - x}{2} + \frac{5x - 2}{3}\right) &< 6(-1) \\ 3(3 - x) + 2(5x - 2) &< -6 \\ 9 - 3x + 10x - 4 &< -6 \\ 7x + 5 &< -6 \\ 7x &< -11 \\ x &< -\frac{11}{7} \end{aligned}$$

51. $$\begin{aligned} 10\left(\frac{2y - 3}{2} + \frac{3y - 1}{5}\right) &< 10(y - 1) \\ 5(2y - 3) + 2(3y - 1) &< 10y - 10 \\ 10y - 15 + 6y - 2 &< 10y - 10 \\ 16y - 17 &< 10y - 10 \\ 16y &< 10y + 7 \\ 6y &< 7 \\ y &< \frac{7}{6} \end{aligned}$$

52. $24\left(\frac{3-4y}{6}-\frac{2y-3}{8}\right) \ge 24(2-y)$

$$\begin{aligned} 4(3-4y)-3(2y-3) &\ge 48-24y \\ 12-16y-6y+9 &\ge 48-24y \\ -22y+21 &\ge 48-24y \\ -22y &\ge 27-24y \\ 2y &\ge 27 \\ y &\ge \frac{27}{2} \end{aligned}$$

53. $2\left[\frac{1}{2}(x-4)-2x\right] \le 2[5(3-x)]$

$$\begin{aligned} x-4-4x &\le 10(3-x) \\ -3x-4 &\le 30-10x \\ -3x &\le 34-10x \\ 7x &\le 34 \\ x &\le \frac{34}{7} \end{aligned}$$

54. $6\left[\frac{1}{2}(x+3)+2(x-4)\right] < 6\left[\frac{1}{3}(x-3)\right]$

$$\begin{aligned} 3(x+3)+12(x-4) &< 2(x-3) \\ 3x+9+12x-48 &< 2x-6 \\ 15x-39 &< 2x-6 \\ 15x &< 2x+33 \\ 13x &< 33 \\ x &< \frac{33}{13} \end{aligned}$$

55. $x^2-2x<0$ for $x=1$

56. $x^2-2x=0$ for $x=0, 2$

57. $x^2-2x>0$ for $x=3, 4, 5, 6$

58. $x^2-2x\le 0$ for $x=0, 1, 2$

59. Multiply both sides of the first equation by 2.

60. Divide both sides of the first equation by 2.

61. (a) No: they have different solutions.

$3x = 6x+9$, $-3x = 9$, $x=-3$ $x = 2x+9$, $-x=9$, $x=-9$

(b) Yes: the solution to both equations is $x=4$.

$6x+2=4x+10$, $6x=4x+8$, $2x=8$, $x=4$ $3x+1=2x+5$, $3x=2x+4$, $x=4$

62. (a) Yes: the solution to both equations is $x=\frac{9}{2}$.

$3x+2=5x-7$, $3x=5x-9$, $-2x=-9$, $x=\frac{9}{2}$ $-2x+2=-7$, $-2x=-9$, $x=\frac{9}{2}$

(b) No: they have different solutions.

$2x+5=x-7$, $2x=x-12$, $x=-12$ $2x=x-7$, $x=-7$

63. False. $6>2$, but $-6<-2$ because -6 lies to the left of -2 on the number line.

64. True. $2 \le \frac{6}{3}$ includes the possibility that $2=\frac{6}{3}$, and this is the case.

65. $3x+5=2x+1$
Subtracting 5 from each side gives $3x=2x-4$.
The answer is E.

66. $-3x<6$
Dividing each side by -3 and reversing the $<$ gives $x>-2$.
The answer is C.

67. $x(x+1)=0$
$x=0$ or $x+1=0$
$x=-1$
The answer is A.

68. $\frac{2x}{3}+\frac{1}{2}=\frac{x}{4}-\frac{1}{3}$
Multiplying each side by 12 gives $8x+6=3x-4$.
The answer is B.

69. (c) $\frac{800}{801}>\frac{799}{800}$

(d) $-\frac{103}{102}>-\frac{102}{101}$

(e) If your calculator returns 0 when you enter $2x+1<4$, you can conclude that the value stored in x is not a solution of the inequality $2x+1<4$.

70.

$$\begin{aligned} P &= 2(L+W) \\ \frac{1}{2}P &= L+W \\ \frac{1}{2}P-L &= W \\ W &= \frac{1}{2}P-L=\frac{P-2L}{2} \end{aligned}$$

71.

$$\begin{aligned} A &= \frac{1}{2}h(b_1+b_2) \\ h(b_1+b_2) &= 2A \\ b_1+b_2 &= \frac{2A}{h} \\ b_1 &= \frac{2A}{h}-b_2 \end{aligned}$$

72.

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ \frac{3}{4\pi}V &= r^3 \\ \sqrt[3]{\frac{3V}{4\pi}} &= r \\ r &= \sqrt[3]{\frac{3V}{4\pi}} \end{aligned}$$

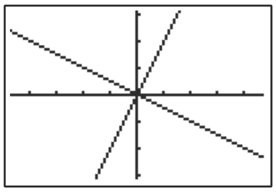
73.

$$\begin{aligned} C &= \frac{5}{9}(F-32) \\ \frac{9}{5}C &= F-32 \\ \frac{9}{5}C+32 &= F \\ F &= \frac{9}{5}C+32 \end{aligned}$$

Section P.4 Lines in the Plane

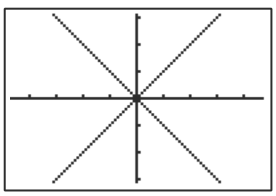
Exploration 1

1. The graphs of $y = mx + b$ and $y = mx + c$ have the same slope but different y-intercepts.

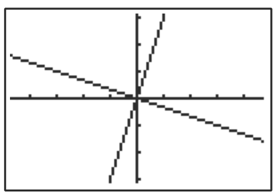
2. 

$[-4.7, 4.7]$ by $[-3.1, 3.1]$
$m = 2$

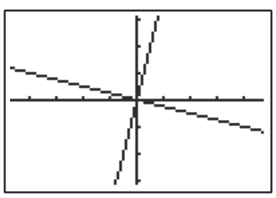
The angle between the two lines appears to be 90°.

3. 

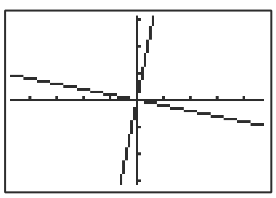
$[-4.7, 4.7]$ by $[-3.1, 3.1]$
$m = 1$



$[-4.7, 4.7]$ by $[-3.1, 3.1]$
$m = 3$



$[-4.7, 4.7]$ by $[-3.1, 3.1]$
$m = 4$



$[-4.7, 4.7]$ by $[-3.1, 3.1]$
$m = 5$

In each case, the two lines appear to be at right angles to one another.

Quick Review P.4

1. $-75x + 25 = 200$
$-75x = 175$
$x = -\frac{7}{3}$

2. $400 - 50x = 150$
$-50x = -250$
$x = 5$

3. $3(1 - 2x) + 4(2x - 5) = 7$
$3 - 6x + 8x - 20 = 7$
$2x - 17 = 7$
$2x = 24$
$x = 12$

4. $2(7x + 1) = 5(1 - 3x)$
$14x + 2 = 5 - 15x$
$29x + 2 = 5$
$29x = 3$
$x = \frac{3}{29}$

5. $2x - 5y = 21$
$-5y = -2x + 21$
$y = \frac{2}{5}x - \frac{21}{5}$

6. $\frac{1}{3}x + \frac{1}{4}y = 2$
$12\left(\frac{1}{3}x + \frac{1}{4}y\right) = 12(2)$
$4x + 3y = 24$
$3y = -4x + 24$
$y = -\frac{4}{3}x + 8$

7. $2x + y = 17 + 2(x - 2y)$
$2x + y = 17 + 2x - 4y$
$y = 17 - 4y$
$5y = 17$
$y = \frac{17}{5}$

8. $x^2 + y = 3x - 2y$
$y = 3x - 2y - x^2$
$3y = 3x - x^2$
$y = x - \frac{1}{3}x^2$

9. $\frac{9 - 5}{-2 - (-8)} = \frac{4}{6} = \frac{2}{3}$

10. $\frac{-4 - 6}{-14 - (-2)} = \frac{-10}{-12} = \frac{5}{6}$

Section P.4 Exercises

1. $m = -2$

2. $m = \frac{2}{3}$

3. $m = \frac{9 - 5}{4 + 3} = \frac{4}{7}$

4. $m = \frac{-3 - 1}{5 + 2} = -\frac{4}{7}$

5. $m = \frac{3 + 5}{-1 + 2} = 8$

6. $m = \frac{12 + 3}{-4 - 5} = -\frac{5}{3}$

7. $2 = \frac{9 - 3}{5 - x} = \frac{6}{5 - x}$, so $x = 2$

8. $-3 = \frac{y - 3}{4 + 2} = \frac{y - 3}{6}$, so $y = -15$

9. $3 = \frac{y + 5}{4 + 3} = \frac{y + 5}{7}$, so $y = 16$

10. $\frac{1}{2} = \frac{2 + 2}{x + 8} = \frac{4}{x + 8}$, so $x = 0$

11. $y - 4 = 2(x - 1)$

12. $y - 3 = -\frac{2}{3}(x + 4)$

13. $y + 4 = -2(x - 5)$

14. $y - 4 = 3(x + 3)$

15. Since $m = 1$, we can choose $A = 1$ and $B = -1$. Since $x = -7$, $y = -2$ solves $x - y + C = 0$, C must equal 5: $x - y + 5 = 0$. Note that the coefficients can be multiplied by any nonzero number, e.g., another answer would be $2x - 2y + 10 = 0$. This comment also applies to the following problems.

16. Since $m = 1$, we can choose $A = 1$ and $B = -1$. Since $x = -3$, $y = -8$ solves $x - y + C = 0$, C must equal -5: $x - y - 5 = 0$. See comment in #15.

17. Since $m = 0$, we can choose $A = 0$ and $B = 1$. Since $x = 1$, $y = -3$ solves $0x + y + C = 0$. C must equal 3: $0x + y + 3 = 0$, or $y + 3 = 0$. See comment in #15.

18. Since $m = -1$, we can choose $A = 1$ and $B = 1$. Since $x = -1$, $y = -5$ solves $x + y + C = 0$, C must equal 6: $x + y + 6 = 0$. See comment in #15.

19. The slope is $m = 1 = -A/B$, so we can choose $A = 1$ and $B = -1$. Since $x = -1$, $y = 2$ solves $x - y + C = 0$, C must equal 3: $x - y + 3 = 0$. See comment in #15.

20. Since m is undefined we must have $B = 0$, and we can choose $A = 1$. Since $x = 4$, $y = 5$ solves $x + 0y + C = 0$, C must equal -4: $x - 4 = 0$. See comment in #15.

21. Begin with point-slope form: $y - 5 = -3(x - 0)$, so $y = -3x + 5$.

22. Begin with point-slope form: $y - 2 = \frac{1}{2}(x - 1)$, so $y = \frac{1}{2}x + \frac{3}{2}$.

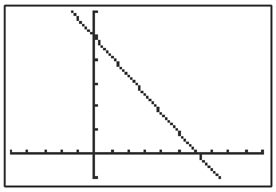
23. $m = -\frac{1}{4}$, so in point-slope form, $y - 5 = -\frac{1}{4}(x + 4)$, and therefore $y = -\frac{1}{4}x + 4$.

24. $m = \frac{1}{7}$, so in point-slope form, $y - 2 = \frac{1}{7}(x - 4)$, and therefore $y = \frac{1}{7}x + \frac{10}{7}$.

25. Solve for y: $y = -\frac{2}{5}x + \frac{12}{5}$.

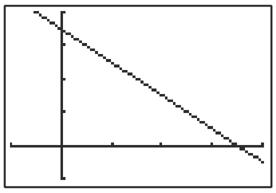
26. Solve for y: $y = \frac{7}{12}x - 8$.

27. Graph $y = 49 - 8x$; window should include (6.125, 0) and (0, 49), for example, $[-5, 10] \times [-10, 60]$.



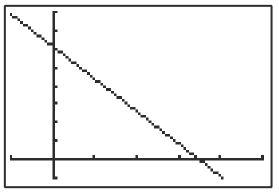
$[-5, 10]$ by $[-10, 60]$

28. Graph $y = 35 - 2x$; window should include (17.5, 0) and (0, 35), for example, $[-5, 20] \times [-10, 40]$.



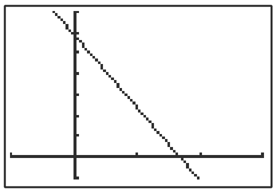
$[-5, 20]$ by $[-10, 40]$

29. Graph $y = (429 - 123x)/7$; window should include (3.488, 0) and (0, 61.29), for example, $[-1, 5] \times [-10, 80]$.



$[-1, 5]$ by $[-10, 80]$

30. Graph $y = (3540 - 2100x)/12 = 295 - 175x$; window should include (1.686, 0) and (0, 295), for example, $[-1, 3] \times [-50, 350]$.



$[-1, 3]$ by $[-50, 350]$

31. **(a)**: The slope is 1.5, compared to 1 in (b).

32. **(b)**: The slopes are $\frac{7}{4}$ and 4, respectively.

33. Substitute and solve: replacing y with 14 gives $x = 4$, and replacing x with 18 gives $y = 21$.

34. Substitute and solve: replacing y with 14 gives $x = 2$, and replacing x with 18 gives $y = -18$.

35. Substitute and solve: replacing y with 14 gives $x = -10$, and replacing x with 18 gives $y = -7$.

36. Substitute and solve: replacing y with 14 gives $x = 14$, and replacing x with 18 gives $y = 20$.

37. Ymin = −30, Ymax = 30, Yscl = 3

38. Ymin = −50, Ymax = 50, Yscl = 5

39. Ymin = −20/3, Ymax = 20/3, Yscl = 2/3

40. Ymin = −12.5, Ymax = 12.5, Yscl = 1.25

In #41–44, use the fact that parallel lines have the same slope, while the slopes of perpendicular lines multiply to give −1.

41. **(a)** Parallel: $y - 2 = 3(x - 1)$, or $y = 3x - 1$.

(b) Perpendicular: $y - 2 = -\frac{1}{3}(x - 1)$, or $y = -\frac{1}{3}x + \frac{7}{3}$.

42. **(a)** Parallel: $y - 3 = -2(x + 2)$, or $y = -2x - 1$.

(b) Perpendicular: $y - 3 = \frac{1}{2}(x + 2)$, or $y = \frac{1}{2}x + 4$.

43. **(a)** Parallel: $2x + 3y = 9$, or $y = -\frac{2}{3}x + 3$.

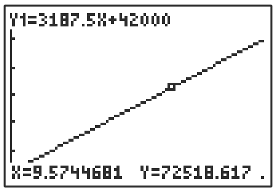
(b) Perpendicular: $3x - 2y = 7$, or $y = \frac{3}{2}x - \frac{7}{2}$.

44. **(a)** Parallel: $3x - 5y = 13$, or $y = \frac{3}{5}x - \frac{13}{5}$.

(b) Perpendicular: $5x + 3y = 33$, or $y = -\frac{5}{3}x + 11$.

45. **(a)** $m = (67{,}500 - 42{,}000)/8 = 3187.5$, the y-intercept is $b = 42{,}000$ so $V = 3187.5t + 42{,}000$.

(b) The house is worth about \$72,500 after 9.57 years.



[0, 15] by [40000, 100000]

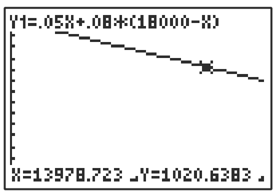
(c) $3187.5t + 42{,}000 = 74{,}000$: $t = 10.04$.

(d) $t = 12$ years.

46. **(a)** $0 \le x \le 18000$

(b) $I = 0.05x + 0.08(18{,}000 - x)$

(c) $x = 14{,}000$ dollars.



[0, 18000] by [0, 1500]

(d) $x = 8500$ dollars.

47. $y = \frac{3}{8}x$, where y is altitude and x is horizontal distance.

The plane must travel $x = 32{,}000$ ft horizontally–just over 6 miles.

48. **(a)** $m = \dfrac{6 \text{ ft}}{100 \text{ ft}} = 0.06$.

(b) $4166.\overline{6}$ ft, or about 0.79 mile.

(c) 2217.6 ft.

49. $m = \dfrac{3}{8} = 0.375 > \dfrac{4}{12} = 0.3\overline{3}$, so asphalt shingles are acceptable.

50. We need to find the value of y when $x = 2004$, 2006, and 2007 using the equation $y = 0.38(x - 2005) + 10.5$.

$y = 0.38(2004 - 2005) + 10.5 = -0.38 + 10.5 = 10.1$

$y = 0.38(2006 - 2005) + 10.5 = 0.38 + 10.5 = 10.9$

$y = 0.38(2007 - 2005) + 10.5 = 0.76 + 10.5 = 11.3$

Americans' income in 2004, 2006, and 2007 was, respectively, 10.1, 10.9, and 11.3 trillion dollars.

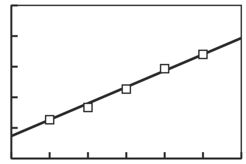
51. **(a)** Slope of the line between the points (1990, 3.8) and (2010, 10.2) is $m = \dfrac{10.2 - 3.8}{20} = \dfrac{6.4}{20} = 0.32$.

Using the point-slope form equation for the line, we have $y - 3.8 = 0.32(x - 1990)$, so $y = 0.32(x - 1990) + 3.8$.

(b) Using $y = 0.32(x - 1990) + 3.8$ and $x = 2005$, the model estimates Americans' expenditures in 2005 were \$8 trillion.

(c) Using $y = 0.32(x - 1990) + 3.8$ and $x = 2015$, the model predicts Americans' expenditures in 2015 will be \$11.8 trillion.

(d)

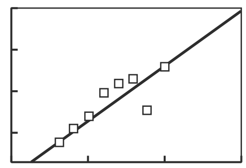


[1985, 2015] by [0, 15]

52. **(a)** Slope of the line between the points (2003, 138.1) and (2010, 229.9) is $m = \dfrac{229.9 - 138.1}{7} = \dfrac{91.8}{7} = 13.11$

Using the point-slope form equation for the line, we have $y - 138.1 = 13.11(x - 2003)$, so $y = 13.11(x - 2003) + 138.1$.

(b)

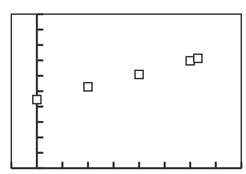


[2000, 2015] by [120, 300]

(c) Using $y = 13.11(x - 2003) + 138.1$ and $x = 2015$, the model predicts U.S. imports from Mexico in 2015 will be approximately \$295.4 billion.

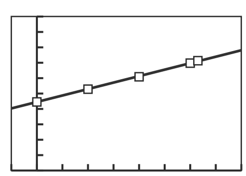
53. **(a)** Slope of the line between the points (0, 4453) and (30, 6707) is $m = \dfrac{6972 - 4453}{30} = \dfrac{2519}{30} = 84.0$

Using the point-slope form equation for the line, we have $y - 4453 = 84.0(x - 0)$, so $y = 84.0x + 4453$.



[–5, 40] by [0, 10,000]

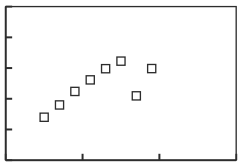
(b)



[–5, 40] by [0, 10,000]

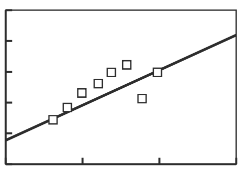
(c) The year 2020 is represented by $x = 40$. Using $y = 84.0x + 4453$ and $x = 40$, the model predicts the midyear world population in 2020 will be 7.813 billion.

54. (a) Using the point-slope form equation for the line, we have $y - 169.9 = 11.31(x - 2003)$, so $y = 11.31(x - 2003) + 169.9$.



[2000, 2015] by [100, 350]

(b) Slope of the line between the points (2003, 169.9) and (2010, 249.1) is $m = \frac{249.1 - 169.9}{7} = \frac{79.2}{7} = 11.31$.



[2000, 2015] by [100, 350]

(c) Using $y = 11.31(x - 2003) + 169.9$ and $x = 2015$, the model predicts total U.S. exports to Canada in 2015 will be 305.6 billion.

55. $\frac{8-0}{a-3} = \frac{4-0}{3-0}$

$\frac{8}{a-3} = \frac{4}{3}$

$24 = 4(a-3)$
$6 = a - 3$
$9 = a$

56. $\frac{a-0}{5-3} = \frac{2-0}{1-0}$

$\frac{a}{2} = 2$

$a = 4$

57. $\overline{AD} \parallel \overline{BC} \Rightarrow b = 5;$

$\overline{AB} \parallel \overline{DC} \Rightarrow \frac{5}{a-4} = \frac{5}{2} \Rightarrow a = 6$

58. $\overline{BC} \parallel \overline{AD} \Rightarrow b = 4;$

$\overline{AB} \parallel \overline{CD} \Rightarrow \frac{4}{a} = \frac{4-0}{8-5} \Rightarrow a = 3$

59. (a) No, it is not possible for two lines with positive slopes to be perpendicular, because if both slopes are positive, they cannot multiply to -1.

(b) No, it is not possible for two lines with negative slopes to be perpendicular, because if both slopes are negative, they cannot multiply to -1.

60. (a) If $b = 0$, both lines are vertical; otherwise, both have slope $m = -a/b$, and are therefore parallel. If $c = d$, the lines are coincident.

(b) If either a or b equals 0, then one line is horizontal and the other is vertical. Otherwise, their slopes are $-a/b$ and b/a, respectively. In either case, they are perpendicular.

61. False. The slope of a vertical line is undefined. For example, the vertical line through (3, 1) and (3, 6) would have a slope of $\frac{6-1}{3-3} = \frac{5}{0}$, which is undefined.

62. True. If $b = 0$, then $a \neq 0$ and the graph of $x = \frac{c}{a}$ is a vertical line. If $b \neq 0$, then the graph of $y = -\frac{a}{b}x + \frac{c}{b}$ is a line with slope $-\frac{a}{b}$ and y-intercept $\frac{c}{b}$. If $b \neq 0$ and $a = 0$, $y = \frac{c}{b}$, which is a horizontal line. An equation of the form $ax + by = c$ is called linear for this reason.

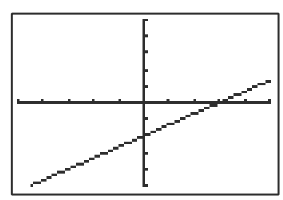
63. With $(x_1, y_1) = (-2, 3)$ and $m = 4$, the point-slope form equation $y - y_1 = m(x - x_1)$ becomes $y - 3 = 4[x - (-2)]$ or $y - 3 = 4(x + 2)$. The answer is A.

64. With $m = 3$ and $b = -2$, the slope-intercept form equation $y = mx + b$ becomes $y = 3x + (-2)$ or $y = 3x - 2$. The answer is B.

65. When a line has a slope of $m_1 = -2$, a perpendicular line must have a slope of $m_2 = -\frac{1}{m_1} = \frac{1}{2}$. The answer is E.

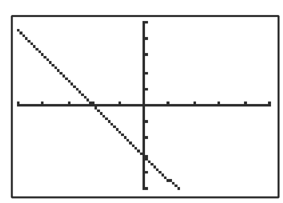
66. The line through $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (1, -4)$ has a slope of $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-1}{1-(-2)} = \frac{-5}{3} = -\frac{5}{3}$. The answer is C.

67. (a)



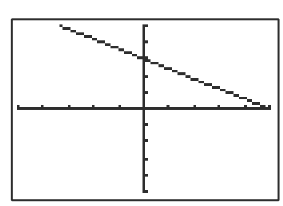
[−5, 5] by [−5, 5]

(b)



[−5, 5] by [−5, 5]

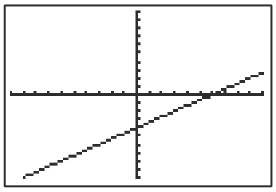
(c)



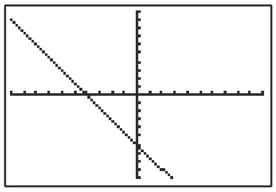
[−5, 5] by [−5, 5]

(d) From the graphs, it appears that a is the x-intercept and b is the y-intercept when $c = 1$.
Proof: The x-intercept is found by setting $y = 0$. When $c = 1$, we have $\frac{x}{a} + \frac{0}{b} = 1$. Hence $\frac{x}{a} = 1$ so $x = a$. The y-intercept is found by setting $x = 0$. When $c = 1$, we have $\frac{0}{a} + \frac{y}{b} = 1$. Hence $\frac{y}{b} = 1$, so $y = b$.

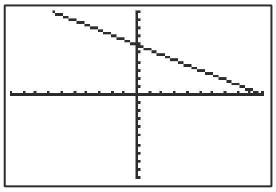
(e)



$[-10, 10]$ by $[-10, 10]$



$[-10, 10]$ by $[-10, 10]$

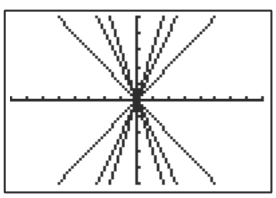


$[-10, 10]$ by $[-10, 10]$

From the graphs, it appears that a is half the x-intercept and b is half the y-intercept when $c = 2$. Proof: When $c = 2$, we can divide both sides by 2 and we have $\frac{x}{2a} + \frac{y}{2b} = 1$. By part **(d)** the x-intercept is $2a$ and the y-intercept is $2b$.

(f) By a similar argument, when $c = -1$, a is the opposite of the x-intercept and b is the opposite of the y-intercept.

68. (a)

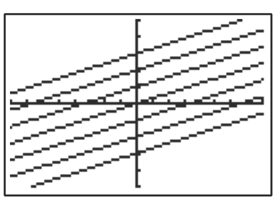


$[-8, 8]$ by $[-5, 5]$

These graphs all pass through the origin. They have different slopes.

(b) If $m > 0$, then the graphs of $y = mx$ and $y = -mx$ have the same steepness, but one increases from left to right, and the other decreases from left to right.

(c)



$[-8, 8]$ by $[-5, 5]$

These graphs have the same slope, but different y-intercepts.

69. As in the diagram, we can choose one point to be the origin, and another to be on the x-axis. The midpoints of the sides, starting from the origin and working around counterclockwise in the diagram, are then $A\left(\frac{a}{2}, 0\right)$, $B\left(\frac{a+b}{2}, \frac{c}{2}\right)$, $C\left(\frac{b+d}{2}, \frac{c+e}{2}\right)$, and $D\left(\frac{d}{2}, \frac{e}{2}\right)$. The opposite sides are therefore parallel, since the slopes of the four lines connecting those points are: $m_{AB} = \frac{c}{b}$; $m_{BC} = \frac{e}{d-a}$; $m_{CD} = \frac{c}{b}$; $m_{DA} = \frac{e}{d-a}$.

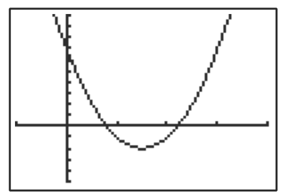
70. The line from the origin to $(3, 4)$ has slope $\frac{4}{3}$, so the tangent line has slope $-\frac{3}{4}$, and in *point-slope form*, the equation is $y - 4 = -\frac{3}{4}(x - 3)$.

71. A has coordinates $\left(\frac{b}{2}, \frac{c}{2}\right)$, while B is $\left(\frac{a+b}{2}, \frac{c}{2}\right)$, so the line containing A and B is the horizontal line $y = c/2$, and the distance from A to B is $\left|\frac{a+b}{2} - \frac{b}{2}\right| = \frac{a}{2}$.

■ Section P.5 Solving Equations Graphically, Numerically, and Algebraically

Exploration 1

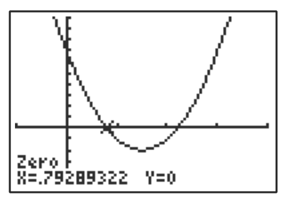
1.



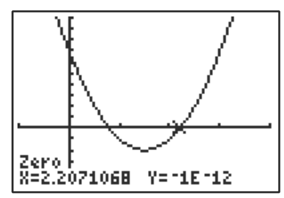
$[-1, 4]$ by $[-5, 10]$

2. Using the numerical zoom, we find the zeros to be 0.79 and 2.21.

3.



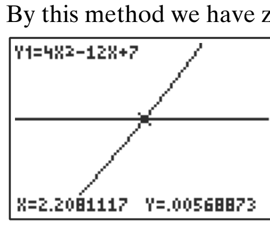
$[-1, 4]$ by $[-5, 10]$



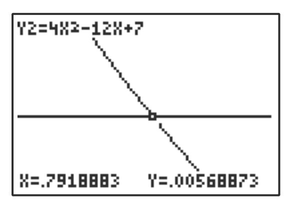
$[-1, 4]$ by $[-5, 10]$

By this method we have zeros at 0.79 and 2.21.

4.



$[2.05, 2.36]$ by $[-0.5, 0.43]$



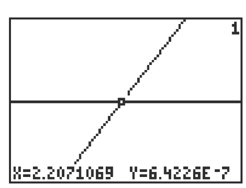
$[0.63, 0.94]$ by $[-0.39, 0.55]$

Zooming in and tracing reveals the same zeros, correct to two decimal places.

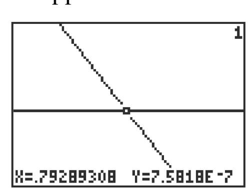
5. The answers in parts 2, 3, and 4 are the same.

6. On a calculator, evaluating $4x^2 - 12x + 7$ when $x = 0.79$ gives $y = 0.0164$ and when $x = 2.21$ gives $y = 0.0164$, so the numbers 0.79 and 2.21 are approximate zeros.

7.



$[2.17, 2.24]$ by $[-0.12, 0.11]$



$[0.75, 0.83]$ by $[-0.11, 0.12]$

Zooming in and tracing reveals zeros of 0.792893 and 2.207107 accurate to six decimal places. If rounded to two decimal places, these would be the same as the answers found in part 3.

Quick Review P.5

1. $(3x - 4)^2 = 9x^2 - 12x - 12x + 16 = 9x^2 - 24x + 16$

2. $(2x + 3)^2 = 4x^2 + 6x + 6x + 9 = 4x^2 + 12x + 9$

3. $(2x + 1)(3x - 5) = 6x^2 - 10x + 3x - 5$
$= 6x^2 - 7x - 5$

4. $(3y - 1)(5y + 4) = 15y^2 + 12y - 5y - 4$
$= 15y^2 + 7y - 4$

5. $25x^2 - 20x + 4 = (5x - 2)(5x - 2) = (5x - 2)^2$

6. $15x^3 - 22x^2 + 8x = x(15x^2 - 22x + 8)$
$= x(5x - 4)(3x - 2)$

7. $3x^3 + x^2 - 15x - 5 = x^2(3x + 1) - 5(3x + 1)$
$= (3x + 1)(x^2 - 5)$

8. $y^4 - 13y^2 + 36 = (y^2 - 4)(y^2 - 9)$
$= (y - 2)(y + 2)(y - 3)(y + 3)$

9. $\dfrac{x}{2x + 1} - \dfrac{2}{x + 3}$

$= \dfrac{x(x + 3)}{(2x + 1)(x + 3)} - \dfrac{2(2x + 1)}{(2x + 1)(x + 3)}$

$= \dfrac{x^2 + 3x - 4x - 2}{(2x + 1)(x + 3)} = \dfrac{x^2 - x - 2}{(2x + 1)(x + 3)}$

$= \dfrac{(x - 2)(x + 1)}{(2x + 1)(x + 3)}$

10. $\dfrac{x + 1}{x^2 - 5x + 6} - \dfrac{3x + 11}{x^2 - x - 6}$

$= \dfrac{x + 1}{(x - 3)(x - 2)} - \dfrac{3x + 11}{(x - 3)(x + 2)}$

$= \dfrac{(x + 1)(x + 2)}{(x - 3)(x - 2)(x + 2)} - \dfrac{(3x + 11)(x - 2)}{(x - 3)(x - 2)(x + 2)}$

$= \dfrac{(x^2 + 3x + 2) - (3x^2 + 5x - 22)}{(x - 3)(x - 2)(x + 2)}$

$= \dfrac{-2x^2 - 2x + 24}{(x - 3)(x - 2)(x + 2)}$

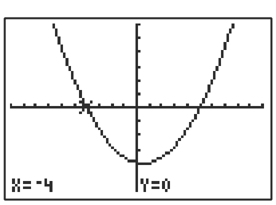
$= \dfrac{-2(x^2 + x - 12)}{(x - 3)(x - 2)(x + 2)}$

$= \dfrac{-2(x + 4)(x - 3)}{(x - 3)(x - 2)(x + 2)}$

$= \dfrac{-2(x + 4)}{(x - 2)(x + 2)}$ if $x \neq 3$

Section P.5 Exercises

1.



[–10, 10] by [–30, 30]

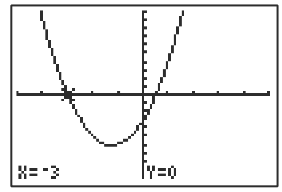
$x = -4$ or $x = 5$

The left side factors to $(x + 4)(x - 5) = 0$:

$x + 4 = 0$ or $x - 5 = 0$

$x = -4$ $\quad x = 5$

2.



[–5, 5] by [–10, 10]

$x = -3$ or $x = 0.5$

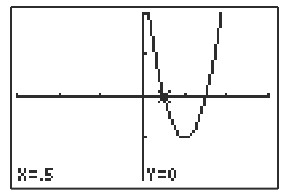
The left side factors to $(x + 3)(2x - 1) = 0$:

$x + 3 = 0$ or $2x - 1 = 0$

$x = -3$ $\quad 2x = 1$

$x = 0.5$

3.



[–3, 3] by [–2, 2]

$x = 0.5$ or $x = 1.5$

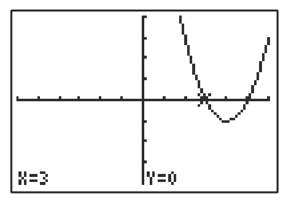
The left side factors to $(2x - 1)(2x - 3) = 0$:

$2x - 1 = 0$ or $2x - 3 = 0$

$2x = 1$ $\quad 2x = 3$

$x = 0.5$ $\quad x = 1.5$

4.



[–6, 6] by [–4, 4]

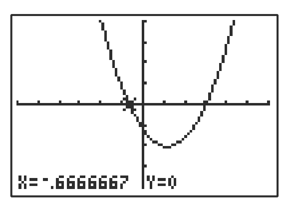
$x = 3$ or $x = 5$

Rewrite as $x^2 - 8x + 15 = 0$; the left side factors to $(x - 3)(x - 5) = 0$:

$x - 3 = 0$ or $x - 5 = 0$

$x = 3$ $\quad x = 5$

5.



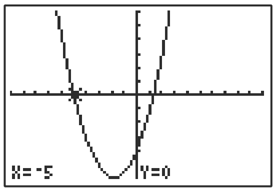
[–6, 6] by [–20, 20]

$x = -\dfrac{2}{3}$ or $x = 3$

Rewrite as $3x^2 - 7x - 6 = 0$; the left side factors to $(3x + 2)(x - 3) = 0$:

$3x + 2 = 0$ or $x - 3 = 0$

$x = -\dfrac{2}{3}$ $\quad x = 3$

6. 

[–10, 10] by [–30, 30]

$x = -5$ or $x = \frac{4}{3}$

Rewrite as $3x^2 + 11x - 20 = 0$; the left side factors to $(3x - 4)(x + 5) = 0$:

$3x - 4 = 0$ or $x + 5 = 0$

$x = \frac{4}{3}$ $\qquad x = -5$

7. Rewrite as $(2x)^2 = 5^2$; then $2x = \pm 5$, or $x = \pm\frac{5}{2}$.

8. Divide both sides by 2 to get $(x - 5)^2 = 8.5$. Then $x - 5 = \pm\sqrt{8.5}$ and $x = 5 \pm \sqrt{8.5}$.

9. Divide both sides by 3 to get $(x + 4)^2 = \frac{8}{3}$. Then $x + 4 = \pm\sqrt{\frac{8}{3}}$ and $x = -4 \pm \sqrt{\frac{8}{3}}$.

10. Divide both sides by 4 to get $(u + 1)^2 = 4.5$. Then $u + 1 = \pm\sqrt{4.5}$ and $u = -1 \pm \sqrt{4.5}$.

11. Adding $2y^2 + 8$ to both sides gives $4y^2 = 14$. Divide both sides by 4 to get $y^2 = \frac{7}{2}$, so $y = \pm\sqrt{\frac{7}{2}}$.

12. $2x + 3 = \pm 13$ so $x = \frac{1}{2}(-3 \pm 13)$, which gives $x = -8$ or $x = 5$.

13.
$$\begin{aligned} x^2 + 6x + 3^2 &= 7 + 3^2 \\ (x + 3)^2 &= 16 \\ x + 3 &= \pm\sqrt{16} \\ x &= -3 \pm 4 \end{aligned}$$
$x = -7$ or $x = 1$

14.
$$\begin{aligned} x^2 + 5x &= 9 \\ x^2 + 5x + \left(\frac{5}{2}\right)^2 &= 9 + \left(\frac{5}{2}\right)^2 \\ (x + 2.5)^2 &= 9 + 6.25 \\ x + 2.5 &= \pm\sqrt{15.25} \end{aligned}$$
$x = -2.5 - \sqrt{15.25} \approx -6.41$ or

$x = -2.5 + \sqrt{15.25} \approx 1.41$

15.
$$\begin{aligned} x^2 - 7x &= -\frac{5}{4} \\ x^2 - 7x + \left(-\frac{7}{2}\right)^2 &= -\frac{5}{4} + \left(-\frac{7}{2}\right)^2 \\ \left(x - \frac{7}{2}\right)^2 &= 11 \\ x - \frac{7}{2} &= \pm\sqrt{11} \\ x &= \frac{7}{2} \pm \sqrt{11} \end{aligned}$$
$x = \frac{7}{2} - \sqrt{11} \approx 0.18$ or $x = \frac{7}{2} + \sqrt{11} \approx 6.82$

16.
$$\begin{aligned} x^2 + 6x &= 4 \\ x^2 + 6x + \left(\frac{6}{2}\right)^2 &= 4 + \left(\frac{6}{2}\right)^2 \\ (x + 3)^2 &= 4 + 9 \\ x + 3 &= \pm\sqrt{13} \\ x &= -3 \pm \sqrt{13} \end{aligned}$$
$x = -3 - \sqrt{13} \approx -6.61$ or $x = -3 + \sqrt{13} \approx 0.61$

17.
$$\begin{aligned} 2x^2 - 7x + 9 &= x^2 - 2x - 3 + 3x \\ 2x^2 - 7x + 9 &= x^2 + x - 3 \\ x^2 - 8x &= -12 \\ x^2 - 8x + (-4)^2 &= -12 + (-4)^2 \\ (x - 4)^2 &= 4 \\ x - 4 &= \pm 2 \\ x &= 4 \pm 2 \end{aligned}$$
$x = 2$ or $x = 6$

18.
$$\begin{aligned} 3x^2 - 6x - 7 &= x^2 + 3x - x^2 - x + 3 \\ 3x^2 - 8x &= 10 \\ x^2 - \frac{8}{3}x &= \frac{10}{3} \\ x^2 - \frac{8}{3}x + \left(-\frac{4}{3}\right)^2 &= \frac{10}{3} + \left(-\frac{4}{3}\right)^2 \\ \left(x - \frac{4}{3}\right)^2 &= \frac{10}{3} + \frac{16}{9} \\ x - \frac{4}{3} &= \pm\sqrt{\frac{46}{9}} \\ x &= \frac{4}{3} \pm \frac{1}{3}\sqrt{46} \end{aligned}$$
$x = \frac{4}{3} - \frac{1}{3}\sqrt{46} \approx -0.93$ or $x = \frac{4}{3} + \frac{1}{3}\sqrt{46} \approx 3.59$

19. $a = 1$, $b = 8$, and $c = -2$:
$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-2)}}{2(1)} = \frac{-8 \pm \sqrt{72}}{2}$$
$$= \frac{-8 \pm 6\sqrt{2}}{2} = -4 \pm 3\sqrt{2}$$
$x \approx -8.24$ or $x \approx 0.24$

20. $a = 2$, $b = -3$, and $c = 1$:
$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(1)}}{2(2)} = \frac{3 \pm \sqrt{1}}{4} = \frac{3}{4} \pm \frac{1}{4}$$
$x = \frac{1}{2}$ or $x = 1$

21. $x^2 - 3x - 4 = 0$, so $a = 1$, $b = -3$, and $c = -4$:
$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-4)}}{2(1)} = \frac{3 \pm \sqrt{25}}{2} = \frac{3}{2} \pm \frac{5}{2}$$
$x = -1$ or $x = 4$

22. $x^2 - \sqrt{3}x - 5 = 0$, so $a = 1$, $b = -\sqrt{3}$, and $c = -5$:
$$x = \frac{\sqrt{3} \pm \sqrt{(-\sqrt{3})^2 - 4(1)(-5)}}{2(1)}$$
$$= \frac{\sqrt{3} \pm \sqrt{23}}{2} = \frac{1}{2}\sqrt{3} \pm \frac{1}{2}\sqrt{23}$$
$x \approx -1.53$ or $x \approx 3.26$

23. $x^2 + 5x - 12 = 0$, so
$a = 1, b = 5, c = -12$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-12)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{73}}{2} = -\frac{5}{2} \pm \frac{\sqrt{73}}{2}$$

$x \approx -6.77$ or $x \approx 1.77$

24. $x^2 - 4x - 32 = 0$, so
$a = 1, b = -4, c = -32$:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-32)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{144}}{2} = 2 \pm 6$$

$x = -4$ or $x = 8$

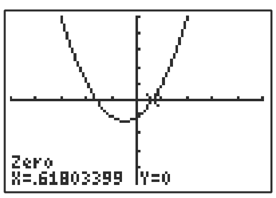
25. x-intercept: 3; y-intercept: -2

26. x-intercepts: 1, 3; y-intercept: 3

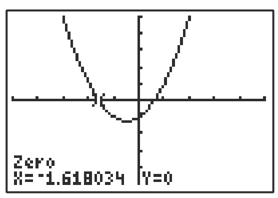
27. x-intercepts: $-2, 0, 2$; y-intercept: 0

28. no x-intercepts; no y-intercepts

29.

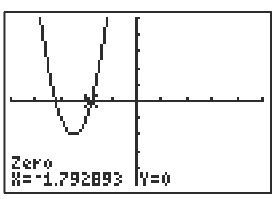


$[-5, 5]$ by $[-5, 5]$

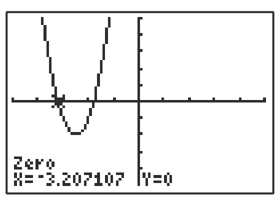


$[-5, 5]$ by $[-5, 5]$

30.

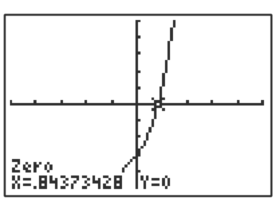


$[-5, 5]$ by $[-5, 5]$



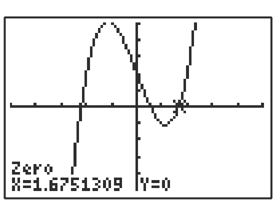
$[-5, 5]$ by $[-5, 5]$

31.

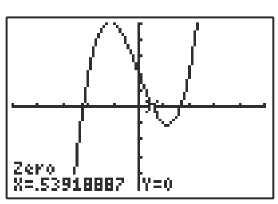


$[-5, 5]$ by $[-5, 5]$

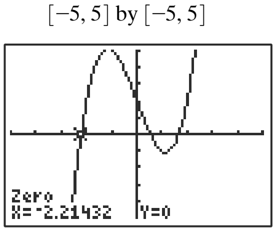
32.



$[-5, 5]$ by $[-5, 5]$

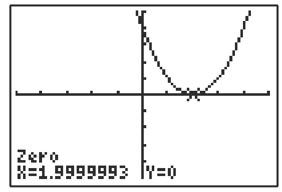


$[-5, 5]$ by $[-5, 5]$



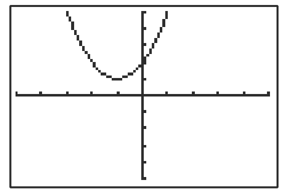
$[-5, 5]$ by $[-5, 5]$

33.



$[-5, 5]$ by $[-5, 5]$

34.


$[-5, 5]$ by $[-5, 5]$

35. $x^2 + 2x - 1 = 0$; $x \approx 0.4$

36. $x^3 - 3x = 0$; $x \approx -1.73$

37. Using TblStart $= 1.61$ and ΔTbl $= 0.001$ gives a zero at 1.62.
Using TblStart $= -0.62$ and ΔTbl $= 0.001$ gives a zero at -0.62.

38. Using TblStart $= 1.32$ and ΔTbl $= 0.001$ gives a zero at 1.32.

39. Graph $y = |x - 8|$ and $y = 2$: $t = 6$ or $t = 10$

40. Graph $y = |x + 1|$ and $y = 4$: $x = -5$ or $x = 3$

41. Graph $y = |2x + 5|$ and $y = 7$: $x = 1$ or $x = -6$

42. Graph $y = |3 - 5x|$ and $y = 4$: $x = -\frac{1}{5}$ or $x = \frac{7}{5}$

43. Graph $y = |2x - 3|$ and $y = x^2$: $x = -3$ or $x = 1$

44. Graph $y = |x + 1|$ and $y = 2x - 3$: $x = 4$

45. **(a)** The two functions are $y_1 = 3\sqrt{x + 4}$ (the one that begins on the x-axis) and $y_2 = x^2 - 1$.

(b) This is the graph of $y = 3\sqrt{x + 4} - x^2 + 1$.

(c) The x-coordinates of the intersections in the first picture are the same as the x-coordinates where the second graph crosses the x-axis.

46. Any number between 1.324 and 1.325 must have the digit 4 in its thousandths position. Such a number would round to 1.32.

47. The left side factors to $(x + 2)(x - 1) = 0$:
$x + 2 = 0$ or $x - 1 = 0$
$x = -2$ $\quad x = 1$

48. Graphing $y = x^2 - 18$ in (e.g.) $[-10, 10] \times [-20, 10]$ and looking for x-intercepts gives $x \approx -4.24$ or $x \approx 4.24$.
$x^2 - 3x = 12 - 3x + 6$
$x^2 - 18 = 0$

49. $2x - 1 = 5$ or $2x - 1 = -5$
$2x = 6$ $\quad 2x = -4$
$x = 3$ $\quad x = -2$

50. $x + 2 = 2\sqrt{x + 3}$
$x^2 + 4x + 4 = 4(x + 3)$
$x^2 = 8$
$x = -\sqrt{8}$ or $x = \sqrt{8}$
$-\sqrt{8}$ is an extraneous solution, $x = \sqrt{8} \approx 2.83$

51. From the graph of $y = x^3 + 4x^2 - 3x - 2$ on $[-10, 10] \times [-10, 10]$, the solutions of the equation (x-intercepts of the graph) are $x \approx -4.56$, $x \approx -0.44$, $x = 1$.

52. From the graph of $y = x^3 - 4x + 2$ on $[-10, 10] \times [-10, 10]$, the solutions of the equation (x-intercepts of the graph) are $x \approx -2.21$, $x \approx 0.54$, and $x \approx 1.68$.

53. $x^2 + 4x - 1 = 7$ or $x^2 + 4x - 1 = -7$

$x^2 + 4x - 8 = 0$ $\qquad$ $x^2 + 4x + 6 = 0$

$x = \dfrac{-4 \pm \sqrt{16 + 32}}{2}$ $\qquad$ $x = \dfrac{-4 \pm \sqrt{16 - 24}}{2}$

$x = -2 \pm 2\sqrt{3}$ $\qquad$ — no real solutions to this equation.

54. Graph $y = |x + 5| - |x - 3|$: $x = -1$

55. Graph $y = |0.5x + 3|$ and $y = x^2 - 4$: $x \approx -2.41$ or $x \approx 2.91$

56. Graph $y = \sqrt{x + 7}$ and $y = -x^2 + 5$: $x \approx -1.64$ or $x \approx 1.45$

57. (a) There must be two distinct real zeros, because $b^2 - 4ac > 0$ implies that $\pm\sqrt{b^2 - 4ac}$ are two distinct real numbers.

(b) There must be one real zero, because $b^2 - 4ac = 0$ implies that $\pm\sqrt{b^2 - 4ac} = 0$, so the root must be $x = -\dfrac{b}{a}$.

(c) There must be no real zeros, because $b^2 - 4ac < 0$ implies that $\pm\sqrt{b^2 - 4ac}$ are not real numbers.

58. For **(a)–(c)**, answers may vary.

(a) $x^2 + 2x - 3$ has discriminant $(2)^2 - 4(1)(-3) = 16$, so it has two distinct real zeros. The graph (or factoring) shows the zeros are at $x = -3$ and $x = 1$.

(b) $x^2 + 2x + 1$ has discriminant $(2)^2 - 4(1)(1) = 0$, so it has one real zero. The graph (or factoring) shows the zero is at $x = -1$.

(c) $x^2 + 2x + 2$ has discriminant $(2)^2 - 4(1)(2) = -4$, so it has no real zeros. The graph lies entirely above the x-axis.

59. Let x be the width of the field (in yd); the length is $x + 30$. Then the field is 80 yd wide and $80 + 30 = 110$ yd long.

$$8800 = x(x + 30)$$
$$0 = x^2 + 30x - 8800$$
$$0 = (x + 110)(x - 80)$$
$$0 = x + 110 \text{ or } 0 = x - 80$$
$$x = -110 \text{ or } x = 80$$

60. Solving $x^2 + (x + 5)^2 = 18^2$, or $2x^2 + 10x - 299 = 0$, gives $x \approx 9.98$ or $x \approx -14.98$. The ladder is about $x + 5 \approx 14.98$ ft up the wall.

61. The area of the square is x^2. The area of the semicircle is $\dfrac{1}{2}\pi r^2 = \dfrac{1}{2}\pi\left(\dfrac{1}{2}x\right)^2$ since the radius of the semicircle is $\dfrac{1}{2}x$. Then $200 = x^2 + \dfrac{1}{2}p\left(\dfrac{1}{2}x\right)^2$. Solving this (graphically is easiest) gives $x \approx 11.98$ ft (since x must be positive).

62. True. If 2 is an x-intercept of the graph of $y = ax^2 + bx + c$, then $y = 0$ when $x = 2$. That is, $ax^2 + bx + c = 0$ when $x = 2$.

63. False. Notice that for $x = -3$, $2x^2 = 2(-3)^2 = 18$. So x could also be -3.

64. $x(x - 3) = 0$ when $x = 0$ and when $x - 3 = 0$ or $x = 3$. The answer is D.

65. For $x^2 - 5x + ?$ to be a perfect square, ? must be replaced by the square of half of -5, which is $\left(-\dfrac{5}{2}\right)^2$. The answer is B.

66. By the quadratic formula, the solutions are

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-1)}}{2(2)} = \frac{3 \pm \sqrt{17}}{4}$$

The answer is B.

67. Since an absolute value cannot be negative, there are no solutions. The answer is E.

68. (a)
$$ax^2 + bx + c = 0$$
$$ax^2 + bx = -c$$
$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

(b)
$$x^2 + \frac{b}{a}x + \left(\frac{1}{2}\cdot\frac{b}{a}\right)^2 = -\frac{c}{a} + \left(\frac{1}{2}\cdot\frac{b}{a}\right)^2$$
$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$
$$\left(x + \frac{b}{2a}\right)\left(x + \frac{b}{2a}\right) = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$$
$$\left(x + \frac{b}{2a}\right) = \frac{b^2 - 4ac}{4a^2}$$

(c)
$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$
$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$
$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

69. Graph $y = |x^2 - 4|$ and $y = c$ for several values of c.

(a) Let $c = 2$. The graph suggests $y = 2$ intersects $y = |x^2 - 4|$ four times.

$|x^2 - 4| = 2 \Rightarrow x^2 - 4 = 2$ or $x^2 - 4 = -2$

$x^2 = 6$ $\qquad$ $x^2 = 2$

$x = \pm\sqrt{6}$ $\qquad$ $x = \pm\sqrt{2}$

$|x^2 - 4| = 2$ has four solutions: $\{\pm\sqrt{2}, \pm\sqrt{6}\}$.

(b) Let $c = 4$. The graph suggests $y = 4$ intersects $y = |x^2 - 4|$ three times.

$|x^2 - 4| = 4 \Rightarrow x^2 - 4 = 4$ or $x^2 - 4 = -4$

$x^2 = 8$ $\qquad$ $x^2 = 0$

$x = \pm\sqrt{8}$ $\qquad$ $x = 0$

(c) Let $c = 5$. The graph suggest $y = 5$ intersects $y = |x^2 - 4|$ twice.

$|x^2 - 4| = 5 \Rightarrow x^2 - 4 = 5$ or $x^2 - 4 = -5$

$x^2 = 9$ $\qquad$ $x^2 = -1$

$x = \pm 3$ $\qquad$ no solution

$|x^2 - 4| = 5$ has two solutions: $\{\pm 3\}$.

(d) Let $c = -1$. The graph suggests $y = -1$ does not intersect $y = |x^2 - 4|$. Since absolute value is never negative, $|x^2 - 4| = -1$ has no solutions.

(e) There is no other possible number of solutions of this equation. For any c, the solution involves solving two quadratic equations, each of which can have 0, 1, or 2 solutions.

70. **(a)** Let $D = b^2 - 4ac$. The two solutions are $\dfrac{-b \pm \sqrt{D}}{2a}$; adding them gives

$$\frac{-b + \sqrt{D}}{2a} + \frac{-b - \sqrt{D}}{2a} = \frac{-2b + \sqrt{D} - \sqrt{D}}{2a} = \frac{-2b}{2a} = -\frac{b}{a}$$

(b) Let $D = b^2 - 4ac$. The two solutions are $\dfrac{-b \pm \sqrt{D}}{2a}$; multiplying them gives

$$\frac{-b + \sqrt{D}}{2a} \cdot \frac{-b - \sqrt{D}}{2a} = \frac{(-b)^2 - (\sqrt{D})^2}{4a^2} = \frac{b^2 - (b^2 - 4ac)}{4a^2} = \frac{c}{a}$$

71. From #70(a), $x_1 + x_2 = -\dfrac{b}{a} = 5$. Since $a = 2$, this means $b = -10$. From #70(b), $x_1 \cdot x_2 = \dfrac{c}{a} = 3$; since $a = 2$, this means $c = 6$. The solutions are $\dfrac{10 \pm \sqrt{100 - 48}}{4}$; this reduces to $2.5 \pm \dfrac{1}{2}\sqrt{13}$, or approximately 0.697 and 4.303.

Section P.6 Complex Numbers

Quick Review P.6

1. $x + 9$
2. $x + 2y$
3. $a + 2d$
4. $5z - 4$
5. $x^2 - x - 6$
6. $2x^2 + 5x - 3$
7. $x^2 - 2$
8. $x^2 - 12$
9. $x^2 - 2x - 1$
10. $x^2 - 4x + 1$

Section P.6 Exercises

In #1–8, add or subtract the real and imaginary parts separately.

1. $(2 - 3i) + (6 + 5i) = (2 + 6) + (-3 + 5)i = 8 + 2i$
2. $(2 - 3i) + (3 - 4i) = (2 + 3) + (-3 - 4)i = 5 - 7i$
3. $(7 - 3i) + (6 - i) = (7 + 6) + (-3 - 1)i = 13 - 4i$
4. $(2 + i) - (9i - 3) = (2 + 3) + (1 - 9)i = 5 - 8i$
5. $(2 - i) + (3 - \sqrt{-3}) = (2 + 3) + (-1 - \sqrt{3})i$ $= 5 - (1 + \sqrt{3})i$
6. $(\sqrt{5} - 3i) + (-2 + \sqrt{-9}) = (\sqrt{5} - 2) + (-3 + 3)i$ $= (\sqrt{5} - 2) + 0i$
7. $(i^2 + 3) - (7 + i^3) = (-1 + 3) - (7 - i)$ $= (2 - 7) + i = -5 + i$
8. $(\sqrt{7} + i^2) - (6 - \sqrt{-81}) = (\sqrt{7} - 1)$ $- (6 - 9i) = (\sqrt{7} - 1 - 6) + 9i = (\sqrt{7} - 7) + 9i$

In #9–16, multiply out and simplify, recalling that $i^2 = -1$.

9. $(2 + 3i)(2 - i) = 4 - 2i + 6i - 3i^2$ $= 4 + 4i + 3 = 7 + 4i$
10. $(2 - i)(1 + 3i) = 2 + 6i - i - 3i^2$ $= 2 + 5i + 3 = 5 + 5i$
11. $(1 - 4i)(3 - 2i) = 3 - 2i - 12i + 8i^2$ $= 3 - 14i - 8 = -5 - 14i$
12. $(5i - 3)(2i + 1) = 10i^2 + 5i - 6i - 3$ $= -10 - i - 3 = -13 - i$
13. $(7i - 3)(2 + 6i) = 14i + 42i^2 - 6 - 18i$ $= -42 - 6 - 4i = -48 - 4i$
14. $(\sqrt{-4} + i)(6 - 5i) = (3i)(6 - 5i) = 18i - 15i^2$ $= 15 + 18i$
15. $(-3 - 4i)(1 + 2i) = -3 - 6i - 4i - 8i^2$ $= -3 - 10i + 8 = 5 - 10i$
16. $(\sqrt{-2} + 2i)(6 + 5i) = (\sqrt{2} + 2)i(6 + 5i)$ $= 6(2 + \sqrt{2})i + 5(2 + \sqrt{2})i^2$ $= -(10 + 5\sqrt{2}) + (12 + 6\sqrt{2})i$
17. $\sqrt{-16} = 4i$
18. $\sqrt{-25} = 5i$
19. $\sqrt{-3} = \sqrt{3}i$
20. $\sqrt{-5} = \sqrt{5}i$

In #21–24, equate the real and imaginary parts.

21. $x = 2, y = 3$
22. $x = 3, y = -7$
23. $x = 1, y = 2$
24. $x = 7, y = -7/2$

In #25–28, multiply out and simplify, recalling that $i^2 = -1$.

25. $(3 + 2i)^2 = 9 + 12i + 4i^2 = 5 + 12i$
26. $(1 - i)^3 = (1 - 2i + i^2)(1 - i) = (-2i)(1 - i)$ $= -2i + 2i^2 = -2 - 2i$
27. $\left(\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i\right)^4 = \left(\dfrac{\sqrt{2}}{2}\right)^4 (1 + i)^4$ $= \dfrac{1}{4}(1 + 2i + i^2)^2 = \dfrac{1}{4}(2i)^2 = \dfrac{1}{4}(-4) = -1 + 0i$
28. $\left(\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i\right)^3 = \left(\dfrac{1}{2}\right)^3 (\sqrt{3} + i)^3$ $= \dfrac{1}{8}(3 + 2\sqrt{3}\,i + i^2)(\sqrt{3} + i)$ $= \dfrac{1}{4}(1 + \sqrt{3}\,i)(\sqrt{3} + i)$ $= \dfrac{1}{4}(\sqrt{3} + i + 3i + \sqrt{3}\,i^2) = \dfrac{1}{4}(4i) = 0 + i$

In #29–32, recall that $(a + bi)(a - bi) = a^2 + b^2$.

29. $2^2 + 3^2 = 13$
30. $5^2 + 6^2 = 61$
31. $3^2 + 4^2 = 25$
32. $1^2 + (\sqrt{2})^2 = 3$

In #33–40, multiply both the numerator and denominator by the complex conjugate of the denominator, recalling that $(a + bi)(a - bi) = a^2 + b^2$.

33. $\frac{1}{2+i} \cdot \frac{2-i}{2-i} = \frac{2-i}{5} = \frac{2}{5} - \frac{1}{5}i$

34. $\frac{i}{2-i} \cdot \frac{2+i}{2+i} = \frac{2i+i^2}{5} = -\frac{1}{5} + \frac{2}{5}i$

35. $\frac{2+i}{2-i} \cdot \frac{2+i}{2+i} = \frac{4+4i+i^2}{5} = \frac{3}{5} + \frac{4}{5}i$

36. $\frac{2+i}{3i} \cdot \frac{-3i}{-3i} = \frac{-6i-3i^2}{9} = \frac{1}{3} - \frac{2}{3}i$

37. $\frac{(2+i)^2(-i)}{1+i} \cdot \frac{1-i}{1-i} = \frac{(4+4i+i^2)(-i+i^2)}{2}$

$= \frac{(3+4i)(-1-i)}{2} = \frac{-3-3i-4i-4i^2}{2} = \frac{1}{2} - \frac{7}{2}i$

38. $\frac{(2-i)(1+2i)}{5+2i} \cdot \frac{5-2i}{5-2i} = \frac{(2+4i-i-2i^2)(5-2i)}{29}$

$= \frac{(4+3i)(5-2i)}{29} = \frac{20-8i+15i-6i^2}{29}$

$= \frac{26}{29} + \frac{7}{29}i$

39. $\frac{(1-i)(2-i)}{1-2i} \cdot \frac{1+2i}{1+2i} = \frac{(2-i-2i+i^2)(1+2i)}{5}$

$= \frac{(1-3i)(1+2i)}{5} = \frac{1+2i-3i-6i^2}{5} = \frac{7}{5} - \frac{1}{5}i$

40. $\frac{(1-\sqrt{2}i)(1+i)}{1+\sqrt{2}i} \cdot \frac{1-\sqrt{2}i}{1-\sqrt{2}i}$

$= \frac{(1+i-\sqrt{2}i-\sqrt{2}i^2)(1-\sqrt{2}i)}{3}$

$= \frac{[1+\sqrt{2}+(1-\sqrt{2})i](1-\sqrt{2}i)}{3}$

$= \frac{1+\sqrt{2}-(\sqrt{2}+2)i+(1-\sqrt{2})i-(\sqrt{2}-2)i^2}{3}$

$= \frac{1+\sqrt{2}+\sqrt{2}-2+(-2\sqrt{2}-1)i}{3}$

$= \frac{2\sqrt{2}-1}{3} - \frac{2\sqrt{2}+1}{3}i$

In #41–44, use the quadratic formula.

41. $x = -1 \pm 2i$

42. $x = -\frac{1}{6} \pm \frac{\sqrt{23}}{6}i$

43. $x = \frac{7}{8} \pm \frac{\sqrt{15}}{8}i$

44. $x = 2 \pm \sqrt{15}i$

45. False. When $a = 0$, $z = a + bi$ becomes $z = bi$, and then $-\bar{z} = -(-bi) = bi = z$.

46. True. Because $i^2 = -1$, $i^3 = i(i^2) = -i$, and $i^4 = (i^2)^2 = 1$, we obtain $i + i^2 + i^3 + i^4 = i + (-1) + (-i) + 1 = 0$.

47. $(2 + 3i)(2 - 3i)$ is a product of conjugates and equals $2^2 + 3^2 = 13 + 0i$. The answer is E.

48. $\frac{1}{i} = \frac{1}{i} \cdot \frac{-i}{-i} = \frac{-i}{1} = -1 + 0i$. The answer is E.

49. Complex, nonreal solutions of polynomials with real coefficients always come in conjugate pairs. So another solution is $2 + 3i$, and the answer is A.

50. $(1 - i)^3 = (-2i)(1 - i) = -2i + 2i^2 = -2 - 2i$. The answer is C.

51. **(a)** $i = i$ $\quad i^5 = i \cdot i^4 = i$

$i^2 = -1$ $\quad i^6 = i^2 \cdot i^4 = -1$

$i^3 = (-1)i = -i$ $\quad i^7 = i^3 \cdot i^4 = -i$

$i^4 = (-1)^2 = 1$ $\quad i^8 = i^4 \cdot i^4 = 1 \cdot 1 = 1$

(b) $i^{-1} = \frac{1}{i} = \frac{1}{i} \cdot \frac{i}{i} = -i$ $\quad i^{-5} = \frac{1}{i} \cdot \frac{1}{i^4} = \frac{1}{i} = -i$

$i^{-2} = \frac{1}{i^2} = -1$ $\quad i^{-6} = \frac{1}{i^2} \cdot \frac{1}{i^4} = -1$

$i^{-3} = \frac{1}{i} \cdot \frac{1}{i^2} = -\frac{1}{i} = i$ $\quad i^{-7} = \frac{1}{i^3} \cdot \frac{1}{i^4} = -\frac{1}{i} = i$

$i^{-4} = \frac{1}{i^2} \cdot \frac{1}{i^2} = (-1)(-1) = 1$ $\quad i^{-8} = \frac{1}{i^4} \cdot \frac{1}{i^4} = 1 \cdot 1 = 1$

(c) $i^0 = 1$

(d) Answers will vary.

52. Answers will vary. One possibility: The graph has the shape of a parabola, but does not cross the x-axis when plotted in the real plane, beacuse it does not have any real zeros. As a result, the function will *always* be positive or *always* be negative.

53. Let a and b be any two real numbers. Then $(a + bi) - (a - bi) = (a - a) + (b + b)i = 0 + 2bi = 2bi$.

54. $(a + bi)\overline{(a + bi)} = (a + bi)(a - bi) = a^2 + b^2$, imaginary part is zero.

55. $\overline{(a + bi)(c + di)} = \overline{(ac - bd) + (ad + bc)i} = (ac - bd) - (ad + bc)i$ and $\overline{(a + bi)}\,\overline{(c + di)} = (a - bi) \cdot (c - di) = (ac - bd) - (ad + bc)i$ are equal.

56. $\overline{(a + bi) + (c + di)} = \overline{(a + c) + (b + d)i} = (a + c) - (b + d)i$ and $\overline{(a + bi)} + \overline{(c + di)} = (a - bi) + (c - di) = (a + c) - (b + d)i$ are equal.

57. $(-i)^2 - i(-i) + 2 = 0$ but $(i)^2 - i(i) + 2 \neq 0$. Because the coefficient of x in $x^2 - ix + 2 = 0$ is not a real number, the complex conjugate, i, of $-i$, need not be a solution.

■ Section P.7 Solving Inequalities Algebraically and Graphically

Quick Review P.7

1. $-7 < 2x - 3 < 7$

$-4 < 2x < 10$

$-2 < x < 5$

2. $5x - 2 \geq 7x + 4$

$-2x \geq 6$

$x \leq -3$

3. $|x + 2| = 3$

$x + 2 = 3$ or $x + 2 = -3$

$x = 1$ or $x = -5$

4. $4x^2 - 9 = (2x - 3)(2x + 3)$

5. $x^3 - 4x = x(x^2 - 4) = x(x - 2)(x + 2)$

6. $9x^2 - 16y^2 = (3x - 4y)(3x + 4y)$

7. $\dfrac{z^2 - 25}{z^2 - 5z} = \dfrac{(z - 5)(z + 5)}{z(z - 5)} = \dfrac{z + 5}{z}$

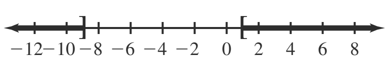
8. $\dfrac{x^2 + 2x - 35}{x^2 - 10x + 25} = \dfrac{(x + 7)(x - 5)}{(x - 5)(x - 5)} = \dfrac{x + 7}{x - 5}$

9. $\dfrac{x}{x - 1} + \dfrac{x + 1}{3x - 4}$
$= \dfrac{x(3x - 4)}{(x - 1)(3x - 4)} + \dfrac{(x + 1)(x - 1)}{(x - 1)(3x - 4)}$
$= \dfrac{4x^2 - 4x - 1}{(x - 1)(3x - 4)}$

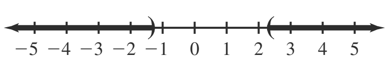
10. $\dfrac{2x - 1}{(x - 2)(x + 1)} + \dfrac{x - 3}{(x - 2)(x - 1)}$
$= \dfrac{(2x - 1)(x - 1) + (x - 3)(x + 1)}{(x - 2)(x + 1)(x - 1)}$
$= \dfrac{(2x^2 - 3x + 1) + (x^2 - 2x - 3)}{(x - 2)(x + 1)(x - 1)}$
$= \dfrac{3x^2 - 5x - 2}{(x - 2)(x + 1)(x - 1)} = \dfrac{(3x + 1)(x - 2)}{(x - 2)(x + 1)(x - 1)}$
if $x \neq 2$ $= \dfrac{(3x + 1)}{(x + 1)(x - 1)}$

Section P.7 Exercises

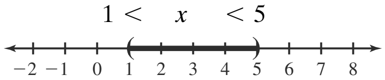
1. $(-\infty, -9] \cup [1, \infty)$:
$x + 4 \geq 5$ or $x + 4 \leq -5$
$x \geq 1$ or $x \leq -9$



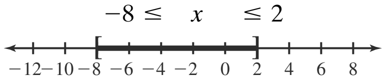
2. $(-\infty, -1.3) \cup (2.3, \infty)$:
$2x - 1 > 3.6$ or $2x - 1 < -3.6$
$2x > 4.6$ or $2x < -2.6$
$x > 2.3$ or $x < -1.3$



3. $(1, 5)$: $-2 < x - 3 < 2$
$1 < x < 5$

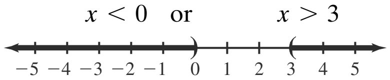


4. $[-8, 2]$: $-5 \leq x + 3 \leq 5$
$-8 \leq x \leq 2$

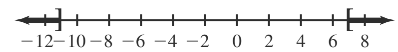


5. $\left(-\dfrac{2}{3}, \dfrac{10}{3}\right)$: $|4 - 3x| < 6$
$-6 < 4 - 3x < 6$
$-10 < -3x < 2$
$\dfrac{10}{3} > x > -\dfrac{2}{3}$

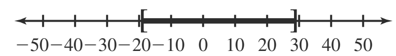
6. $(-\infty, 0) \cup (3, \infty)$: $|3 - 2x| > 3$
$3 - 2x > 3$ or $3 - 2x < -3$
$-2x > 0$ or $-2x < -6$
$x < 0$ or $x > 3$



7. $(-\infty, -11] \cup [7, \infty)$:
$\dfrac{x + 2}{3} \leq -3$ or $\dfrac{x + 2}{3} \geq 3$
$x + 2 \leq -9$ or $x + 2 \geq 9$
$x \leq -11$ or $x \geq 7$



8. $[-19, 29]$: $\left|\dfrac{x - 5}{4}\right| \leq 6$
$-6 \leq \dfrac{x - 5}{4} \leq 6$
$-24 \leq x - 5 \leq 24$
$-19 \leq x \leq 29$



9. $2x^2 + 17x + 21 = 0$
$(2x + 3)(x + 7) = 0$
$2x + 3 = 0$ or $x + 7 = 0$
$x = -\dfrac{3}{2}$ or $x = -7$

The graph of $y = 2x^2 + 17x + 21$ lies below the x-axis for $-7 < x < -\dfrac{3}{2}$. Hence $\left[-7, -\dfrac{3}{2}\right]$ is the solution since the endpoints are included.

10. $6x^2 - 13x + 6 = 0$
$(2x - 3)(3x - 2) = 0$
$2x - 3 = 0$ or $3x - 2 = 0$
$x = \dfrac{3}{2}$ or $x = \dfrac{2}{3}$

The graph of $y = 6x^2 - 13x + 6$ lies above the x-axis for $x < \dfrac{2}{3}$ and for $x > \dfrac{3}{2}$. Hence $\left(-\infty, \dfrac{2}{3}\right] \cup \left[\dfrac{3}{2}, \infty\right)$ is the solution since the endpoints are included.

11. $2x^2 + 7x - 15 = 0$
$(2x - 3)(x + 5) = 0$
$2x - 3 = 0$ or $x + 5 = 0$
$x = \dfrac{3}{2}$ or $x = -5$

The graph of $y = 2x^2 + 7x - 15$ lies above the x-axis for $x < -5$ and for $x > \dfrac{3}{2}$. Hence $(-\infty, -5) \cup \left(\dfrac{3}{2}, \infty\right)$ is the solution.

12. $4x^2 - 9x + 2 = 0$
$(4x - 1)(x - 2) = 0$
$4x - 1 = 0$ or $x - 2 = 0$
$x = \dfrac{1}{4}$ or $x = 2$

The graph of $y = 4x^2 - 9x + 2$ lies below the x-axis for $\dfrac{1}{4} < x < 2$. Hence $\left(\dfrac{1}{4}, 2\right)$ is the solution.

13. $2 - 5x - 3x^2 = 0$
$(2 + x)(1 - 3x) = 0$
$2 + x = 0$ or $1 - 3x = 0$
$x = -2$ or $x = \dfrac{1}{3}$

The graph of $y = 2 - 5x - 3x^2$ lies below the x-axis for $x < -2$ and for $x > \dfrac{1}{3}$. Hence $(-\infty, -2) \cup \left(\dfrac{1}{3}, \infty\right)$ is the solution.

14.
$$21 + 4x - x^2 = 0$$
$$(7 - x)(3 + x) = 0$$
$$7 - x = 0 \quad \text{or} \quad 3 + x = 0$$
$$x = 7 \quad \text{or} \quad x = -3$$
The graph of $y = 21 + 4x - x^2$ lies above the x-axis for $-3 < x < 7$. Hence $(-3, 7)$ is the solution.

15.
$$x^3 - x = 0$$
$$x(x^2 - 1) = 0$$
$$x(x + 1)(x - 1) = 0$$
$$x = 0 \quad \text{or} \quad x + 1 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = 0 \quad \text{or} \quad x = -1 \quad \text{or} \quad x = 1$$
The graph of $y = x^3 - x$ lies above the x-axis for $x > 1$ and for $-1 < x < 0$. Hence $[-1, 0] \cup [1, \infty)$ is the solution.

16.
$$x^3 - x^2 - 30x = 0$$
$$x(x^2 - x - 30) = 0$$
$$x(x - 6)(x + 5) = 0$$
$$x = 0 \quad \text{or} \quad x - 6 = 0 \quad \text{or} \quad x + 5 = 0$$
$$x = 0 \quad \text{or} \quad x = 6 \quad \text{or} \quad x = -5$$
The graph of $y = x^3 - x^2 - 30x$ lies below the x-axis for $x < -5$ and for $0 < x < 6$. Hence $(-\infty, -5] \cup [0, 6]$ is the solution.

17. The graph of $y = x^2 - 4x - 1$ is zero for $x \approx -0.24$ and $x \approx 4.24$, and lies below the x-axis for $-0.24 < x < 4.24$. Hence $(-0.24, 4.24)$ is the approximate solution.

18. The graph of $y = 12x^2 - 25x + 12$ is zero for $x = \frac{4}{3}$ and $x = \frac{3}{4}$ and lies above the x-axis for $x < \frac{3}{4}$ and for $x > \frac{4}{3}$. Hence $\left(-\infty, \frac{3}{4}\right] \cup \left[\frac{4}{3}, \infty\right)$ is the solution.

19.
$$6x^2 - 5x - 4 = 0$$
$$(3x - 4)(2x + 1) = 0$$
$$3x - 4 = 0 \quad \text{or} \quad 2x + 1 = 0$$
$$x = \frac{4}{3} \quad \text{or} \quad x = -\frac{1}{2}$$
The graph of $y = 6x^2 - 5x - 4$ lies above the x-axis for $x < -\frac{1}{2}$ and for $x > \frac{4}{3}$. Hence $\left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{4}{3}, \infty\right)$ is the solution.

20.
$$4x^2 - 1 = 0$$
$$(2x + 1)(2x - 1) = 0$$
$$2x + 1 = 0 \quad \text{or} \quad 2x - 1 = 0$$
$$x = -\frac{1}{2} \quad \text{or} \quad x = \frac{1}{2}$$
The graph of $y = 4x^2 - 1$ lies below the x-axis for $-\frac{1}{2} < x < \frac{1}{2}$. Hence $\left[-\frac{1}{2}, \frac{1}{2}\right]$ is the solution.

21. The graph of $y = 9x^2 + 12x - 1$ appears to be zero for $x \approx -1.41$ and $x \approx 0.08$ and lies above the x-axis for $x < -1.41$ and $x > 0.08$. Hence $(-\infty, -1.41] \cup [0.08, \infty)$ is the approximate solution.

22. The graph of $y = 4x^2 - 12x + 7$ appears to be zero for $x \approx 0.79$ and $x \approx 2.21$ and lies below the x-axis for $0.79 < x < 2.21$. Hence $(0.79, 2.21)$ is the approximate solution.

23.
$$4x^2 - 4x + 1 = 0$$
$$(2x - 1)(2x - 1) = 0$$
$$(2x - 1)^2 = 0$$
$$2x - 1 = 0$$
$$x = \frac{1}{2}$$
The graph of $y = 4x^2 - 4x + 1$ lies entirely above the x-axis, except at $x = \frac{1}{2}$. Hence $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$ is the solution set.

24.
$$x^2 - 6x + 9 = 0$$
$$(x - 3)(x - 3) = 0$$
$$(x - 3)^2 = 0$$
$$x - 3 = 0$$
$$x = 3$$
The graph of $y = x^2 - 6x + 9$ lies entirely above the x-axis, except at $x = 3$. Hence $x = 3$ is the only solution.

25.
$$x^2 - 8x + 16 = 0$$
$$(x - 4)(x - 4) = 0$$
$$(x - 4)^2 = 0$$
$$x - 4 = 0$$
$$x = 4$$
The graph of $y = x^2 - 8x + 16$ lies entirely above the x-axis, except at $x = 4$. Hence there is no solution.

26.
$$9x^2 + 12x + 4 = 0$$
$$(3x + 2)(3x + 2) = 0$$
$$(3x + 2)^2 = 0$$
$$3x + 2 = 0$$
$$x = -\frac{2}{3}$$
The graph of $y = 9x^2 + 12x + 4$ lies entirely above the x-axis, except at $x = -\frac{2}{3}$. Hence every real number satisfies the inequality. The solution is $(-\infty, \infty)$.

27. The graph of $y = 3x^3 - 12x + 2$ is zero for $x \approx -2.08$, $x \approx 0.17$, and $x \approx 1.91$ and lies above the x-axis for $-2.08 < x < 0.17$ and $x > 1.91$. Hence, $[-2.08, 0.17] \cup [1.91, \infty)$ is the approximate solution.

28. The graph of $y = 8x - 2x^3 - 1$ is zero for $x \approx -2.06$, $x \approx 0.13$, and $x \approx 1.93$ and lies below the x-axis for $-2.06 < x < 0.13$ and $x > 1.93$. Hence, $(-2.06, 0.13) \cup (1.93, \infty)$ is the approximate solution.

29. $2x^3 + 2x > 5$ is equivalent to $2x^3 + 2x - 5 > 0$. The graph of $y = 2x^3 + 2x - 5$ is zero for $x \approx 1.11$ and lies above the x-axis for $x > 1.11$. So, $(1.11, \infty)$ is the approximate solution.

30. $4 \le 2x^3 + 8x$ is equivalent to $2x^3 + 8x - 4 \ge 0$. The graph of $y = 2x^3 + 8x - 4$ is zero for $x \approx 0.47$ and lies above the x-axis for $x > 0.47$. So, $[0.47, \infty)$ is the approximate solution.

31. Answers may vary. Here are some possibilities.

(a) $x^2 + 1 > 0$

(b) $x^2 + 1 < 0$

(c) $x^2 \le 0$

(d) $(x + 2)(x - 5) \le 0$

(e) $(x + 1)(x - 4) > 0$

(f) $x(x - 4) \ge 0$

32. $-16t^2 + 288t - 1152 = 0$
$t^2 - 18t + 72 = 0$
$(t - 6)(t - 12) = 0$
$t - 6 = 0$ or $t - 12 = 0$
$t = 6$ or $t = 12$
The graph of $-16t^2 + 288t - 1152$ lies above the t-axis for $6 < t < 12$. Hence $[6, 12]$ is the solution. This agrees with the result obtained in Example 10.

33. $s = -16t^2 + 256t$

(a) $-16t^2 + 256t = 768$
$-16t^2 + 256t - 768 = 0$
$t^2 - 16t + 48 = 0$
$(t - 12)(t - 4) = 0$
$t - 12 = 0$ or $t - 4 = 0$
$t = 12$ or $t = 4$
The projectile is 768 ft above ground twice: at $t = 4$ sec, on the way up, and $t = 12$ sec, on the way down.

(b) The graph of $s = -16t^2 + 256t$ lies above the graph of $s = 768$ for $4 < t < 12$. Hence the projectile's height will be at least 768 ft when t is in the interval $[4, 12]$.

(c) The graph of $s = -16t^2 + 256t$ lies below the graph of $s = 768$ for $0 < t < 4$ and $12 < t < 16$. Hence the projectile's height will be less than or equal to 768 ft when t is in the interval $(0, 4]$ or $[12, 16)$.

34. $s = -16t^2 + 272t$

(a) $-16t^2 + 272t = 960$
$-16t^2 + 272t - 960 = 0$
$t^2 - 17t + 60 = 0$
$(t - 12)(t - 5) = 0$
$t - 12 = 0$ or $t - 5 = 0$
$t = 12$ or $t = 5$
The projectile is 960 ft above ground twice: at $t = 5$ sec, on the way up, and $t = 12$ sec, on the way down.

(b) The graph of $s = -16t^2 + 272t$ lies above the graph of $s = 960$ for $5 < t < 12$. Hence the projectile's height will be more than 960 ft when t is in the interval $(5, 12)$.

(c) The graph of $s = -16t^2 + 272t$ lies below the graph of $s = 960$ for $0 < t < 5$ and $12 < t < 17$. Hence the projectile's height will be less than or equal to 960 ft when t is in the interval $(0, 5]$ or $[12, 17)$.

35. Solving the corresponding equation in the process of solving an inequality reveals the boundaries of the solution set. For example, to solve the inequality $x^2 - 4 \le 0$, we first solve the corresponding equation $x^2 - 4 = 0$ and find that $x = \pm 2$. The solution, $[-2, 2]$, of inequality has ± 2 as its boundaries.

36. Let x be her average speed; then $105 < 2x$. Solving this gives $x > 52.5$, so her least average speed is 52.5 mph.

37. (a) Let $x > 0$ be the width of a rectangle; then the length is $2x - 2$ and the perimeter is $P = 2[x + (2x - 2)]$. Solving $P < 200$ and $2x - 2 > 0$ gives 1 in. $< x <$ 34 in.
$2[x + (2x - 2)] < 200$ and $2x - 2 > 0$
$2(3x - 2) < 200$ $\quad 2x > 2$
$6x - 4 < 200$ $\quad x > 1$
$6x < 204$
$x < 34$

(b) The area is $A = x(2x - 2)$. We already know $x > 1$ from **(a)**. Solve $A \le 1200$.
$x(2x - 2) = 1200$
$2x^2 - 2x - 1200 = 0$
$x^2 - x - 600 = 0$
$(x - 25)(x + 24) = 0$
$x - 25 = 0$ or $x + 24 = 0$
$x = 25$ or $x = -24$
The graph of $y = 2x^2 - 2x - 1200$ lies below the x-axis for $1 < x < 25$, so $A \le 1200$ when x is in the interval $(1, 25]$.

38. Substitute 20 and 40 into the equation $P = \frac{400}{V}$ to find the range for P: $P = \frac{400}{20} = 20$ and $P = \frac{400}{40} = 10$. The pressure can range from 10 to 20, or $10 \le P \le 20$. Alternatively, solve graphically: graph $y = \frac{400}{x}$ on $[20, 40] \times [0, 30]$ and observe that all y-values are between 10 and 20.

39. Let x be the amount borrowed; then $\frac{200{,}000 + x}{50{,}000 + x} \ge 2$. Solving for x reveals that the company can borrow no more than \$100,000.

40. False. If b is negative, there are no solutions, because the absolute value of a number is always nonnegative and every nonnegative real number is greater than any negative real number.

41. True. The absolute value of any real number is always nonnegative, i.e., greater than or equal to zero.

42. $|x - 2| < 3$
$-3 < x - 2 < 3$
$-1 < x < 5$
$(-1, 5)$
The answer is E.

43. The graph of $y = x^2 - 2x + 2$ lies entirely above the x-axis, so $x^2 - 2x + 2 \ge 0$ for all real numbers x. The answer is D.

44. $x^2 > x$ is true for all negative x, and for positive x when $x > 1$. So the solution is $(-\infty, 0) \cup (1, \infty)$. The answer is A.

45. $x^2 \le 1$ implies $-1 \le x \le 1$, so the solution is $[-1, 1]$. The answer is D.

46. (a) The lengths of the sides of the box are x, $12 - 2x$, and $15 - 2x$, so the volume is $x(12 - 2x)(15 - 2x)$. To solve $x(12 - 2x)(15 - 2x) = 125$, graph $y = x(12 - 2x)(15 - 2x)$ and $y = 125$ and find where the graphs intersect: Either $x \approx 0.94$ in or $x \approx 3.78$ in.

(b) The graph of $y = x(12 - 2x)(15 - 2x)$ lies above the graph of $y = 125$ for $0.94 < y < 3.78$ (approximately). So choosing x in the interval (0.94, 3.78) will yield a box with volume greater than 125 in^3.

(c) The graph of $y = x(12 - 2x)(15 - 2x)$ lies below the graph of $y = 125$ for $0 < y < 0.94$ and for $3.78 < x < 6$ (approximately). So choosing x in either interval $(0, 0.94)$ or interval $(3.78, 6)$ will yield a box with volume at most 125 in^3.

47. $2x^2 + 7x - 15 = 10$ or $2x^2 + 7x - 15 = -10$
$2x^2 + 7x - 25 = 0$ $\quad$ $2x^2 + 7x - 5 = 0$
The graph of $y = 2x^2 + 7x - 25$ appears to be zero for $x \approx -5.69$ and $x \approx 2.19$. The graph of $y = 2x^2 + 7x - 5$ appears to be zero for $x \approx -4.11$ and $x \approx 0.61$
Now look at the graphs of $y = |2x^2 + 7x - 15|$ and $y = 10$. The graph of $y = |2x^2 + 7x - 15|$ lies below the graph of $y = 10$ when $-5.69 < x < -4.11$ and when $0.61 < x < 2.19$. Hence $(-5.69, -4.11) \cup (0.61, 2.19)$ is the approximate solution.

48. $2x^2 + 3x - 20 = 10$ or $2x^2 + 3x - 20 = -10$
$2x^2 + 3x - 30 = 0$ $\quad$ $2x^2 + 3x - 10 = 0$
The graph of $y = 2x^2 + 3x - 30$ appears to be zero for $x \approx -4.69$ and $x \approx 3.19$. The graph of $y = 2x^2 + 3x - 10$ appears to be zero for $x \approx -3.11$ and $x \approx 1.61$
Now look at the graphs of $y = |2x^2 + 3x - 20|$ and $y = 10$. The graph of $y = |2x^2 + 7x - 20|$ lies above the graph of $y = 10$ when $x < -4.69, -3.11 < x < 1.61$, and $x > 3.19$. Hence $(-\infty, -4.69] \cup [-3.11, 1.61] \cup [3.19, \infty)$ is the (approximate) solution.

■ Chapter P Review

1. Endpoints 0 and 5; bounded

2. Endpoint 2; unbounded

3. $2(x^2 - x) = 2x^2 - 2x$

4. $2x^3 + 4x^2 = 2x^2 \cdot x + 2x^2 \cdot 2 = 2x^2(x + 2)$

5. $\dfrac{(uv^2)^3}{v^2u^3} = \dfrac{u^3v^6}{u^3v^2} = v^4$

6. $(3x^2y^3)^{-2} = \dfrac{1}{(3x^2y^3)^2} = \dfrac{1}{3^2(x^2)^2(y^3)^2} = \dfrac{1}{9x^4y^6}$

7. 3.68×10^9

8. 7×10^{-6}

9. 5,000,000,000

10. 0.000 000 000 000 000 000 000 000 000 910 94
(27 zeros between the decimal point and the first 9)

11. **(a)** 1.45×10^{13}
(b) 5.456×10^8
(c) 1.15×10^{10}
(d) 9.7×10^7
(e) 7.1×10^9

12. $-0.\overline{45}$ (repeating)

13. **(a)** Distance: $|14 - (-5)| = |19| = 19$
(b) Midpoint: $\dfrac{-5 + 14}{2} = \dfrac{9}{2} = 4.5$

14. **(a)** Distance:
$\sqrt{[5 - (-4)]^2 + (-1 - 3)^2} = \sqrt{9^2 + (-4)^2}$
$= \sqrt{81 + 16} = \sqrt{97} \approx 9.85$
(b) Midpoint:
$\left(\dfrac{-4 + 5}{2}, \dfrac{3 + (-1)}{2}\right) = \left(\dfrac{1}{2}, \dfrac{2}{2}\right) = \left(\dfrac{1}{2}, 1\right)$

15. The three side lengths (distances between pairs of points) are
$\sqrt{[3 - (-2)]^2 + (11 - 1)^2} = \sqrt{5^2 + 10^2}$
$= \sqrt{25 + 100} = \sqrt{125} = 5\sqrt{5}$
$\sqrt{(7 - 3)^2 + (9 - 11)^2} = \sqrt{4^2 + (-2)^2}$
$= \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$
$\sqrt{[7 - (-2)]^2 + (9 - 1)^2} = \sqrt{9^2 + 8^2}$
$= \sqrt{81 + 64} = \sqrt{145}$.
Since $(2\sqrt{5})^2 + (5\sqrt{5})^2 = 20 + 125 = 145 = (\sqrt{145})^2$ — the sum of the squares of the two shorter side lengths equals the square of the long side length — the points determine a right triangle.

16. The three side lengths (distances between pairs of points) are
$\sqrt{(4 - 0)^2 + (1 - 1)^2} = \sqrt{4^2 + 0^2} = \sqrt{16} = 4$
$\sqrt{(2 - 0)^2 + [(1 - 2\sqrt{3}) - 1]^2} = \sqrt{2^2 + (-2\sqrt{3})^2}$
$= \sqrt{4 + 12} = \sqrt{16} = 4$
$\sqrt{(4 - 2)^2 + [1 - (1 - 2\sqrt{3})]^2\}} = \sqrt{2^2 + (2\sqrt{3})^2}$
$= \sqrt{4 + 12} = \sqrt{16} = 4$.
Since all three sides have the same length, the figure is an equilateral triangle.

17. $(x - 0)^2 + (y - 0)^2 = 2^2$, or $x^2 + y^2 = 4$

18. $(x - 5)^2 + [y - (-3)]^2 = 4^2$, or
$(x - 5)^2 + (y + 3)^2 = 16$

19. $[x - (-5)]^2 + [y - (-4)]^2 = 3^2$, so the center is $(-5, -4)$ and the radius is 3.

20. $(x - 0)^2 + (y - 0)^2 = 1^2$, so the center is $(0, 0)$ and the radius is 1.

21. **(a)** Distance between $(-3, 2)$ and $(-1, -2)$:
$\sqrt{(-2 - 2)^2 + [-1 - (-3)]^2} = \sqrt{(-4)^2 + (2)^2}$
$= \sqrt{16 + 4} = \sqrt{20} \approx 4.47$
Distance between $(-3, 2)$ and $(5, 6)$:
$\sqrt{(6 - 2)^2 + [5 - (-3)]^2} = \sqrt{4^2 + 8^2}$
$= \sqrt{16 + 64} = \sqrt{80} \approx 8.94$
Distance between $(5, 6)$ and $(-1, -2)$:
$\sqrt{(-2 - 6)^2 + (-1 - 5)^2} = \sqrt{(-8)^2 + (-6)^2}$
$= \sqrt{64 + 36} = \sqrt{100} = 10$
(b) $(\sqrt{20})^2 + (\sqrt{80})^2 = 20 + 80 = 100 = 10^2$, so the Pythagorean Theorem guarantees the triangle is a right triangle.

22. $|z - (-3)| \le 1$, or $|z + 3| \le 1$

23. $\dfrac{-1 + a}{2} = 3$ and $\dfrac{1 + b}{2} = 5$
$-1 + a = 6 \qquad 1 + b = 10$
$a = 7 \qquad b = 9$

24. $m = \dfrac{-5 + 2}{4 + 1} = -\dfrac{3}{5}$

25. $y + 1 = -\dfrac{2}{3}(x - 2)$

26. The slope is $m = -\frac{9}{7} = -\frac{A}{B}$, so we can choose $A = 9$ and $B = 7$. Since $x = -5$, $y = 4$ solves $9x + 7y + C = 0$, C must equal 17: $9x + 7y + 17 = 0$. Note that the coefficients can be multiplied by any nonzero number, e.g., another answer would be $18x + 14y + 34 = 0$.

27. Beginning with point-slope form: $y + 2 = \frac{4}{5}(x - 3)$, so $y = \frac{4}{5}x - 4.4$.

28. $m = \frac{2 + 4}{3 + 1} = \frac{3}{2}$, so in point-slope form, $y + 4 = \frac{3}{2}(x + 1)$, and therefore $y = \frac{3}{2}x - \frac{5}{2}$.

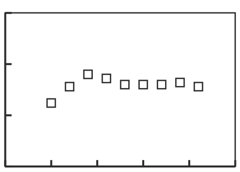
29. $y = 4$

30. Solve for y: $y = \frac{3}{4}x - \frac{7}{4}$.

31. The slope of the given line is the same as the line we want: $m = -\frac{2}{5}$, so $y + 3 = -\frac{2}{5}(x - 2)$, and therefore $y = -\frac{2}{5}x - \frac{11}{5}$.

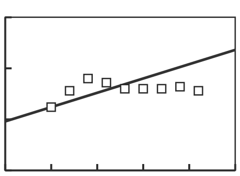
32. The slope of the given line is $-\frac{2}{5}$, so the slope of the line we seek is $m = \frac{5}{2}$. Then $y + 3 = \frac{5}{2}(x - 2)$, and therefore $y = \frac{5}{2}x - 8$.

33. **(a)**



[1990, 2015] by [475, 550]

(b) Slope of the line between the points (1995, 506) and (2005, 520) is $m = \frac{520 - 506}{10} = \frac{14}{10} = 1.4$.
Using the point-slope form equation for the line, we have $y - 506 = 1.4(x - 1995)$, so $y = 1.4(x - 1995) + 506$.



[1990, 2015] by [475, 550]

(c) Using $y = 1.4(x - 1995) + 506$ and $x = 2015$, the model estimates the average SAT score in 2007 was 534.

(d) No; the superimposed graphs in **(b)** show that data for 2005–2011 do not follow the pattern for 1995–2005, so it is unlikely that the average SAT math score in 2015 will be nearly as high as 534.

34. **(a)** $4x - 3y = -33$, or $y = \frac{4}{3}x + 11$

(b) $3x + 4y = -6$, or $y = -\frac{3}{4}x - \frac{3}{2}$

35. $m = \frac{25}{10} = \frac{5}{2} = 2.5$

36. Both graphs look the same, but the graph on the left has slope $\frac{2}{3}$ — less than the slope of the one on the right, which is $\frac{12}{15} = \frac{4}{5}$. The different horizontal and vertical scales for the two windows make it difficult to judge by looking at the graphs.

37.
$$\begin{aligned} 3x - 4 &= 6x + 5 \\ -3x &= 9 \\ x &= -3 \end{aligned}$$

38.
$$\begin{aligned} \frac{x-2}{3} + \frac{x+5}{2} &= \frac{1}{3} \\ 2(x - 2) + 3(x + 5) &= 2 \\ 2x - 4 + 3x + 15 &= 2 \\ 5x + 11 &= 2 \\ 5x &= -9 \\ x &= -\frac{9}{5} \end{aligned}$$

39.
$$\begin{aligned} 2(5 - 2y) - 3(1 - y) &= y + 1 \\ 10 - 4y - 3 + 3y &= y + 1 \\ 7 - y &= y + 1 \\ -2y &= -6 \\ y &= 3 \end{aligned}$$

40.
$$\begin{aligned} &3(3x - 1)^2 = 21 \\ &(3x - 1)^2 = 7 \\ &3x - 1 = \pm\sqrt{7} \\ &3x - 1 = -\sqrt{7} \quad \text{or} \quad 3x - 1 = \sqrt{7} \\ &x = \frac{1}{3} - \frac{\sqrt{7}}{3} \approx -0.55 \quad x = \frac{1}{3} + \frac{\sqrt{7}}{3} \approx 1.22 \end{aligned}$$

41.
$$\begin{aligned} x^2 - 4x - 3 &= 0 \\ x^2 - 4x &= 3 \\ x^2 - 4x + (2)^2 &= 3 + (2)^2 \\ (x - 2)^2 &= 7 \\ x - 2 &= \pm\sqrt{7} \end{aligned}$$
$$x - 2 = -\sqrt{7} \quad \text{or} \quad x - 2 = \sqrt{7}$$
$$x = 2 - \sqrt{7} \approx -0.65 \quad x = 2 + \sqrt{7} \approx 4.65$$

42. $16x^2 - 24x + 7 = 0$
Using the quadratic formula:
$$x = \frac{24 \pm \sqrt{24^2 - 4(16)(7)}}{2(16)}$$
$$= \frac{24 \pm \sqrt{128}}{32} = \frac{3}{4} \pm \frac{\sqrt{2}}{4}$$
$$x = \frac{3}{4} - \frac{\sqrt{2}}{4} \approx 0.40 \quad \text{or} \quad x = \frac{3}{4} + \frac{\sqrt{2}}{4} \approx 1.10$$

43.
$$\begin{aligned} 6x^2 + 7x &= 3 \\ 6x^2 + 7x - 3 &= 0 \\ (3x - 1)(2x + 3) &= 0 \end{aligned}$$
$$3x - 1 = 0 \quad \text{or} \quad 2x + 3 = 0$$
$$x = \frac{1}{3} \quad \text{or} \quad x = -\frac{3}{2}$$

44. $2x^2 + 8x = 0$

$2x(x + 4) = 0$

$2x = 0$ or $x + 4 = 0$

$x = 0$ or $x = -4$

45. $x(2x + 5) = 4(x + 7)$

$2x^2 + 5x = 4x + 28$

$2x^2 + x - 28 = 0$

$(2x - 7)(x + 4) = 0$

$2x - 7 = 0$ or $x + 4 = 0$

$x = \frac{7}{2}$ or $x = -4$

46. $4x + 1 = 3$ or $4x + 1 = -3$

$4x = 2$ or $4x = -4$

$x = \frac{1}{2}$ or $x = -1$

47. $4x^2 - 20x + 25 = 0$

$(2x - 5)(2x - 5) = 0$

$(2x - 5)^2 = 0$

$2x - 5 = 0$

$x = \frac{5}{2}$

48. $-9x^2 + 12x - 4 = 0$

$9x^2 - 12x + 4 = 0$

$(3x - 2)(3x - 2) = 0$

$(3x - 2)^2 = 0$

$3x - 2 = 0$

$x = \frac{2}{3}$

49. $x^2 = 3x$

$x^2 - 3x = 0$

$x(x - 3) = 0$

$x = 0$ or $x - 3 = 0$

$x = 0$ or $x = 3$

50. Solving $4x^2 - 4x + 2 = 0$ by using the quadratic formula with $a = 4$, $b = -4$, and $c = 2$ gives

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(4)(2)}}{2(4)} = \frac{4 \pm \sqrt{-16}}{8}$$

$$= \frac{4 \pm 4i}{8} = \frac{1}{2} \pm \frac{1}{2}i$$

51. Solving $x^2 - 6x + 13 = 0$ by using the quadratic formula with $a = 1$, $b = -6$, and $c = 13$ gives

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)} = \frac{6 \pm \sqrt{-16}}{2}$$

$$= \frac{6 \pm 4i}{2} = 3 \pm 2i$$

52. Solving $x^2 - 2x + 4 = 0$ by using the quadratic formula with $a = 1$, $b = -2$, and $c = 4$ gives

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)} = \frac{2 \pm \sqrt{-12}}{2}$$

$$= \frac{2 \pm 2i\sqrt{3}}{2} = 1 \pm \sqrt{3}i$$

53. $2x^2 - 3x - 1 = 0$

$$x^2 - \frac{3}{2}x - \frac{1}{2} = 0$$

$$x^2 - \frac{3}{2}x + \left(-\frac{3}{4}\right)^2 = \frac{1}{2} + \left(-\frac{3}{4}\right)^2$$

$$\left(x - \frac{3}{4}\right)^2 = \frac{17}{16}$$

$$x - \frac{3}{4} = \pm\frac{\sqrt{17}}{4}$$

$$x = \frac{3}{4} \pm \frac{\sqrt{17}}{4}$$

$$x = \frac{3}{4} - \frac{\sqrt{17}}{4} \approx -0.28 \quad \text{or} \quad x = \frac{3}{4} + \frac{\sqrt{17}}{4} \approx 1.78$$

54. $3x^2 + 4x - 1 = 0$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(3)(-1)}}{2(3)}$$

$$= \frac{-4 \pm \sqrt{28}}{6} = -\frac{2}{3} \pm \frac{\sqrt{7}}{3}$$

$$x = -\frac{2}{3} - \frac{\sqrt{7}}{3} \approx -1.55 \quad \text{or} \quad x = -\frac{2}{3} + \frac{\sqrt{7}}{3} \approx 0.22$$

55. The graph of $y = 3x^3 - 19x^2 - 14x$ is zero for $x = 0$, $x = -\frac{2}{3}$, and $x = 7$.

56. The graph of $y = x^3 + 2x^2 - 4x - 8$ is zero for $x = -2$, and $x = 2$.

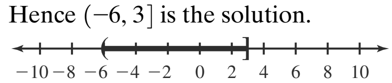
57. The graph of $y = x^3 - 2x^2 - 2$ is zero for $x \approx 2.36$.

58. The graph of $y = |2x - 1| - 4 + x^2$ is zero for $x = -1$ and for $x \approx 1.45$.

59. $-2 < x + 4 \le 7$

$-6 < x \le 3$

Hence $(-6, 3]$ is the solution.



60. $5x + 1 \ge 2x - 4$

$3x \ge -5$

$x \ge -\frac{5}{3}$

Hence $\left[-\frac{5}{3}, \infty\right)$ is the solution.

61. $\frac{3x - 5}{4} \le -1$

$3x - 5 \le -4$

$3x \le 1$

$x \le \frac{1}{3}$

Hence $\left(-\infty, \frac{1}{3}\right]$ is the solution.

62. $-7 < 2x - 5 < 7$

$-2 < 2x < 12$

$-1 < x < 6$

Hence $(-1, 6)$ is the solution.

63. $3x + 4 \geq 2$ or $3x + 4 \leq -2$

$3x \geq -2$ or $3x \leq -6$

$x \geq -\frac{2}{3}$ or $x \leq -2$

Hence $\left(-\infty, -2\right] \cup \left[-\frac{2}{3}, \infty\right)$ is the solution.

64.
$$4x^2 + 3x - 10 = 0$$
$$(4x - 5)(x + 2) = 0$$
$$4x - 5 = 0 \text{ or } x + 2 = 0$$
$$x = \frac{5}{4} \text{ or } x = -2$$

The graph of $y = 4x^2 + 3x - 10$ lies above the x-axis for $x < -2$ and for $x > \frac{5}{4}$. Hence $(-\infty, -2) \cup \left(\frac{5}{4}, \infty\right)$ is the solution.

65. The graph of $y = 2x^2 - 2x - 1$ is zero for $x \approx -0.37$ and $x \approx 1.37$, and lies above the x-axis for $x < -0.37$ and for $x > 1.37$. Hence $(-\infty, -0.37) \cup (1.37, \infty)$ is the approximate solution.

66. The graph of $y = 9x^2 - 12x - 1$ is zero for $x \approx -0.08$, and $x \approx 1.41$, and lies below the x-axis for $-0.08 < x < 1.41$. Hence $[-0.08, 1.41]$ is the approximate solution.

67. $x^3 - 9x \leq 3$ is equivalent to $x^3 - 9x - 3 \leq 0$. The graph of $y = x^3 - 9x - 3$ is zero for $x \approx -2.82$, $x \approx -0.34$, and $x \approx 3.15$, and lies below the x-axis for $x < -2.82$ and for $-0.34 < x < 3.15$. Hence the approximate solution is $(-\infty, -2.82] \cup [-0.34, 3.15]$.

68. The graph of $y = 4x^3 - 9x + 2$ is zero for $x \approx -1.60$, $x \approx 0.23$, and $x \approx 1.37$, and lies above the x-axis for $-1.60 < x < 0.23$ and for $x > 1.37$. Hence the approximate solution is $(-1.60, 0.23) \cup (1.37, \infty)$.

69.
$$\frac{x + 7}{5} > 2 \text{ or } \frac{x + 7}{5} < -2$$
$$x + 7 > 10 \text{ or } x + 7 < -10$$
$$x > 3 \text{ or } x < -17$$

Hence $(-\infty, -17) \cup (3, \infty)$ is the solution.

70.
$$2x^2 + 3x - 35 = 0$$
$$(2x - 7)(x + 5) = 0$$
$$2x - 7 = 0 \text{ or } x + 5 = 0$$
$$x = \frac{7}{2} \text{ or } x = -5$$

The graph of $y = 2x^2 + 3x - 35$ lies below the x-axis for $-5 < x < \frac{7}{2}$. Hence $\left(-5, \frac{7}{2}\right)$ is the solution.

71.
$$4x^2 + 12x + 9 = 0$$
$$(2x + 3)(2x + 3) = 0$$
$$(2x + 3)^2 = 0$$
$$2x + 3 = 0$$
$$x = -\frac{3}{2}$$

The graph of $y = 4x^2 + 12x + 9$ lies entirely above the x-axis except for $x = -\frac{3}{2}$. Hence all real numbers satisfy the inequality. So $(-\infty, \infty)$ is the solution.

72.
$$x^2 - 6x + 9 = 0$$
$$(x - 3)(x - 3) = 0$$
$$(x - 3)^2 = 0$$
$$x - 3 = 0$$
$$x = 3$$

The graph of $y = x^2 - 6x + 9$ lies entirely above the x-axis except for $x = 3$. Hence no real number satisfies the inequality. There is no solution.

73. $(3 - 2i) + (-2 + 5i) = (3 - 2) + (-2 + 5)i$
$= 1 + 3i$

74. $(5 - 7i) - (3 - 2i) = (5 - 3) + (-7 + 2)i$
$= 2 - 5i$

75. $(1 + 2i)(3 - 2i) = 3 - 2i + 6i - 4i^2$
$= 3 + 4i + 4$
$= 7 + 4i$

76. $(1 + i)^3 = ((1 + i)(1 + i))(1 + i)$
$= (1 + 2i + i^2)(1 + i) = 2i(1 + i)$
$= 2i + 2i^2 = -2 + 2i$

77. $(1 + 2i)^2(1 - 2i)^2 = (1 + 4i + 4i^2)(1 - 4i + 4i^2)$
$= (-3 + 4i)(-3 - 4i)$
$= 9 - 12i + 12i - 16i^2 = 25 + 0i$

78. $i^{29} = i^{28}i = (i^2)^{14}\, i = (-1)^{14}\, i = 0 + i$

79. $\sqrt{-16} = \sqrt{(16)(-1)} = 4\sqrt{-1} = 0 + 4i$

80. $\frac{2 + 3i}{1 - 5i} = \frac{2 + 3i}{1 - 5i} \cdot \frac{1 + 5i}{1 + 5i} = \frac{2 + 10i + 3i + 15i^2}{1 + 5i - 5i - 25i^2}$
$= \frac{-13 + 13i}{26} = -\frac{1}{2} + \frac{1}{2}i$

81. $s = -16t^2 + 320t$

(a) $-16t^2 + 320t = 1538$
$-16t^2 + 320t - 1538 = 0$
The graph of $s = -16t^2 + 320t - 1538$ is zero at

$$t = \frac{-320 \pm \sqrt{320^2 - 4(-16)(-1538)}}{2(-16)}$$
$$= \frac{-320 \pm \sqrt{3968}}{-32} = \frac{40 \pm \sqrt{62}}{4}.$$

So $t = \frac{40 - \sqrt{62}}{4} \approx 8.03$ sec or

$t = \frac{40 + \sqrt{62}}{4} \approx 11.97$ sec.

The projectile is 1538 ft above ground twice: at $t \approx 8$ sec, on the way up, and at $t \approx 12$ sec, on the way down.

(b) The graph of $s = -16t^2 + 320t$ lies below the graph of $s = 1538$ for $0 < t < 8$ and for $12 < t < 20$ (approximately). Hence the projectile's height will be at most 1538 ft when t is in the interval $(0, 8]$ or $[12, 20)$ (approximately).

(c) The graph of $s = -16t^2 + 320t$ lies above the graph of $s = 1538$ for $8 < t < 12$ (approximately). Hence the projectile's height will be greater than or equal to 1538 when t is in the interval $[8, 12]$ (approximately).

82. Let the take-off point be located at $(0, 0)$. We want the slope between $(0, 0)$ and $(d, 20{,}000)$ to be $\frac{4}{9}$.

$$\frac{20{,}000 - 0}{d - 0} = \frac{4}{9}$$

$$180{,}000 = 4d$$

$$45{,}000 = d$$

The airplane must fly 45,000 ft horizontally to reach an altitude of 20,000 ft.

83. **(a)** Let $w > 0$ be the width of a rectangle; the length is $3w + 1$ and the perimeter is $P = 2[w + (3w + 1)]$. Solve $P \le 150$.

$$2[w + (3w + 1)] \le 150$$

$$2(4w + 1) \le 150$$

$$8w + 2 \le 150$$

$$8w \le 148$$

$$w \le 18.5$$

Thus $P \le 150$ cm when w is in the interval $(0, 18.5]$.

(b) The area is $A = w(3w + 1)$. Solve $A > 1500$.

$$w(3w + 1) > 1500$$

$$3w^2 + w - 1500 > 0$$

The graph of $A = 3w^2 + w - 1500$ appears to be zero for $w \approx 22.19$ when w is positive, and lies above the w-axis for $w > 22.19$. Hence, $A > 1500$ when w is in the interval $(22.19, \infty)$ (approximately).