

## Matrices

For 1-5, complete the matrix operation. If it is not possible, write "not possible". No Calculator.

$$1. \begin{bmatrix} 5 & 6 & 1 & 0 \\ 2 & -2 & 3 & 4 \\ 1 & 5 & -2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 4 & -1 & 1 \\ -2 & 0 & 5 & 8 \\ 10 & -3 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 2 & -1 \\ 4 & -2 & -2 & -4 \\ -9 & 8 & -7 & -6 \end{bmatrix}$$

times!

$$2. 4 \cdot \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 7 & 9 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 12 & -4 \\ 8 & 16 \end{bmatrix} + \begin{bmatrix} 7 & 9 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 19 & 5 \\ 7 & 17 \end{bmatrix}$$

$$3. \begin{bmatrix} 4 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} 5 & 3 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 20 & 12 & -4 & 8 \\ 25 & 15 & -5 & 10 \end{bmatrix}$$

$2 \times 1 \cdot 1 \times 4$

$$4. \begin{bmatrix} 3 & 2 & -1 & 0 & 4 \\ 5 & 9 & -2 & -3 & 5 \\ 1 & 0 & -4 & -1 & 3 \\ 7 & 8 & 1 & 2 & 4 \\ 0 & -3 & 4 & -3 & 1 \\ 8 & 10 & -2 & 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 5 \\ 4 & -1 \end{bmatrix} = \text{Not possible :)$$

$6 \times 5$  No  $2 \times 2$

$$5. \begin{bmatrix} 1 & 2 & 0 \\ 4 & 1 & -2 \\ 3 & 7 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -10 & 1 \\ -6 & 9 \end{bmatrix}$$

$3 \times 3$   $\checkmark$   $3 \times 2$   
 $3 \times 2$

For 6-8, determine if the inverse of the matrix exists. If it does exist, find it! (#6 and #7 No Calculator, #8 Calculator OK)

$$6. \begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix} \quad \det A = 2(-2) - (-1)(4) = -4 + 4 = 0$$

No inverse

$$7. \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \quad \det A = 2(5) - 3(4) = 10 - 12 = -2$$

Yes

$$A^{-1} = \frac{1}{-2} \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} -\frac{5}{2} & \frac{3}{2} \\ 2 & -1 \end{bmatrix}$$

$$8. \begin{bmatrix} 4 & -1 & 3 \\ 2 & 1 & 4 \\ 5 & -2 & 0 \end{bmatrix} \quad \begin{bmatrix} -.53 & .4 & .47 \\ -1.33 & 1 & .67 \\ .6 & -.2 & -.4 \end{bmatrix}$$

9. Explain in words how you would prove that two matrices are inverses of each other.

① Multiply the 2 matrices together to see if their product is Identity matrix.  $AB = BA = I$ ?

OR  
② Find inverse of one matrix and see if it equals the other matrix.  $A^{-1} = B$ ?

10. Explain in words what the "identity matrix" is.

A square matrix that, when multiplied by a matrix of the same size, gives the given matrix.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad AI = A \quad IA = A$$

For 11-13, solve the system of equations using matrices (Calculator OK). You must use each method at least once (Inverses and Reduced Row Echelon Form).

11.  $2x - 3y = -10$   
 $x + 2y = 16$

$$\begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -10 \\ 16 \end{bmatrix}$$

$$A \cdot X = B$$

$$X = A^{-1}B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$\boxed{x=4 \quad y=6}$$

$$x + y + z = 2$$

12.  $2x - 3y + z = -5$

$$3x + 2y + 4z = 3$$

$$\text{RREF} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & -3 & 1 & -5 \\ 3 & 2 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

So  $x=1 \quad y=2 \quad z=-1$

$$x + y + z = -2$$

13.  $2x + z = -1$

$$3y + 3z = -12$$

$$\text{RREF} \begin{bmatrix} 1 & 1 & 1 & -2 \\ 2 & 0 & 1 & -1 \\ 0 & 3 & 3 & -12 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -5 \end{bmatrix}$$

$$\boxed{x=2 \quad y=-1 \quad z=-5}$$

$$\begin{matrix} L & - & - & - & - & 1 & 0 & 0 & 1 & - & 5 \\ \hline x=2 & y=1 & z=-5 \end{matrix}$$

14. Mrs. Billz has paper money in her wallet consisting of \$1 bills, \$5 bills, \$10 bills, and \$20 dollar bills. On Friday she had 19 total bills in her wallet that adds up to \$125. She also has one more \$10 bill than the total number of \$5 bills. The number of \$20 bills is equal to the number of \$5 bills minus the number of \$1 bills. How many of each type of bill does she have?

$x = \# \text{ of } \$1 \text{ bills}$   
 $y = \# \text{ of } \$5 \text{ bills}$   
 $z = \# \text{ of } \$10 \text{ bills}$   
 $w = \# \text{ of } \$20 \text{ bills}$

$$\begin{aligned} x + y + z + w &= 19 && : \text{total \# bills} \\ 1x + 5y + 10z + 20w &= 125 && : \text{total \$} \\ z - 1 &= y && : \text{one more \$10 than \$5s} \\ w &= y - x && : \$20s = \$5s minus \$1s \end{aligned}$$

$$\begin{aligned} & \begin{cases} 0 - y + z + 0 = 1 \\ x - y + 0 + w = 0 \end{cases} \\ \text{RREF} \begin{bmatrix} 1 & 1 & 1 & 1 & 19 \\ 1 & 5 & 10 & 20 & 125 \\ 0 & -1 & 1 & 0 & 1 \\ 1 & -1 & 0 & 1 & 0 \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{aligned}$$

5	\$1 bills
6	\$5 bills
7	\$10 bills
1	\$20 bills

15. The Gaussians Math team has made it to State! After they compete at State, they take home a total of 10 trophies (1<sup>st</sup>, 2<sup>nd</sup>, and 3<sup>rd</sup> place finishes in each event earns the team a trophy). The number of 1<sup>st</sup> place trophies is the same as the number of 2<sup>nd</sup> and 3<sup>rd</sup> place trophies combined. Also, the number of 1<sup>st</sup> place trophies is one less than twice the number of 2<sup>nd</sup> place trophies. How many 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> place trophies do they take home?

$a = \# \text{ of } 1^{\text{st}} \text{ place trophies}$   
 $b = \# \text{ of } 2^{\text{nd}} \text{ place trophies}$   
 $c = \# \text{ of } 3^{\text{rd}} \text{ place trophies}$

$$\begin{aligned} a + b + c &= 10 \\ a &= b + c \Rightarrow a - b - c = 0 \\ a &= 2b - 1 \quad a - 2b + 0 = -1 \end{aligned}$$

$$\text{RREF} \begin{bmatrix} 1 & 1 & 1 & 10 \\ 1 & -1 & -1 & 0 \\ 1 & -2 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

5	1 <sup>st</sup> place trophies
3	2 <sup>nd</sup> place trophies

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 1 & -2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

0 1st place trophies  
3 2nd place trophies  
2 3rd place trophies