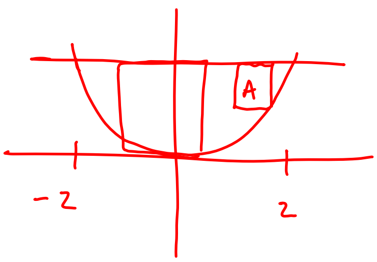
AP Calc AB   Chapter 7 Volumes review   Name___Solutions___

1. The base of a region between the line y=4 and the parabola $y = x^2$. The cross sections of the solid are perpendicular to the x-axis are:

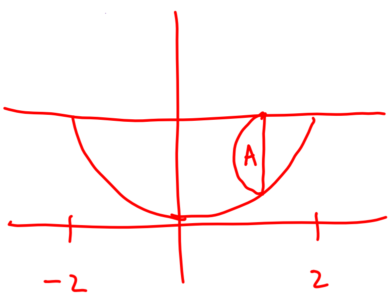
a. Squares:



$$A(x) = (4-x^2)^2 = 16 - 8x^2 + x^4$$

$$V = 2\int_0^2 (16 - 8x^2 + x^4)\,dx = \frac{512}{15} \approx 34.133$$

b. Semicircles:



$$A(x) = \frac{1}{2}\pi\left(\frac{4-x^2}{2}\right)^2 = \frac{\pi}{8}(4-x^2)^2 = \frac{\pi}{8}(16-8x^2+x^4)$$

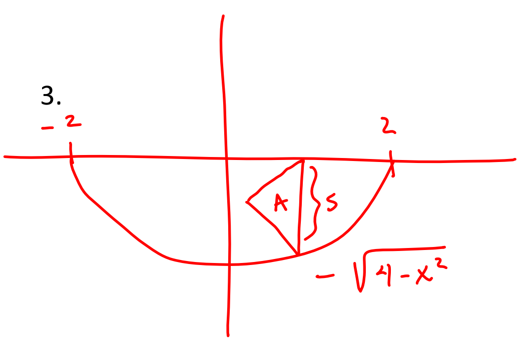
$$V = 2\int_0^2 \frac{\pi}{8}(16-8x^2+x^4)\,dx = \frac{\pi}{8}\cdot\frac{512}{15} = \frac{64\pi}{15}$$

$$\approx 13.404$$

2. Find the volume of the solid that lies between planes perpendicular to the x-axis at x=2 and x= -2. The cross sections perpendicular to the x-axis between the planes are equilateral triangles whose bases run from y=0 to $y = -\sqrt{4-x^2}$.

$$s = \left(0 - \left(-\sqrt{4-x^2}\right)\right) = \sqrt{4-x^2}$$

3.



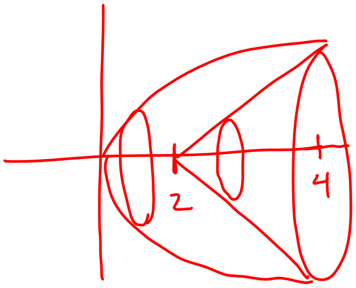
$$A(x) = \frac{s^2}{4}\sqrt{3} = \frac{4-x^2}{4}\sqrt{3}$$

$$V = 2\int_0^2 \frac{\sqrt{3}}{4}(4-x^2)\,dx = \frac{\sqrt{3}}{2}\cdot\frac{16}{3} = \frac{16\sqrt{3}}{6}$$

$$= \frac{8\sqrt{3}}{3} \approx 4.619$$

A region is bounded by $y=\sqrt{x}$, $y=x-2$, *and*, $y=0$. What is the volume of the solid generated when this region is rotated around the x-axis?

$$V = \int_0^2 \pi \left(\sqrt{x}\right)^2 dx + \int_2^4 \pi\left[\left(\sqrt{x}\right)^2 - (x-2)^2\right] dx$$



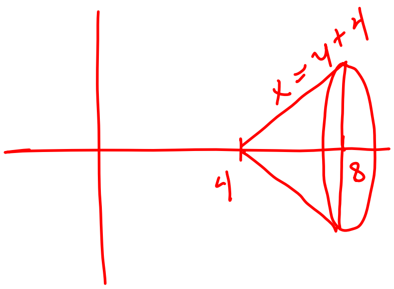
$$= 2\pi + \frac{10}{3}\pi = \frac{16}{3}\pi$$

4. A region is bounded by $f(y)=4+y$, *and* $[0,4]$ Find the volume of the solid if it is rotated about:

y

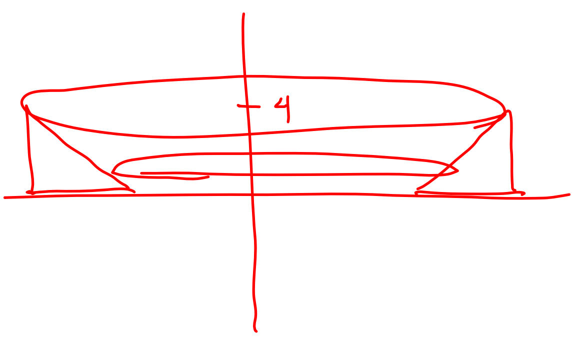
a. The x-axis

$$\int_4^8 \pi (x-4)^2 dx = \frac{64\pi}{3} \approx 67.021$$



b. The y-axis

$$A(y) = \pi\left(8^2 - (4+y)^2\right)$$

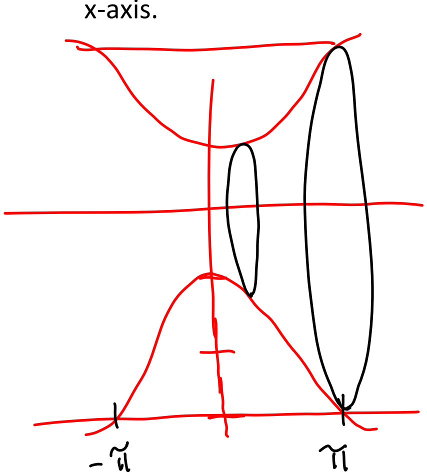


$$V = \int_0^4 \pi\left(8^2 - (4+y)^2\right) dy = \frac{320\pi}{3}$$

$$\approx 335.103$$

5. A region bounded by $y=\cos x-2$, $y=-3$, *and* $[-\pi,\pi]$. Find the volume of the solid if it is rotated about the x-axis.

$$A(x) = \pi\left[(-3)^2 - (\cos x - 2)^2\right]$$



$$V = 2\pi \int_0^{\pi} \left[(-3)^2 - (\cos x - 2)^2\right] dx = \frac{103{,}993}{3678}\pi$$

$$\approx 88.826$$