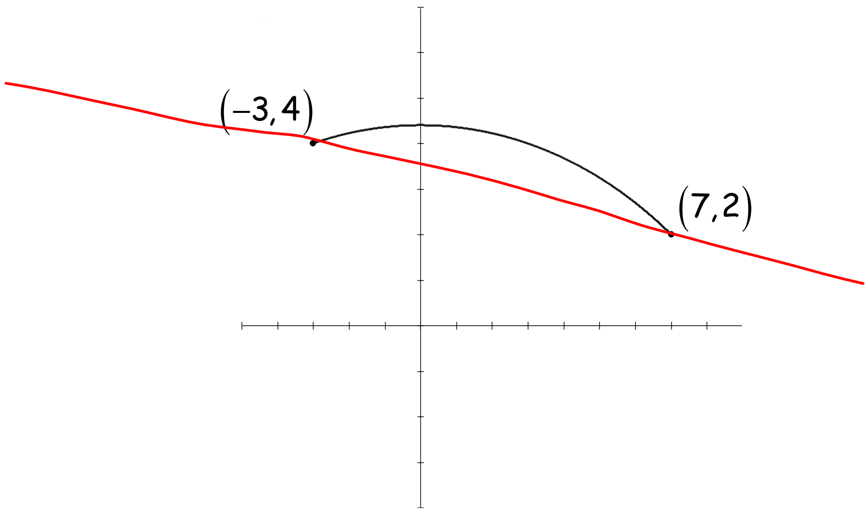
Ch 5 AP Problems Name Solutions

NO CALCULATOR



1. The graph of $y = f(x)$ on the closed interval $[-3,7]$ is shown in the figure above. If *f* is continuous on $[-3,7]$ and differentiable on $(-3,7)$, then there exists a *c*, $-3 < c < 7$, such that

A. $f(c) = 0$

B. $f'(c) = 0$

C. $f'(c) = \frac{1}{5}$

D. $f'(c) = -\frac{1}{5}$

E. $f'(c) = -5$

$$\frac{f(7)-f(-3)}{7-(-3)} = \frac{2-4}{7-(-3)} = \frac{-2}{10} = -\frac{1}{5}$$

2. Let *f* be the function given by $f(x) = x^3$. What are all values of *c* that satisfy the conclusion of the Mean Value Theorem on the closed interval $[-1,2]$?

A. 0 only

B. 1 only

C. $\sqrt{3}$ only

D. -1 and 1

E. $-\sqrt{3}$ and $\sqrt{3}$

$$f'(x) = 3x^2 = \frac{f(2)-f(-1)}{2-(-1)}$$

$$3x^2 = \frac{8-(-1)}{3} = 3$$

$$3x^2 = 3$$

$$x = \pm 1$$

$$x = 1$$

because −1 is not on (−1, 2)

3. Let $f(x)$ be a differentiable function defined only on the interval $-2 \le x \le 10$. The table below gives the value of $f(x)$ and its derivative $f'(x)$ at several points of the domain.

x	-2	0	2	4	6	8	10
$f(x)$	26	27	26	23	18	11	2
$f'(x)$	1	0	-1	-2	-3	-4	-5

The line tangent to the graph of $f(x)$ and parallel to the segment between the endpoints intersects the y-axis at the point

A. (0, 27)
B. (0, 28)
C. (0, 31)
D. (0, 36)
E. (0, 43)

$$\frac{f(10)-f(-2)}{10-(-2)} = \frac{2-26}{12} = -2 = f'(x)$$

$f'(x) = -2$ @ $x = 4$ on $(-2, 10)$

$f(4) = 23$

$y - 23 = -2(x-4)$ set $x = 0$

$y = 23 + 8 = 31$ $(0, 31)$

4. If $f(x) = \ln x - k\sqrt{x}$ has a local minimum at $x = 4$ then the value of k is

A. -1 B. $\frac{1}{2}$ C. 1 D. 4 E. none of these

5.

How many extrema (maximum and minimum) does the function $f(x) = (x+2)^3(x-5)^2$ have on the interval $-3 \le x \le 6$?

(A) None

(B) One

(C) Two

(D) Three

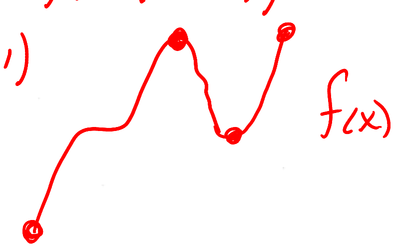
(E) Four

$f'(x) = 3(x+2)^2(x-5)^2 + 2(x+2)^3(x-5)$

$= (x+2)^2(x-5)\left(3(x-5)+2(x+2)\right)$

$= (x+2)^2(x-5)(5x-11)$

$f'(x)$: $+$ $+$ $-$ $+$ at -3, -2, $11/5$, 5, 6



6.

The graph of $y = 2x^3 + 24x - 18$ is

(A) increasing for all x

(B) decreasing for all x

(C) only increasing for all x such that $|x| > 2$

(D) only increasing for all x such that $|x| < 2$

(E) only decreasing for all x such that $x < -2$

$y' = 6x^2 + 24 > 0$ for All x

$\therefore$ A

7.

A particle moves along the x-axis so that at any time t its position is given by $x(t) = (t+1)(t-3)^3$. For what values of t is the velocity of the particle increasing ?

(A) $t > 3$ only

(B) $0 < t < 3$ only

(C) $1 < t < 3$ only

(D) $t < 1$ or $t > 3$

(E) $0 < t < 3$ or $t > 3$

$x'(t) = 1(t-3)^3 + 3(t-3)^2(t+1)$

$= (t-3)^2(t-3+3(t+1)) = (t-3)^2(4t)$

$x''(t) = 2(t-3)(4t) + (t-3)^2 \cdot 4$

$= (t-3)(8t + 4(t-3)) = (t-3)(12t-12)$

$= 12(t-3)(t-1)$

$x''(t)$: $+$ | $-$ | $+$ at 1, 3

8.

In which interval is the function $f(x) = x^3 + 6x^2 + 9x + 1$ increasing ?

(A) $(-\infty, -3)$ only

(B) $(-3, -1)$ only

(C) $(-1, \infty)$ only

(D) $(-\infty, -3) \cup (-1, \infty)$

(E) $(-\infty, -3) \cup (1, \infty)$

$f'(x) = 3x^2 + 12x + 9 = 0$

$3(x^2 + 4x + 3) = 0$

$3(x+3)(x+1) = 0$

$f'(x)$: $+$ | $-$ | $+$ at -3, -1

9.

Let f be the function given by $f(x) = x^3$. What are all values of c that satisfy the conclusion of the Mean Value Theorem on the closed interval $[-1, 2]$?

(A) 0 only

(B) 1 only

(C) $\sqrt{3}$ only

(D) -1 and 1

(E) $-\sqrt{3}$ and $\sqrt{3}$

$3x^2 = \dfrac{f(2)-f(-1)}{2-(-1)}$

$3x^2 = \dfrac{9}{3}$

$x = \pm 1$ -1 not on $(-1,2)$

10.

At what value(s) of x does $f(x) = x^4 - 8x^2$ have a relative minimum ?

(A) 0 and -2 only

(B) 0 and 2 only

(C) 0 only

(D) -2 and 2 only

(E) -2, 0, and 2

$f'(x) = 4x^3 - 16x = 4x(x^2-4) = 0$

$x = 0, \quad x = \pm 2$

$f'(x)$: $-$ | $+$ | $-$ | $+$ at -2, 0, 2

11.

The maximum value of $f(x) = 2x^3 - 9x^2 + 12x - 1$ on $[-1, 2]$ is

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

$f'(x) = 6x^2 - 18x + 12 = 0$

$6(x^2 - 3x + 2) = 0$

$6(x-2)(x-1) = 0$

$f(1) = 2 - 9 + 12 - 1 = 4$

$f'(x)$: $+$ | $-$ at -1, 1, 2

12.

For what value of k will $\dfrac{8x+k}{x^2}$ have a relative maximum at $x = 4$?

(A) -32 (B) -16 (C) 0   (D) 16   (E) 32

$\dfrac{8x^2 - (8x+k)\cdot 2x}{x^4}$ @ $x=4$ $\dfrac{8\cdot 16 - (32+k)\cdot 8}{4^4} = 0$ $k = \dfrac{-128+256}{-8}$

$= -16$

13. Which of the following statements is *false*?

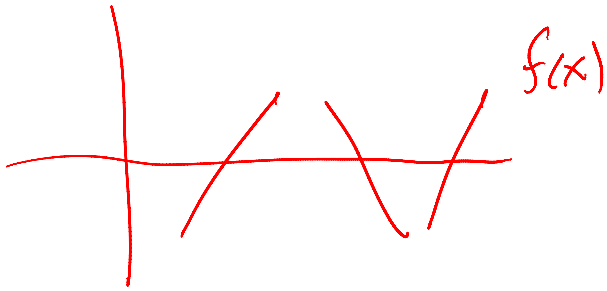
A. If c is a critical value of the function f, then it is also a critical value of the function $g(x) = f(x) + k$ where k is a constant.

B. If a function f is continuous on a closed interval then it must have a minimum value on the interval.

C. If a function not necessarily continuous has 3 zeros, then it must have at least 2 points where the tangent line is horizontal.

D. The maximum value of a continuous function on a closed interval can occur at more than one input in the interval.

E. The graph of a function can have at most two horizontal asymptotes.



CALCULATOR ALLOWED

1. The graph of the function $f(x) = 2x^{\frac{5}{3}} - 5x^{\frac{2}{3}}$ is increasing on which of the following intervals?

I. $1 < x$ II. $0 < x < 1$ III. $x < 0$

$f'(x) = \frac{10}{3}x^{2/3} - \frac{10}{3}x^{-1/3}$ graph $f'(x)$ And find

A. I only   B. II only   C. III only   D. I and II only   E. I and III only

where $f'(x) > 0$

2. Let $f(x) = x^5 - 3x^2 + 4$. For how many inputs c between $a = -2$ and $b = 2$ is it true that $\frac{f(b) - f(a)}{b - a} = f'(c)$?

$\frac{f(2) - f(-2)}{2-(-2)} = \frac{24+40}{4} = 16$ graph $f'(x) = 5x^4 - 6x$ AND $g(x) = 16$

find how many times they intersect on $(-2, 2)$

A. 0   B. 1   C. 2   D. 3   E. 4

3. Suppose a particle is moving along a coordinate line and its position at time t is given by $s(t) = \frac{9t^2}{t^2 + 2}$. For what values of t in the interval $[1, 4]$ is the instantaneous velocity equal to the average velocity?

find where $s'(t) = \frac{s(4) - s(1)}{4-1} = \frac{8-3}{3}$ on $(1, 4)$

A. 2.00   B. 2.11   C. 2.22   D. 2.33   E. 2.44

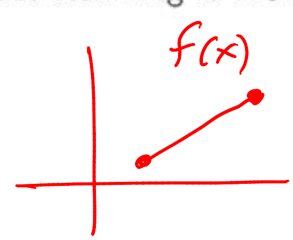
4.

If f is a continuous function on the closed interval $[a, b]$, which of the following is NOT necessarily true ?

I. f has a minimum on $[a, b]$ T

II. f has a maximum on $[a, b]$ T

III. $f'(c) = 0$ for $a < c < b$ F



(A) I only   (B) II only   (C) III only

(D) I and II only   (E) I, II, and III

5.

Let $f(x)$ be a differentiable function defined for all real numbers. The table below gives the value of $f(x)$ and its derivative $f'(x)$ for several values of x.

x	-3	-2	-1	0	1	2	3
$f(x)$	8	5	0	1	0	5	8
$f'(x)$	-6	-4	-2	0	2	4	6

Which of the following statements is true?

I. At $x = 2$, the function is increasing. T

II. There is a relative minimum in the interval $-1 \leq x \leq 1$, but not necessarily at $x = 0$. T

III. There is a relative maximum in the interval $-1 \leq x \leq 1$. F

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

6.

If the derivative of a function f is given by $f'(x) = \sin(x^x)$, then how many critical points does the function $f(x)$ have on the interval $[0.2, 2.6]$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

find how many times $f'(x) = 0$ on $[0.2, 2.6]$

7.

The derivative of f is given by $f'(x) = e^x(-x^3 + 3x) - 3$ for $0 \leq x \leq 5$.

At what value of x is $f(x)$ an absolute minimum?

(A) For no value of x

(B) 0

(C) 0.653

(D) 1.604

(E) 5

graph $f'(x)$ and look for $-$ to $+$ sign changes and right end pt

8. If $f(x) = \left|(x^2 - 12)(x^2 + 4)\right|$, how many numbers in the interval $-2 \leq x \leq 3$ satisfy the conclusion of the Mean Value Theorem?

A. None
B. One
C. Two
D. Three
E. Four

$$\frac{f(3) - f(-2)}{3-(-2)} = \frac{39 - 64}{5} = -5$$

graph $f'(x)$ using NDERIV and find how many times $f'(x) = -5$ on $(-2, 3)$.