21. $(0, -2)$

22. $(-3, 0)$

23. $\left(2, \frac{\pi}{6} + 2n\pi\right)$ and $\left(-2, \frac{\pi}{6} + (2n + 1)\pi\right)$, n an integer

24. $\left(1, -\frac{\pi}{4} + 2n\pi\right)$ and $\left(-1, -\frac{\pi}{4} + (2n + 1)\pi\right)$, n an integer

25. $(1.5, -20° + 360n°)$ and $(-1.5, 160° + 360n°)$, n an integer

26. $(-2.5, 50° + 360n°)$ and $(2.5, 230° + 360n°)$, n an integer

27. **(a)** $\left(\sqrt{2}, \frac{\pi}{4}\right)$ or $\left(-\sqrt{2}, \frac{5\pi}{4}\right)$

(b) $\left(\sqrt{2}, \frac{\pi}{4}\right)$ or $\left(-\sqrt{2}, -\frac{3\pi}{4}\right)$

(c) The answers from **(a)**, and also $\left(\sqrt{2}, \frac{9\pi}{4}\right)$ or $\left(-\sqrt{2}, \frac{13\pi}{4}\right)$

28. **(a)** $(\sqrt{10}, \tan^{-1} 3) \approx (\sqrt{10}, 1.25)$ or $(-\sqrt{10}, \tan^{-1} 3 + \pi) \approx (-\sqrt{10}, 4.39)$

(b) $(\sqrt{10}, \tan^{-1} 3) \approx (\sqrt{10}, 1.25)$ or $(-\sqrt{10}, \tan^{-1} 3 - \pi) \approx (-\sqrt{10}, -1.89)$

(c) The answers from **(a)**, and also $(\sqrt{10}, \tan^{-1} 3 + 2\pi) \approx (\sqrt{10}, 7.53)$ or $(-\sqrt{10}, \tan^{-1} 3 + 3\pi) \approx (-\sqrt{10}, 10.67)$

29. **(a)** $(\sqrt{29}, \tan^{-1}(-2.5) + \pi) \approx (\sqrt{29}, 1.95)$ or $(-\sqrt{29}, \tan^{-1}(-2.5) + 2\pi) \approx (-\sqrt{29}, 5.09)$

(b) $(-\sqrt{29}, \tan^{-1}(-2.5)) \approx (-\sqrt{29}, -1.19)$ or $(\sqrt{29}, \tan^{-1}(-2.5) + \pi) \approx (\sqrt{29}, 1.95)$

(c) The answers from **(a)**, plus $(\sqrt{29}, \tan^{-1}(-2.5) + 3\pi) \approx (\sqrt{29}, 8.23)$ or $(-\sqrt{29}, \tan^{-1}(-2.5) + 4\pi) \approx (-\sqrt{29}, 11.38)$

30. **(a)** $(-\sqrt{5}, \tan^{-1} 2) \approx (-\sqrt{5}, 1.11)$ or $(\sqrt{5}, \tan^{-1} 2 + \pi) \approx (\sqrt{5}, 4.25)$

(b) $(-\sqrt{5}, \tan^{-1} 2) \approx (-\sqrt{5}, 1.11)$ or $(\sqrt{5}, \tan^{-1} 2 - \pi) \approx (\sqrt{5}, -2.03)$

(c) The answers from **(a)**, plus $(-\sqrt{5}, \tan^{-1} 2 + 2\pi) \approx (-\sqrt{5}, 7.39)$ or $(\sqrt{5}, \tan^{-1} 2 + 3\pi) \approx (\sqrt{5}, 10.53)$

31. **(b)**

32. **(d)**

33. **(c)**

34. **(a)**

35. $x = 3$ — a vertical line

36. $y = -2$ — a horizontal line

37. $r^2 + 3r \sin\theta = 0$, or $x^2 + y^2 + 3y = 0$. Completing the square gives $x^2 + \left(y + \frac{3}{2}\right)^2 = \frac{9}{4}$ — a circle centered at $\left(0, -\frac{3}{2}\right)$ with radius $\frac{3}{2}$.

38. $r^2 + 4r \cos\theta = 0$, or $x^2 + y^2 + 4x = 0$. Completing the square gives $(x + 2)^2 + y^2 = 4$ — a circle centered at $(-2, 0)$ with radius 2

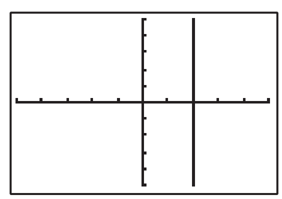
39. $r^2 - r \sin\theta = 0$, or $x^2 + y^2 - y = 0$. Completing the square gives $x^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}$ — a circle centered at $\left(0, \frac{1}{2}\right)$ with radius $\frac{1}{2}$.

40. $r^2 - 3r \cos\theta = 0$, or $x^2 + y^2 - 3x = 0$. Completing the square gives $\left(x - \frac{3}{2}\right)^2 + y^2 = \frac{9}{4}$ — a circle centered at $\left(\frac{3}{2}, 0\right)$ with radius $\frac{3}{2}$.

41. $r^2 - 2r \sin\theta + 4r \cos\theta = 0$, or $x^2 + y^2 - 2y + 4x = 0$. Completing the square gives $(x + 2)^2 + (y - 1)^2 = 5$ — a circle centered at $(-2, 1)$ with radius $\sqrt{5}$.

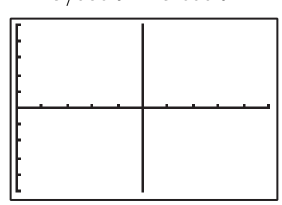
42. $r^2 - 4r \cos\theta + 4r \sin\theta = 0$, or $x^2 + y^2 - 4x + 4y = 0$. Completing the square gives $(x - 2)^2 + (y + 2)^2 = 8$ — a circle centered at $(2, -2)$ with radius $2\sqrt{2}$.

43. $r = 2/\cos\theta = 2 \sec\theta$ — a vertical line



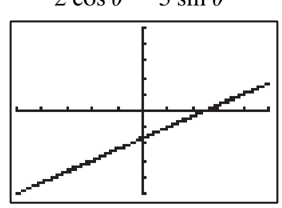
[−5, 5] by [−5, 5]

44. $r = 5/\cos\theta = 5 \sec\theta$



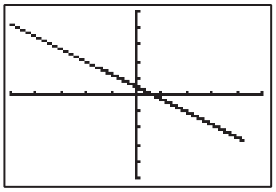
[0, 10] by [−5, 5]

45. $r = \dfrac{5}{2\cos\theta - 3\sin\theta}$



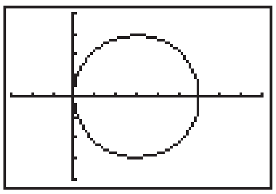
[−5, 5] by [−5, 5]

46. $r = \dfrac{2}{3\cos\theta + 4\sin\theta}$



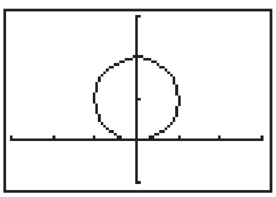
$[-5, 5]$ by $[-5, 5]$

47. $r^2 - 6r\cos\theta = 0$, so $r = 6\cos\theta$



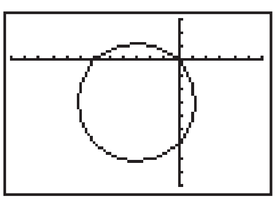
$[-3, 9]$ by $[-4, 4]$

48. $r^2 - 2r\sin\theta = 0$, so $r = 2\sin\theta$



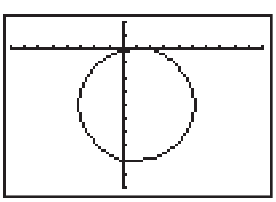
$[-3, 3]$ by $[-1, 3]$

49. $r^2 + 6r\cos\theta + 6r\sin\theta = 0$, so $r = -6\cos\theta - 6\sin\theta$



$[-12, 6]$ by $[-9, 3]$

50. $r^2 - 2r\cos\theta + 8r\sin\theta = 0$, so $r = 2\cos\theta - 8\sin\theta$



$[-8, 10]$ by $[-10, 2]$

51. $d = \sqrt{3^2 + 5^2 - 2\cdot 3\cdot 5\cos(170° - 150°)}$
$= \sqrt{34 - 30\cos 20°} \approx 3.46$ mi

52. $d = \sqrt{4^2 + 2^2 - 2\cdot 4\cdot 2\cos(12° - 72°)}$
$= \sqrt{20 - 16\cos 60°} = \sqrt{12} = 2\sqrt{3} \approx 2.41$ mi

53. Using the Pythagorean theorem, the center-to-vertex distance is $\dfrac{a}{\sqrt{2}}$. The four vertices are then $\left(\dfrac{a}{\sqrt{2}}, \dfrac{\pi}{4}\right)$, $\left(\dfrac{a}{\sqrt{2}}, \dfrac{3\pi}{4}\right)$, $\left(\dfrac{a}{\sqrt{2}}, \dfrac{5\pi}{4}\right)$, and $\left(\dfrac{a}{\sqrt{2}}, \dfrac{7\pi}{4}\right)$. Other polar coordinates for these points are possible, of course.

54. The vertex on the x-axis has polar coordinates $(a, 0)$. All other vertices must also be a units from the origin; their coordinates are $\left(a, \dfrac{2\pi}{5}\right)$, $\left(a, \dfrac{4\pi}{5}\right)$, $\left(a, \dfrac{6\pi}{5}\right)$, and $\left(a, \dfrac{8\pi}{5}\right)$. Other polar coordinates for these points are possible, of course.

55. False. Point (r, θ) is the same as point $(r, \theta + 2n\pi)$ for any integer n. So each point has an infinite number of distinct polar coordinates.

56. True. For (r_1, θ) and $(r_2, \theta + \pi)$ to represent the same point, $(r_2, \theta + \pi)$ has to be the reflection across the origin of $(r_1, \theta + \pi)$, and this is accomplished by setting $r_2 = -r_1$.

57. For point (r, θ), changing the sign on r and adding 3π to θ constitutes a twofold reflection across the origin.
The answer is C.

58. The rectangular coordinates are
$(-2\cos(-\pi/3), -2\sin(-\pi/3)) = (-1, \sqrt{3})$.
The answer is C.

59. For point (r, θ), changing the sign on r and subtracting $180°$ from θ constitutes a twofold reflection across the origin. The answer is A.

60. $(-2, 2)$ lies in Quadrant III, whereas $(-2\sqrt{2}, 135°)$ lies in Quadrant IV. The answer is E.

61. **(a)** If $\theta_1 - \theta_2$ is an odd integer multiple of π, then the distance is $|r_1 + r_2|$. If $\theta_1 - \theta_2$ is an even integer multiple of π, then the distance is $|r_1 - r_2|$.

(b) Consider the triangle formed by O_1, P_1, and P_2 (ensuring that the angle at the origin is less than 180°), then by the law of cosines,
$\overline{P_1P_2}^{\,2} = \overline{OP_1}^{\,2} + \overline{OP_2}^{\,2} - 2\cdot\overline{OP_1}\cdot\overline{OP2}\cos\theta$,
where θ is the angle between $\overline{OP_1}$ and $\overline{OP_2}$. In polar coordinates, this formula translates very nicely into
$d^2 = r_1^{\,2} + r_2^{\,2} - 2r_1r_2\cos(\theta_2 - u_1)$ (or $\cos(\theta_1 - \theta_2)$ since $\cos(\theta_2 - \theta_1) = \cos(\theta_1 - \theta_2)$), so
$d = \sqrt{r_1^{\,2} + r_2^{\,2} - 2r_1 r_2\cos(\theta_2 - u_1)}$.

(c) Yes. If $\theta_1 - \theta_2$ is an odd integer multiple of π, then $\cos(\theta_1 - \theta_2) = -1 \Rightarrow d = \sqrt{r_1^{\,2} + r_2^{\,2} + 2r_1r_2}$
$= |r_1 + r_2|$. If $\theta_1 - \theta_2$ is an even integer multiple of π, then $\cos(\theta_1 - u_2) = 1 \Rightarrow$
$d = \sqrt{r_1^{\,2} + r_2^{\,2} - 2r_1r_2} = |r_1 - r_2|$.

62. **(a)** The right half of a circle centered at $(0, 2)$ of radius 2

(b) Three quarters of the same circle, starting at $(0, 0)$ and moving counterclockwise

(c) The full circle (plus another half circle found through the TRACE function)

(d) 4 counterclockwise rotations of the same circle

63. $d = \sqrt{2^2 + 5^2 - 2(2)(5)\cos 120°} \approx 6.24$

64. $d = \sqrt{4^2 + 6^2 - 2(4)(6)\cos 45°} \approx 4.25$

65. $d = \sqrt{(-3)^2 + (-5)^2 - 2(-3)(-5)\cos 135°} \approx 7.43$

66. $d = \sqrt{6^2 + 8^2 - 2(6)(8)\cos 30°} \approx 4.11$

67. Since $x = r\cos\theta$ and $y = r\sin\theta$, the parametric equation would be $x = f(\theta)\cos(\theta)$ and $y = f(\theta)\sin(\theta)$.

68. $x = 2\cos^2\theta$
$y = 2(\cos\theta)(\sin\theta)$

69. $x = 5(\cos\theta)(\sin\theta)$ $y = 5\sin^2\theta$

70. $x = 2(\cos\theta)(\sec\theta) = 2$
$y = 2(\sin\theta)(\sec\theta) = 2\tan\theta$

71. $x = 4(\cos\theta)(\csc\theta) = 4\cot\theta$
$y = 4(\sin\theta)(\csc\theta) = 4$

■ Section 6.5 Graphs of Polar Equations

Exploration 1

Answers will vary.

Exploration 2

1. If $r^2 = 4\cos(2\theta)$, then r does not exist when $\cos(2\theta) < 0$. Since $\cos(2\theta) < 0$ whenever θ is in the interval $\left(\frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi\right)$, n is any integer, the domain of r does not include these intervals.

2. $-r\sqrt{\cos(2\theta)}$ draws the same graph, but in the opposite direction.

3. $(r)^2 - 4\cos(-2\theta) = r^2 - 4\cos(2\theta)$
(since $\cos(\theta) = \cos(-\theta)$)

4. $(-r)^2 - 4\cos(-2\theta) = r^2 - 4\cos(2\theta)$

5. $(-r)^2 - 4\cos(2\theta) = r^2 - 4\cos(2\theta)$

Quick Review 6.5

For #1–4, use your grapher's TRACE function to solve.

1. Minimum: −3 at $x = \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$; Maximum: 3 at $x = \{0, \pi, 2\pi\}$

2. Minimum: −1 at $x = \pi$; Maximum: 5 at $x = \{0, 2\pi\}$

3. Minimum: 0 at $x = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$; Maximum: 2 at $x = \{0, \pi, 2\pi\}$

4. Minimum: 0 at $x = \frac{\pi}{2}$; Maximum: 6 at $x = \frac{3\pi}{2}$

5. **(a)** No **(b)** No **(c)** Yes

6. **(a)** No **(b)** Yes **(c)** No

7. $\sin(\pi - \theta) = \sin\theta$

8. $\cos(\pi - \theta) = -\cos\theta$

9. $\cos 2(\pi + \theta) = \cos(2\pi + 2\theta) = \cos 2\theta$
$= \cos^2\theta - \sin^2\theta$

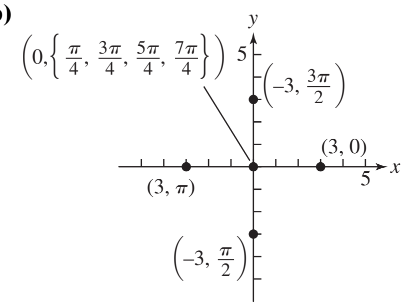
10. $\sin 2(\pi + \theta) = \sin(2\pi + 2\theta) = \sin 2\theta$
$= 2\sin\theta\cos\theta$

Section 6.5 Exercises

1. **(a)**

θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$
r	3	0	−3	0	3	0	−3	0

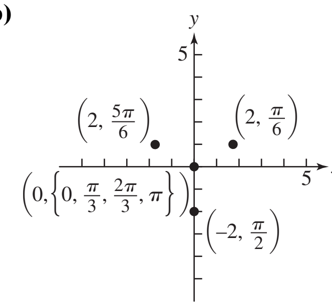
(b)



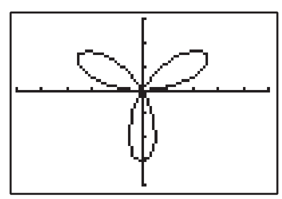
2. **(a)**

θ	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
r	0	2	0	−2	0	2	0

(b)

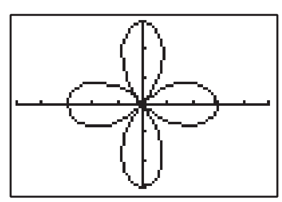


3. $k = \pi$



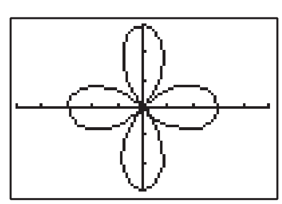
[−5, 5] by [−4, 3]

4. $k = 2\pi$



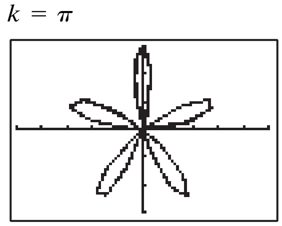
[−5, 5] by [−3, 3]

5. $k = 2\pi$



[−5, 5] by [−3, 3]

6. $k = \pi$



[−5, 5] by [−3, 3]

7. r_1 is not shown (this is a 12-petal rose). r_2 is not shown (this is a 6-petal rose), r_3 is graph **(b)**.

8. $6 \cos 2\theta \sin 2\theta = 3(2 \cos u \sin u)$ where $u = 2\theta$; this equals $3 \sin 2u = 3 \sin 4\theta$. $r = 3 \sin 4\theta$ is the equation for the 8-petal rose shown in graph **(a)**.

9. Graph **(b)** is $r = 2 - 2 \cos \theta$: Taking $\theta = 0$ and $\theta = \frac{\pi}{2}$, we get $r = 2$ and $r = 4$ from the first equation, and $r = 0$ and $r = 2$ from the second. No graph matches the first of these (r, θ) pairs, but **(b)** matches the latter (and any others one might choose).

10. Graph **(c)** is $r = 2 + 3 \cos \theta$: Taking $\theta = 0$, we get $r = -1$ from the other equation, which matches nothing. Any (r, θ) pair from the first equation matches **(c)**, however.

11. Graph **(a)** is $r = 2 - 2 \sin \theta$ — where $\theta = \frac{\pi}{2}$, $2 + 2 \cos \theta = 2$, but $\left(2, \frac{\pi}{2}\right)$ is clearly not on graph **(a)**; meanwhile $2 - 2 \sin \frac{\pi}{2} = 0$, and $\left(0, \frac{\pi}{2}\right)$ (the origin) is part of graph **(a)**.

12. Graph **(d)** is $r = 2 - 1.5 \sin \theta$ — where $\theta = \frac{\pi}{2}$, $2 + 1.5 \cos \theta = 2$, but $\left(2, \frac{\pi}{2}\right)$ is clearly not on graph **(d)**; meanwhile $2 - 1.5 \sin \frac{\pi}{2} = 0.5$, and $\left(0.5, \frac{\pi}{2}\right)$ is part of graph **(d)**.

13. Symmetric about the y-axis: Replacing (r, θ) with $(r, \pi - \theta)$ gives the same equation, since $\sin(\pi - \theta) = \sin \theta$.

14. Symmetric about the x-axis: Replacing (r, θ) with $(r, -\theta)$ gives the same equation, since $\cos(-\theta) = \cos \theta$.

15. Symmetric about the x-axis: Replacing (r, θ) with $(r, -\theta)$ gives the same equation, since $\cos(-\theta) = \cos \theta$.

16. Symmetric about the y-axis: Replacing (r, θ) with $(r, \pi - \theta)$ gives the same equation, since $\sin(\pi - \theta) = \sin \theta$.

17. All three symmetries. Polar axis: Replacing (r, θ) with $(r, -\theta)$ gives the same equation, since $\cos(-2\theta) = \cos 2\theta$. y-axis: replacing (r, θ) with $(r, \pi - \theta)$ gives the same equation, since $\cos[2(\pi - \theta)] = \cos(2\pi - 2\theta) = \cos(-2\theta) = \cos 2\theta$. Pole: Replacing (r, θ) with $(r, \theta + \pi)$ gives the same equation, since $\cos[2(\theta + \pi)] = \cos(2\theta + 2\pi) = \cos 2\theta$.

18. Symmetric about the y-axis: Replacing (r, θ) with $(-r, -\theta)$ gives the same equation, since $\sin(-3\theta) = -\sin 3\theta$.

19. Symmetric about the y-axis: Replacing (r, θ) with $(r, \pi - \theta)$ gives the same equation, since $\sin(\pi - \theta) = \sin \theta$.

20. Symmetric about the x-axis: Replacing (r, θ) with $(r, -\theta)$ gives the same equation, since $\cos(-\theta) = \cos \theta$.

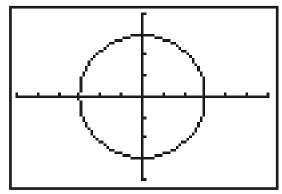
21. Maximum $|r|$ is 5 — when $\theta = 2n\pi$ for any integer n.

22. Maximum $|r|$ is 5 (when $r = -5$) — when $\theta = \frac{3\pi}{2} + 2n\pi$ for any integer n.

23. Maximum $|r|$ is 3 (when $r = \pm 3$) — when $\theta = 2n\pi/3$ for any integer n.

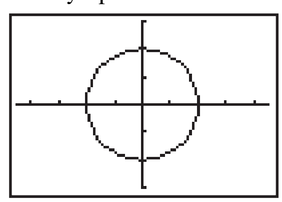
24. Maximum $|r|$ is 4 (when $r = \pm 4$) — when $\theta = n\pi/4$ for any odd integer n.

25. Domain: $(-\infty, \infty)$
Range: $r = 3$
Symmetric about the x-axis, y-axis, and origin
Continuous
Bounded
Maximum $|r|$ value: 3
No asymptotes



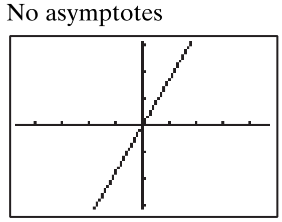
[–6, 6] by [–4, 4]

26. Domain: $(-\infty, \infty)$
Range: $r = 2$
Symmetric about the x-axis, y-axis, and origin
Continuous
Bounded
Maximum $|r|$ value: 2
No asymptotes



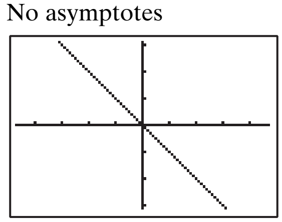
[–4.5, 4.5] by [–3, 3]

27. Domain: $\theta = \pi/3$
Range: $(-\infty, \infty)$
Symmetric about the origin
Continuous
Unbounded
Maximum $|r|$ value: none
No asymptotes



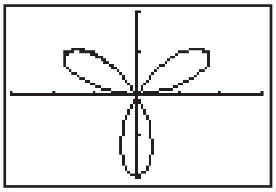
[−4.7, 4.7] by [−3.1, 3.1]

28. Domain: $\theta = -\pi/4$
Range: $(-\infty, \infty)$
Symmetric about the origin
Continuous
Unbounded
Maximum $|r|$ value: none
No asymptotes



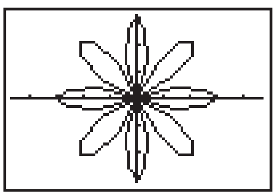
[−4.7, 4.7] by [−3.1, 3.1]

29. Domain: $(-\infty, \infty)$
Range: $[-2, 2]$
Symmetric about the y-axis
Continuous
Bounded
Maximum $|r|$ value: 2
No asymptotes



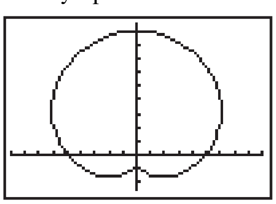
[–3, 3] by [–2, 2]

30. Domain: $(-\infty, \infty)$
Range: $[-3, 3]$
Symmetric about the x-axis, y-axis, and origin
Continuous
Bounded
Maximum $|r|$ value: 3
No asymptotes



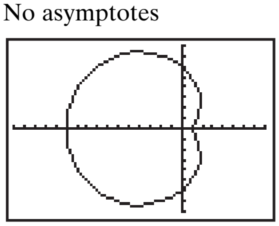
[–4.7, 4.7] by [–3.1, 3.1]

31. Domain: $(-\infty, \infty)$
Range: $[1, 9]$
Symmetric about the y-axis
Continuous
Bounded
Maximum $|r|$ value: 9
No asymptotes



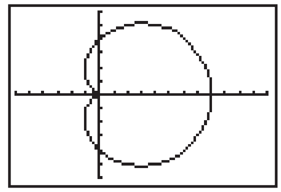
[–9, 9] by [–2.5, 9.5]

32. Domain: $(-\infty, \infty)$
Range: $[1, 11]$
Symmetric about the x-axis
Continuous
Bounded
Maximum $|r|$ value: 11
No asymptotes



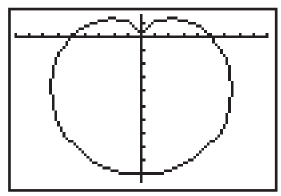
[–16, 8] by [–8, 8]

33. Domain: $(-\infty, \infty)$
Range: $[0, 8]$
Symmetric about the x-axis
Continuous
Bounded
Maximum $|r|$ value: 8
No asymptotes



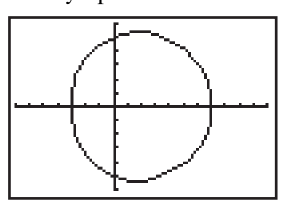
[–6, 12] by [–6, 6]

34. Domain: $(-\infty, \infty)$
Range: $[0, 10]$
Symmetric about the y-axis
Continuous
Bounded
Maximum $|r|$ value: 10
No asymptotes



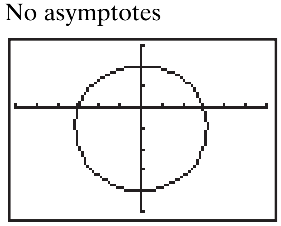
[–9, 9] by [–10.5, 1.5]

35. Domain: $(-\infty, \infty)$
Range: $[3, 7]$
Symmetric about the x-axis
Continuous
Bounded
Maximum $|r|$ value: 7
No asymptotes



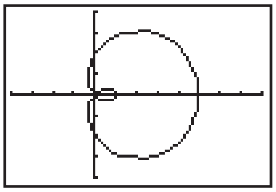
[–7, 11] by [–6, 6]

36. Domain: $(-\infty, \infty)$
Range: $[2, 4]$
Symmetric about the y-axis
Continuous
Bounded
Maximum $|r|$ value: 4
No asymptotes



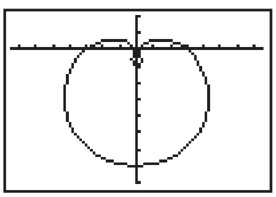
[–6, 6] by [–6, 3]

37. Domain: $(-\infty, \infty)$
Range: $[-3, 7]$
Symmetric about the x-axis
Continuous
Bounded
Maximum $|r|$ value: 7
No asymptotes



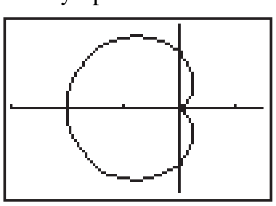
[–4, 8] by [–4, 4]

38. Domain: $(-\infty, \infty)$
Range: $[-1, 7]$
Symmetric about the y-axis
Continuous
Bounded
Maximum $|r|$ value: 7
No asymptotes



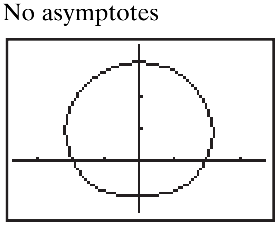
[–7.5, 7.5] by [–8, 2]

39. Domain: $(-\infty, \infty)$
Range: $[0, 2]$
Symmetric about the x-axis
Continuous
Bounded
Maximum $|r|$ value: 2
No asymptotes



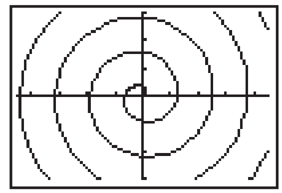
[–3, 1.5] by [–1.5, 1.5]

40. Domain: $(-\infty, \infty)$
Range: $[1, 3]$
Symmetric about the y-axis
Continuous
Bounded
Maximum $|r|$ value: 3
No asymptotes



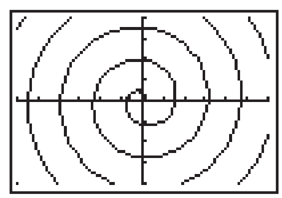
[–3.75, 3.75] by [–1.5, 3.5]

41. Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Continuous
No symmetry
Unbounded
Maximum $|r|$ value: none
No asymptotes
Graph for $\theta \geq 0$:



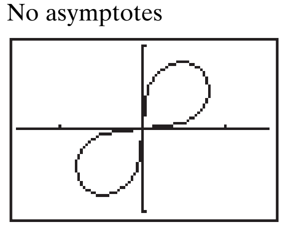
[–45, 45] by [–30, 30]

42. Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Continuous
No symmetry
Unbounded
Maximum $|r|$ value: none
No asymptotes
Graph for $\theta \geq 0$:



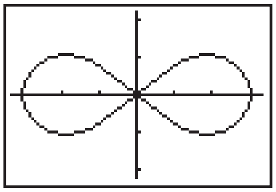
[–6, 6] by [–4, 4]

43. Domain: $\left[0, \frac{\pi}{2}\right] \cup \left[\pi, \frac{3\pi}{2}\right]$
Range: $[0, 1]$
Symmetric about the origin
Continuous on each interval in domain
Bounded
Maximum $|r|$ value: 1
No asymptotes



[–1.5, 1.5] by [–1, 1]

44. Domain: $\left[0, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \frac{5\pi}{4}\right] \cup \left[\frac{7\pi}{4}, 2\pi\right]$
Range: $[0, 3]$
Symmetric about the x-axis, y-axis, and origin
Continuous on each interval in domain
Bounded
Maximum $|r|$ value: 3
No asymptotes



[–3.3, 3.3] by [–2.2, 2.2]

For #45–48, recall that the petal length is the maximum $|r|$ value over the interval that creates the petal.

45. $r = -2$ when $\theta = \left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$ and $r = 6$ when $\theta = \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$. There are four petals with lengths $\{6, 2, 6, 2\}$.

46. $r = -2$ when $\theta = \{0, \pi\}$ and $r = 8$ when $\theta = \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$. There are four petals with lengths $\{2, 8, 2, 8\}$.

47. $r = -3$ when $\theta = \left\{0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}\right\}$ and $r = 5$ when $\theta = \left\{\frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}\right\}$.
There are ten petals with lengths $\{3, 5, 3, 5, 3, 5, 3, 5, 3, 5\}$.

48. $r = 7$ when $\theta = \left\{\frac{\pi}{10}, \frac{\pi}{2}, \frac{9\pi}{10}, \frac{13\pi}{10}, \frac{17\pi}{10}\right\}$ and $r = -1$ when $\theta = \left\{\frac{3\pi}{10}, \frac{7\pi}{10}, \frac{11\pi}{10}, \frac{3\pi}{2}, \frac{19\pi}{10}\right\}$.
There are ten petals with lengths $\{7, 1, 7, 1, 7, 1, 7, 1, 7, 1\}$.

49. r_1 and r_2 produce identical graphs — r_1 begins at $(1, 0)$ and r_2 begins at $(-1, 0)$.

50. r_1 and r_3 produce identical graphs — r_1 begins at $(3, 0)$ and r_2 begins at $(1, 0)$.

51. r_2 and r_3 produce identical graphs — r_1 begins at $(3, 0)$ and r_3 begins at $(-3, 0)$.

52. r_1 and r_2 produce identical graphs — r_1 begins at $(2, 0)$ and r_2 begins at $(-2, 0)$.

53. **(a)** A 4-petal rose curve with 2 short petals of length 1 and 2 long petals of length 3.
(b) Symmetric about the origin.
(c) Maximum $|r|$ value: 3.

54. **(a)** A 4-petal rose curve with petals of about length 1, 3.3, and 4 units.
(b) Symmetric about the y-axis.
(c) Maximum $|r|$ value: 4.

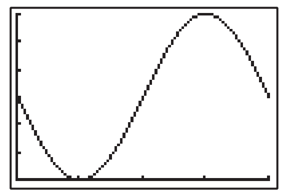
55. **(a)** A 6-petal rose curve with three short petals of length 2 and three long petals of length 4.
(b) Symmetric about the x-axis.
(c) Maximum $|r|$ value: 4.

56. **(a)** A 6-petal rose curve with three short petals of length 2 and three long petals of length 4.
(b) Symmetric about the y-axis.
(c) Maximum $|r|$ value: 4.

57. Answers will vary but generally students should find that a controls the length of the rose petals and n controls both the number of rose petals and symmetry. If n is odd, n rose petals are formed, with the cosine curve symmetric about the polar x-axis and sine curve symmetric about the y-axis. If n is even, $2n$ rose petals are formed, with both the cosine and sine functions having symmetry about the polar x-axis, y-axis, and origin.

58. Symmetry about y-axis: $r - 3\sin(4\theta) = 0 \Rightarrow$ $-r - 3\sin(4(-\theta)) = -r + 3\sin(4\theta)$ (since $\sin(\theta)$ is odd, i.e., $\sin(-\theta) = -\sin(\theta)$) $= r - 3\sin(4\theta) = 0$.
Symmetry about the origin: $r - 3\sin(4\theta) = 0$ $\Rightarrow r - 3\sin(4\theta + 4\pi) = r - 3\sin(4\theta) = 0$.

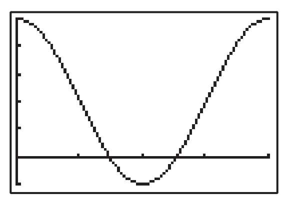
59.



$[0, 2\pi]$ by $[0, 6]$

$y = 3 - 3\sin x$ has minimum and maximum values of 0 and 6 on $[0, 2\pi]$. So the range of the polar function $r = 3 - 3\sin\theta$ is also $[0, 6]$.

60.



$[0, 2\pi]$ by $[-1, 5]$

$y = 2 + 3\cos x$ has minimum and maximum values of -1 and 5 on $[0, 2\pi]$. So the range of the polar function $r = 2 + 3\cos\theta$ is also $[-1, 5]$.

In general, this works because any polar graph can also be plotted using rectangular coordinates. Here, we have y representing r and x representing θ on a rectangular coordinate graph. Since y is exactly equal to r, the range of y and range of r will be exactly the same.

61. False. The spiral $r = \theta$ is unbounded, since a point on the curve can be found at any arbitrarily large distance from the origin by setting θ numerically equal to that distance.

62. True. If point (r, θ) satisfies the equation $r = 2 + \cos\theta$, then point $(r, -\theta)$ does also, since $2 + \cos(-\theta) = 2 + \cos\theta = r$.

63. With $r = a\cos n\theta$, if n is even there are $2n$ petals.
The answer is D.

64. The four petals lie along the x- and y-axis, because $\cos 2\theta$ takes on its extreme values at multiples of $\pi/2$.
The answer is D.

65. When $\cos\theta = -1$, $r = 5$. The answer is B.

66. With $r = a\sin n\theta$, if n is odd there are n petals.
The answer is B.

67. **(a)** Symmetry about the polar x-axis: $r - a\cos(n\theta) = 0$ $\Rightarrow r - a\cos(-n\theta) = r - a\cos(n\theta)$ (since $\cos(\theta)$ is even, i.e., $\cos(\theta) = \cos(-\theta)$ for all θ) $= 0$.

(b) No symmetry about y-axis: $r - a\cos(n\theta) = 0 \Rightarrow -r - a\cos(-n\theta) = -r - a\cos(n\theta)$ (since $\cos(\theta)$ is even) $\neq r - a\cos(n\theta)$ unless $r = 0$. As a result, the equation is not symmetric about the y-axis.

(c) No symmetry about origin: $r - a\cos(n\theta) = 0 \Rightarrow -r - a\cos(n\theta) \neq -r - a\cos(n\theta)$ unless $r = 0$. As a result, the equation is not symmetric about the origin.

(d) Since $|\cos(n\theta)| \leq 1$ for all θ, the maximum $|r|$ value is $|a|$.

(e) Domain: $(-\infty, \infty)$
Range: $[-|a|, |a|]$
Symmetric about the x-axis
Continuous
Bounded
Maximum $|r|$ value: $|a|$
No asymptotes

68. (a) Symmetry about the y-axis: $r - a\sin(n\theta) = 0 \Rightarrow -r - a\sin(-n\theta) = -r + a\sin(n\theta)$ (since $\sin(\theta)$ is odd, $\sin(-\theta) = -\sin(\theta)) = -1(r - a\sin(n\theta)) = (-1)(0) = 0$.

(b) Not symmetric about polar x-axis: $r - a\sin(n\theta) = 0 \Rightarrow r - a\sin(-n\theta) = r + a\sin(n\theta)$. The two functions are equal only when $-\sin(n\theta) = \sin(n\theta) = 0$, or $\theta = \{0, \pi\}$, so $r - a\sin(n\theta)$ is not symmetric about the polar x-axis.

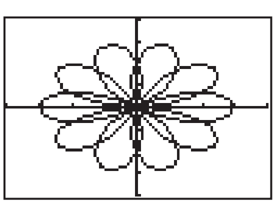
(c) Not symmetric about origin: $r - a\sin(n\theta) \Rightarrow -r - a\sin(n\theta) = -(r + a\sin(n\theta))$. The two functions are equal only when $r = 0$, so $r - a\sin(n\theta))$ is not symmetric about the origin.

(d) Since $|\sin(n\theta)| \leq 1$ for all θ, the maximum $|r|$ value is $|a|$.

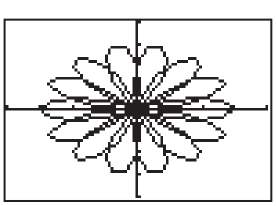
(e) Domain: $(-\infty, \infty)$
Range: $[-|a|, |a|]$
Symmetric about y-axis
Continuous
Bounded
Maximum $|r|$ value: $|a|$
No asymptotes

69. (a) For r_1: $0 \leq \theta \leq 4\pi$ (or any interval that is 4π units long). For r_2: same answer.

(b) r_1: 10 (overlapping) petals. r_2: 14 (overlapping) petals.



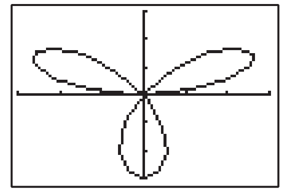
[–4, 4] by [–4, 4]



[–4, 4] by [–4, 4]

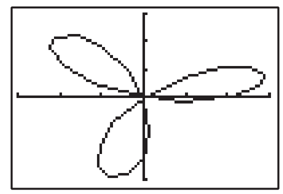
70. Starting with the graph of r_1, if we rotate clockwise (centered at the origin) by $\pi/12$ radians (15°), we get the graph of r_2; rotating r_1 clockwise by $\pi/4$ radians (45°) gives the graph of r_3.

(a)



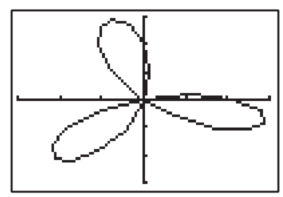
$[-3, 3]$ by $[-3, 3]$

(b)



$[-3, 3]$ by $[-3, 3]$

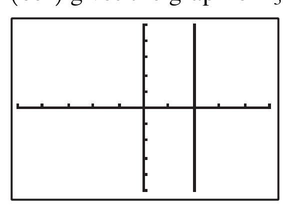
(c)



$[-3, 3]$ by $[-3, 3]$

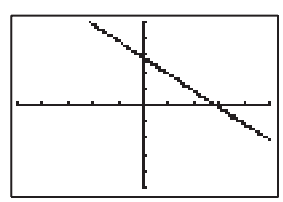
71. Starting with the graph of r_1, if we rotate counterclockwise (centered at the origin) by $\pi/4$ radians (45°), we get the graph of r_2; rotating r_1 counterclockwise by $\pi/3$ radians (60°) gives the graph of r_3.

(a)



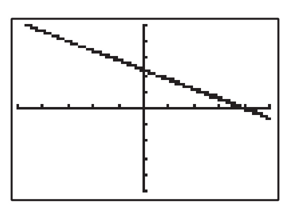
$[-5, 5]$ by $[-5, 5]$

(b)



$[-5, 5]$ by $[-5, 5]$

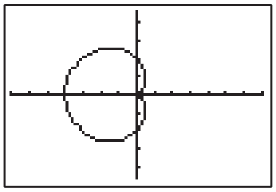
(c)



$[-5, 5]$ by $[-5, 5]$

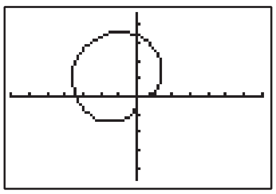
72. Starting with the graph of r_1, if we rotate clockwise (centered at the origin) by $\pi/4$ radians (45°), we get the graph of r_2; rotating r_1 clockwise by $\pi/3$ radians (60°) gives the graph of r_3.

(a)



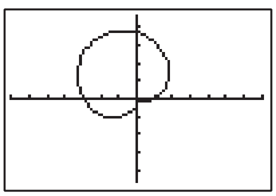
[−7.05, 7.05] by [−4.65, 4.65]

(b)



[−7.05, 7.05] by [−4.65, 4.65]

(c)



[−7.05, 7.05] by [−4.65, 4.65]

73. The second graph is the result of rotating the first graph clockwise (centered at the origin) through an angle of α. The third graph results from rotating the first graph counterclockwise through the same angle. One possible explanation: the radius r achieved, for example, when $\theta = 0$ in the first equation is achieved instead when $\theta = -\alpha$ for the second equation, and when $\theta = \alpha$ for the third equation.

■ Section 6.6 DeMoivre's Theorem and *n*th Roots

Quick Review 6.6

1. Using the quadratic equation to find the roots of $x^2 + 13 = 4x$, we have $x^2 - 4x + 13 = 0$ with $a = 1$, $b = -4$, and $c = 13$.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 52}}{2} = \frac{4 \pm \sqrt{-36}}{2} = \frac{4 \pm 6i}{2}$$

$$x = \frac{4 + 6i}{2} = \frac{4}{2} + \frac{6i}{2} = 2 + 3i \text{ and}$$

$$x = \frac{4 - 6i}{2} = \frac{4}{2} - \frac{6i}{2} = 2 - 3i$$

The roots are $2 + 3i$ and $2 - 3i$.

2. Using the quadratic equation to find the roots of $5(x^2 + 1) = 6x$, we have $5x^2 - 6x + 5 = 0$ with $a = 5$, $b = -6$, and $c = 5$.

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(5)(5)}}{2(5)}$$

$$= \frac{6 \pm \sqrt{36 - 100}}{10} = \frac{6 \pm \sqrt{-64}}{10} = \frac{6 \pm 8i}{10}$$

$$x = \frac{6 + 8i}{10} = \frac{6}{10} + \frac{8i}{10} = 0.6 + 0.8i \text{ and}$$

$$x = \frac{6 - 8i}{10} = \frac{6}{10} + \frac{8i}{10} = 0.6 - 0.8i$$

The roots are $0.6 + 0.8i$ and $0.6 - 0.8i$.

3. $(1 + i)^5 = (1 + i) \cdot [(1 + i)^2]^2 = (1 + i) \cdot (2i)^2$
$= -4(1 + i) = -4 - 4i$

4. $(1 - i)^4 = [(1 - i)^2]^2 = (-2i)^2 = -4 = -4 + 0i$

For #5–8, use the given information to find a point P on the terminal side of the angle, which in turn determines the quadrant of the terminal side.

5. $P(-\sqrt{3}, 1)$, in Quadrant II: $\theta = \frac{5\pi}{6}$

6. $P(1, -1)$, in Quadrant IV: $\theta = \frac{7\pi}{4}$

7. $P(-1, -\sqrt{3},)$, in Quadrant III: $\theta = \frac{4\pi}{3}$

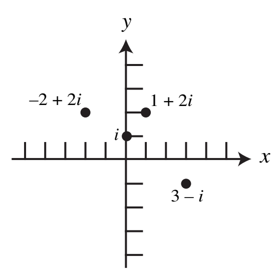
8. $P(-1, -1)$, in Quadrant III: $\theta = \frac{5\pi}{4}$

9. $x^3 = 1$ when $x = 1$

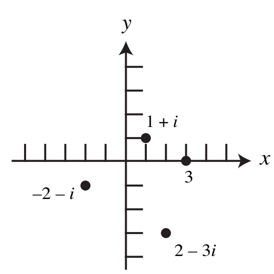
10. $x^4 = 1$ when $x = \pm 1$

Section 6.6 Exercises

1.



2.



For #3–12, $a + bi = r(\cos\theta + i\sin\theta)$, where $r = |a + bi| = \sqrt{a^2 + b^2}$ and θ is chosen so that $\cos\theta = \frac{a}{\sqrt{a^2 + b^2}}$ and $\sin\theta = \frac{b}{\sqrt{a^2 + b^2}}$.

3. $r = |3i| = 3$; $\cos\theta = 0$ and $\sin\theta = 1$, so $\theta = \frac{\pi}{2}$:

$$3i = 3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

4. $r = |-2i| = 2$; $\cos\theta = 0$ and $\sin\theta = -1$, so $\theta = \frac{3\pi}{2}$: $-2i = 2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

5. $r = |2 + 2i| = 2\sqrt{2}$; $\cos\theta = \frac{\sqrt{2}}{2}$ and $\sin\theta = \frac{\sqrt{2}}{2}$, so $\theta = \frac{\pi}{4}$: $2 + 2i = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

6. $r = |\sqrt{3} + i| = 2$; $\cos\theta = \frac{\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{2}$, so $\theta = \frac{\pi}{6}$: $\sqrt{3} + i = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

7. $r = |-2 + 2i\sqrt{3}| = 4$; $\cos\theta = -\frac{1}{2}$ and $\sin\theta = \frac{\sqrt{3}}{2}$, so $\theta = \frac{2\pi}{3}$: $-2 + 2i\sqrt{3} = 4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

8. $r = |3 - 3i| = 3\sqrt{2}$; $\cos\theta = \frac{\sqrt{2}}{2}$ and $\sin\theta = -\frac{\sqrt{2}}{2}$, so $\theta = \frac{7\pi}{4}$: $3 - 3i = 3\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

9. $r = |3 + 2i| = \sqrt{13}$; $\cos\theta = \frac{3}{\sqrt{13}}$ and $\sin\theta = \frac{2}{\sqrt{13}}$, so $\theta \approx 0.588$: $3 + 2i \approx \sqrt{13}(\cos 0.59 + i\sin 0.59)$

10. $r = |4 - 7i| = \sqrt{65}$; $\cos\theta = \frac{4}{\sqrt{65}}$ and $\sin\theta = -\frac{7}{\sqrt{65}}$, so $\theta \approx 5.232$: $4 - 7i \approx \sqrt{65}\,(\cos 5.23 + i\sin 5.23)$

11. $r = 3$; $30° = \frac{\pi}{6}$; $3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

12. $r = 3$; $225° = \frac{5\pi}{4}$; $4\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

13. $3\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \frac{3\sqrt{3}}{2} - \frac{3}{2}i$

14. $8\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = -4\sqrt{3} - 4i$

15. $5\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = \frac{5}{2} - \frac{5\sqrt{3}}{2}i$

16. $5\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) = \frac{5\sqrt{2}}{2} + \frac{5\sqrt{2}}{2}i$

17. $\sqrt{2}\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i$

18. $\approx 2.56 + 0.68i$

19. $(2\cdot 7)[\cos(25° + 130°) + i\sin(25° + 130°)]$
$= 14\,(\cos 155° + i\sin 155°)$

20. $(\sqrt{2}\cdot 0.5)[\cos(188° - 19°) + i\sin(118° - 19°)]$
$= \frac{\sqrt{2}}{2}\,(\cos 99° + i\sin 99°)$

21. $(5\cdot 3)\left[\cos\left(\frac{\pi}{4} + \frac{5\pi}{3}\right) + i\sin\left(\frac{\pi}{4} + \frac{5\pi}{3}\right)\right]$
$= 15\left(\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}\right)$

22. $\left(\sqrt{3}\cdot\frac{1}{3}\right)\left[\cos\left(\frac{3\pi}{4} + \frac{\pi}{6}\right) + i\sin\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)\right]$
$= \frac{\sqrt{3}}{3}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$

23. $\frac{2}{3}[\cos(30° - 60°) + i\sin(30° - 60°)]$
$= \frac{2}{3}[\cos(-30°) + i\sin(-30°)] = \frac{2}{3}(\cos 30° - i\sin 30°)$

24. $\frac{5}{2}[\cos(220° - 115°) + i\sin(220° - 115°)]$
$= \frac{5}{2}(\cos 105° + i\sin 105°)$

25. $\frac{6}{3}[\cos(5\pi - 2\pi) + i\sin(5\pi - 2\pi)]$
$= 2(\cos 3\pi + i\sin 3\pi) = 2(\cos\pi + i\sin\pi)$

26. $1\left[\cos\left(\frac{\pi}{2} - \frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{2} - \frac{\pi}{4}\right)\right] = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$

27. (a) $3 - 2i \approx \sqrt{13}\,[\cos(5.695) + i\sin(5.695)]$ and $1 + i = \sqrt{2}\left(\cos\frac{\pi}{4} + \text{i}\sin\frac{\pi}{4}\right)$, so

$\sqrt{13}\,[\cos(5.695) + i\sin(5.695)]\cdot\sqrt{2}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]$

$= \sqrt{26}\left[\cos\left(5.695 + \frac{\pi}{4}\right) + i\sin\left(5.695 + \frac{\pi}{4}\right)\right] = 5 + i$

$\frac{\sqrt{13}[\cos(5.695) + i\sin(5.695)]}{\sqrt{2}[\cos(\pi/4) + \text{i}\sin(\pi/4)]}$

$\approx \sqrt{6.5}\left[\cos\left(5.695 - \frac{\pi}{4}\right) + i\sin\left(5.695 - \frac{\pi}{4}\right)\right]$

$= \frac{1}{2} - \frac{5}{2}i$

(b) $(3 - 2i)(1 + i) = 3 + 3i - 2i - 2i^2 = 5 + i$

$\frac{3 - 2i}{1 + i} = \frac{3 - 2i}{1 + i}\cdot\frac{1 - i}{1 - i} = \frac{1 - 5i}{2} = \frac{1}{2} - \frac{5}{2}i$

28. (a) $1 - i = \sqrt{2}\,(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4})$ and $\sqrt{3} + i = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$, so

$\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)\cdot 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$= 2\sqrt{2}\left(\cos\frac{23\pi}{12} + \sin\frac{23\pi}{12}\right) \approx 2.73 - 0.73i$

$\frac{\sqrt{2}[\cos(7\pi/4) + i\sin(7\pi/4)]}{2[\cos(\pi/6) + i\sin(\pi/6)]}$

$= \frac{1}{\sqrt{2}}\left(\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12}\right) \approx 0.18 - 0.68i$

(b) $(1 - i)(\sqrt{3} + i) = \sqrt{3} + i - \sqrt{3}i - i^2$
$(1 + \sqrt{3}) + (1 - \sqrt{3})i \approx 2.73 - 0.73i$
$\frac{1 - i}{\sqrt{3} + i} = \frac{1 - i}{\sqrt{3} + i} \cdot \frac{\sqrt{3} - i}{\sqrt{3} - i} = \frac{(1 - i)(\sqrt{3} - i)}{4}$
$= \frac{1}{4}\left[\sqrt{3} - 1 - (\sqrt{3} + 1)i\right] \approx 0.18 - 0.68i$

29. (a) $3 + i \approx \sqrt{10}[\cos(0.321) + i\sin(0.321)]$
and $5 - 3i \approx \sqrt{34}[\cos(-0.540) + i\sin(-0.540)]$, so
$\sqrt{10}[\cos(0.321) + i\sin(0.321)] \cdot \sqrt{34}[\cos(-0.540) + i\sin(-0.540)]$
$= 2\sqrt{85}[\cos(-0.219) + i\sin(-0.219)] = 18 - 4i$
$\frac{\sqrt{10}[\cos(0.321) + i\sin(0.321)]}{\sqrt{34}[\cos(-0.540) + i\sin(-0.540)]}$
$\approx \sqrt{\frac{5}{17}}[\cos(0.862) + i\sin(0.862)]$
$\approx 0.35 + 0.41i$

(b) $(3 + i)(5 - 3i) = 15 - 9i + 5i - 3i^2 = 18 - 4i$
$\frac{3 + i}{5 - 3i} = \frac{3 + i}{5 - 3i} \cdot \frac{5 + 3i}{5 + 3i} = \frac{(3 + i)(5 + 3i)}{34}$
$\frac{1}{17}(6 + 7i) \approx 0.35 + 0.41i$

30. (a) $2 - 3i \approx \sqrt{13}[\cos(-0.982) + i\sin(-0.982)]$,
and $1 - \sqrt{3}i = 2\left[\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right]$, so
$\sqrt{13}[\cos(-0.983) + i\sin(-0.983)] \cdot \left[\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right] = 2\sqrt{13}\left[\cos\left(-0.983 - \frac{\pi}{3}\right) + i\sin\left(-0.983 - \frac{\pi}{3}\right)\right]$
$\approx -3.20 - 6.46i$
$\frac{\sqrt{13}[\cos(-0.983) + i\sin(-0.983)]}{2[\cos(-\pi/3) + i\sin(-\pi/3)]}$
$\approx \frac{\sqrt{13}}{2}[\cos(0.064) + i\sin(0.064)]$
$\approx 1.80 + 0.12i$

(b) $(2 - 3i)(1 - \sqrt{3}i) = 2 - 2\sqrt{3}i - 3i + 3\sqrt{3}i^2$
$= (2 - 3\sqrt{3}) - (2\sqrt{3} + 3)i \approx -3.196 - 6.464i$
$\frac{2 - 3i}{1 - \sqrt{3}i} = \frac{2 - 3i}{1 - \sqrt{3}i} \cdot \frac{1 + \sqrt{3}i}{1 + \sqrt{3}i}$
$= \frac{(2 - 3i)(1 + \sqrt{3}i)}{4}$
$= \frac{1}{4}\left[2 + 3\sqrt{3} + (2\sqrt{3} - 3)i\right] \approx 1.80 + 0.12i$

31. $\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^3 = \cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}$
$= -\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

32. $\left[3\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)\right]^5 = 243\left(\cos\frac{15\pi}{2} + i\sin\frac{15\pi}{2}\right)$
$= -243i$

33. $\left[2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\right]^3 = 8\left(\cos\frac{9\pi}{4} + i\sin\frac{9\pi}{4}\right)$
$= 4\sqrt{2} + 4\sqrt{2}i$

34. $\left[6\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)\right]^4$
$= 1296\left(\cos\frac{20\pi}{6} + i\sin\frac{20\pi}{6}\right)$
$= -648 - 648\sqrt{3}i$

35. $(1 + i)^5 = \left[\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]^5$
$= (\sqrt{2})^5\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$
$= 4\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right) = -4 - 4i$

36. $(3 + 4i)^{20}$
$= \left\{5\left[\cos\tan^{-1}\left(\frac{4}{3}\right) + i\sin\tan^{-1}\left(\frac{4}{3}\right)\right]\right\}^{20}$
$= 5^{20}\left\{\cos\left[20\tan^{-1}\left(\frac{4}{3}\right)\right] + i\sin\left[20\tan^{-1}\left(\frac{4}{3}\right)\right]\right\}$
$= 5^{20}[\cos(5.979) + i\sin(5.979)] \approx 5^{20}(0.95 - 0.30i)$

37. $(1 - \sqrt{3}i)^3 = \left[2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)\right]^3$
$= 8(\cos 5\pi + i\sin 5\pi) = 8(\cos\pi + i\sin\pi)$
$= -8 + 0i = -8$

38. $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^3 = \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^3$
$= \cos\pi + i\sin\pi = -1 + 0i = -1$

For #39–44, the cube roots of $r(\cos\theta + i\sin\theta)$ are
$\sqrt[3]{r} = \left(\cos\frac{\theta + 2k\pi}{3} + i\sin\frac{\theta + 2k\pi}{3}\right), k = 0, 1, 2.$

39. $\sqrt[3]{2}\left(\cos\frac{2k\pi + 2\pi}{3} + i\sin\frac{2k\pi + 2\pi}{3}\right)$
$= \sqrt[3]{2}\left(\cos\frac{2\pi(k + 1)}{3} + i\sin\frac{2\pi(k + 1)}{3}\right)$,
$k = 0, 1, 2$:
$\sqrt[3]{2}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = \sqrt[3]{2}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$
$= \frac{-1 + \sqrt{3}\,i}{\sqrt[3]{4}}$,
$\sqrt[3]{2}\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$
$= \sqrt[3]{2}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = \frac{-1 - \sqrt{3}\,i}{\sqrt[3]{4}}$,
$\sqrt[3]{2}(\cos 2\pi + i\sin 2\pi) = \sqrt[3]{2}$

40. $\sqrt[3]{2}\left(\cos\frac{2k\pi + \pi/4}{3} + i\sin\frac{2k\pi + \pi/4}{3}\right)$
$= \sqrt[3]{2}\left(\cos\frac{\pi(8k + 1)}{12} + i\sin\frac{\pi(8k + 1)}{12}\right)$,
$k = 0, 1, 2$:

$\sqrt[3]{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$,

$\sqrt[3]{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

$\sqrt[3]{2}\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) = \frac{-1+i}{\sqrt[6]{2}}$,

$\sqrt[3]{2}\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right)$

41. $\sqrt[3]{3}\left(\cos\frac{2k\pi + 4\pi/3}{3} + i\sin\frac{2k\pi + 4\pi/3}{3}\right)$

$= \sqrt[3]{3}\left(\cos\frac{2\pi(3k+2)}{9} + i\sin\frac{2\pi(3k+2)}{9}\right)$,

$k = 0, 1, 2$:

$\sqrt[3]{3}\left(\cos\frac{4\pi}{9} + i\sin\frac{4\pi}{9}\right)$, $\sqrt[3]{3}\left(\cos\frac{10\pi}{9} + i\sin\frac{10\pi}{9}\right)$,

$\sqrt[3]{3}\left(\cos\frac{16\pi}{9} + i\sin\frac{16\pi}{9}\right)$

42. $\sqrt[3]{27}\left(\cos\frac{2k\pi + 11\pi/6}{3} + i\sin\frac{2k\pi + 11\pi/6}{3}\right)$

$= 3\left(\cos\frac{\pi(12k+11)}{18} + i\sin\frac{\pi(12k+11)}{18}\right)$,

$k = 0, 1, 2$:

$3\left(\cos\frac{11\pi}{18} + i\sin\frac{11\pi}{18}\right)$, $3\left(\cos\frac{23\pi}{18} + i\sin\frac{23\pi}{18}\right)$,

$3\left(\cos\frac{35\pi}{18} + i\sin\frac{35\pi}{18}\right)$

43. $3 - 4i \approx 5\,(\cos 5.355 + i\sin 5.355)$

$\sqrt[3]{5}\left(\cos\frac{2k\pi + 5.355}{3} + i\sin\frac{2k\pi + 5.355}{3}\right)$

$k = 0, 1, 2$:

$\approx \sqrt[3]{5}(\cos 1.79 + i\sin 1.79)$,

$\approx \sqrt[3]{5}\,(\cos 3.88 + i\sin 3.88)$,

$\approx \sqrt[3]{5}(\cos 5.97 + i\sin 5.97)$

44. $-2 + 2i = 2\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$. Note that

$\sqrt[3]{2\sqrt{2}} = \sqrt{2}$.

$\sqrt{2}\left(\cos\frac{2k\pi + 3\pi/4}{3} + i\sin\frac{2k\pi + 3\pi/4}{3}\right)$

$= \sqrt{2}\left(\cos\frac{\pi(8k+3)}{12} + i\sin\frac{\pi(8k+3)}{12}\right)$,

$k = 0, 1, 2$:

$\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = \sqrt{2}\left(\frac{\sqrt{2}}{2} + \frac{i\sqrt{2}}{2}\right) = 1 + i$,

$\sqrt{2}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$,

$\sqrt{2}\left(\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12}\right)$

For #45–50, the fifth roots of $r\,(\cos\theta + i\sin\theta)$ are

$\sqrt[5]{r}\left(\cos\frac{\theta + 2k\pi}{5} + i\sin\frac{\theta + 2k\pi}{5}\right), k = 0, 1, 2, 3, 4.$

45. $\cos\frac{2k\pi + \pi}{5} + i\sin\frac{2k\pi + \pi}{5}$

$= \cos\frac{\pi(2k+1)}{5} + i\sin\frac{\pi(2k+1)}{5}, k = 0, 1, 2, 3, 4$:

$\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}, \cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5}, -1,$

$\cos\frac{7\pi}{5} + i\sin\frac{7\pi}{5}, \cos\frac{9\pi}{5} + i\sin\frac{9\pi}{5}$

46. $\sqrt[5]{32}\left(\cos\frac{2k\pi + \pi/2}{5} + i\sin\frac{2k\pi + \pi/2}{5}\right)$

$= 2\left(\cos\frac{\pi(4k+1)}{10} + i\sin\frac{\pi(4k+1)}{10}\right)$,

$k = 0, 1, 2, 3, 4$:

$2\left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right), 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = 2i,$

$2\left(\cos\frac{9\pi}{10} + i\sin\frac{9\pi}{10}\right), 2\left(\cos\frac{13\pi}{10} + i\sin\frac{13\pi}{10}\right),$

$2\left(\cos\frac{17\pi}{10} + i\sin\frac{17\pi}{10}\right)$

47. $\sqrt[5]{2}\left(\cos\frac{2k\pi + \pi/6}{5} + i\sin\frac{2k\pi + \pi/6}{5}\right)$

$\sqrt[5]{2}\left(\cos\frac{\pi(12k+1)}{30} + \sin\frac{\pi(12k+1)}{30}\right)$,

$k = 0, 1, 2, 3, 4$:

$\sqrt[5]{2}\left(\cos\frac{\pi}{30} + i\sin\frac{\pi}{30}\right), \sqrt[5]{2}\left(\cos\frac{13\pi}{30} + i\sin\frac{13\pi}{30}\right),$

$\sqrt[5]{2}\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right), \sqrt[5]{2}\left(\cos\frac{37\pi}{30} + i\sin\frac{37\pi}{30}\right),$

$\sqrt[5]{2}\left(\cos\frac{49\pi}{30} + i\sin\frac{49\pi}{30}\right)$

48. $\sqrt[5]{2}\left(\cos\frac{2k\pi + \pi/4}{5} + i\sin\frac{2k\pi + \pi/4}{5}\right)$

$= \sqrt[5]{2}\left(\cos\frac{\pi(8k+1)}{20} + i\sin\frac{\pi(8k+1)}{20}\right)$,

$k = 0, 1, 2, 3, 4$:

$\sqrt[5]{2}\left(\cos\frac{\pi}{20} + i\sin\frac{\pi}{20}\right), \sqrt[5]{2}\left(\cos\frac{9\pi}{20} + i\sin\frac{9\pi}{20}\right),$

$\sqrt[5]{2}\left(\cos\frac{17\pi}{20} + i\sin\frac{17\pi}{20}\right), \sqrt[5]{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right) =$

$\sqrt[5]{2}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right) = \frac{-2^{1/5} - 2^{1/5}i}{2^{1/2}} =$

$\frac{-1-i}{2^{3/10}} = \frac{-1-i}{\sqrt[10]{8}}$,

$\sqrt[5]{2}\left(\cos\frac{33\pi}{20} + i\sin\frac{33\pi}{20}\right)$

49. $\sqrt[5]{2}\left(\cos\frac{2k\pi + \pi/2}{5} + i\sin\frac{2k\pi + \pi/2}{5}\right)$

$= \sqrt[5]{2}\left(\cos\frac{\pi(4k+1)}{10} + i\sin\frac{\pi(4k+1)}{10}\right)$,

$k = 0, 1, 2, 3, 4$:

$\sqrt[5]{2}\left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right), \sqrt[5]{2}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = \sqrt[5]{2}i,$

$\sqrt[5]{2}\left(\cos\frac{9\pi}{10} + i\sin\frac{9\pi}{10}\right), \sqrt[5]{2}\left(\cos\frac{13\pi}{10} + i\sin\frac{13\pi}{10}\right),$

$\sqrt[5]{2}\left(\cos\frac{17\pi}{10} + i\sin\frac{17\pi}{10}\right)$

50. $\sqrt[5]{2}\left(\cos\frac{2k\pi + \pi/3}{5} + i\sin\frac{2k\pi + \pi/3}{5}\right)$

$= \sqrt[5]{2}\left(\cos\frac{\pi(6k+1)}{15} + i\sin\frac{\pi(6k+1)}{15}\right),$

$k = 0, 1, 2, 3, 4:$

$\sqrt[5]{2}\left(\cos\frac{\pi}{15} + i\sin\frac{\pi}{15}\right), \sqrt[5]{2}\left(\cos\frac{7\pi}{15} + i\sin\frac{7\pi}{15}\right),$

$\sqrt[5]{2}\left(\cos\frac{13\pi}{15} + i\sin\frac{13\pi}{15}\right), \sqrt[5]{2}\left(\cos\frac{19\pi}{15} + i\sin\frac{19\pi}{15}\right),$

$\sqrt[5]{2}\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right) = 2^{1/5}\left(\frac{1}{2} - \frac{\sqrt{3}i}{2}\right) =$

$\frac{1-\sqrt{3}i}{2^{4/5}} = \frac{1-\sqrt{3}i}{\sqrt[5]{16}}$

For #51–56, the nth roots of $r(\cos\theta + i\sin\theta)$are

$\sqrt[n]{r}\left(\cos\frac{\theta + 2k\pi}{n} + i\sin\frac{\theta + 2k\pi}{n}\right), k = 0, 1, 2, \ldots, n-1.$

51. $1 + i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$, so the roots are

$\sqrt[4]{2}\left(\cos\frac{2k\pi + \pi/4}{4} + i\sin\frac{2k\pi + \pi/4}{4}\right)$

$= \sqrt[8]{2}\left(\cos\frac{\pi(8k+1)}{16} + i\sin\frac{\pi(8k+1)}{16}\right),$

$k = 0, 1, 2, 3:$

$\sqrt[8]{2}\left(\cos\frac{\pi}{16} + i\sin\frac{\pi}{16}\right), \sqrt[8]{2}\left(\cos\frac{9\pi}{16} + i\sin\frac{9\pi}{16}\right),$

$\sqrt[8]{2}\left(\cos\frac{17\pi}{16} + i\sin\frac{17\pi}{16}\right), \sqrt[8]{2}\left(\cos\frac{25\pi}{16} + i\sin\frac{25\pi}{16}\right)$

52. $1 - i = \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$, so the roots are

$\sqrt[6]{2}\left(\cos\frac{2k\pi + 7\pi/4}{6} + i\sin\frac{2k\pi + 7\pi/4}{6}\right)$

$\sqrt[12]{2}(\cos\frac{\pi(8k+7)}{24} + i\sin\frac{\pi(8k+7)}{24}\Big),$

$k = 0, 1, 2, 3, 4, 5:$

$\sqrt[12]{2}\left(\cos\frac{7\pi}{24} + i\sin\frac{7\pi}{24}\right), \sqrt[12]{2}\left(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8}\right),$

$\sqrt[12]{2}\left(\cos\frac{23\pi}{24} + i\sin\frac{23\pi}{24}\right), \sqrt[12]{2}\left(\cos\frac{31\pi}{24} + i\sin\frac{31\pi}{24}\right),$

$\sqrt[12]{2}\left(\cos\frac{13\pi}{8} + i\sin\frac{13\pi}{8}\right), \sqrt[12]{2}\left(\cos\frac{47\pi}{24} + i\sin\frac{47\pi}{24}\right)$

53. $2 + 2i = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$, so the roots are

$\sqrt[3]{2}\left(\cos\frac{2k\pi + p/4}{3} + i\sin\frac{2k\pi + \pi/4}{3}\right)$

$= \sqrt{2}\left(\cos\frac{\pi(8k+1)}{12} + i\sin\frac{\pi(8k+1)}{12}\right), k = 0, 1, 2:$

$\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right), -1 + i,$

$\sqrt{2}\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right)$

54. $-2 + 2i = 2\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$, so the roots are

$\sqrt[4]{2\sqrt{2}}\left(\cos\frac{2k\pi + 3\pi/4}{4} + i\sin\frac{2k\pi + 3\pi/4}{4}\right)$

$\sqrt[8]{8}\left(\cos\frac{\pi(8k+3)}{16} + i\sin\frac{\pi(8k+3)}{16}\right),$

$k = 0, 1, 2, 3:$

$\sqrt[8]{8}\left(\cos\frac{3\pi}{16} + i\sin\frac{3\pi}{16}\right), \sqrt[8]{8}\left(\cos\frac{11\pi}{16} + i\sin\frac{11\pi}{16}\right),$

$\sqrt[8]{8}\left(\cos\frac{19\pi}{16} + i\sin\frac{19\pi}{16}\right), \sqrt[8]{8}\left(\cos\frac{27\pi}{16} + i\sin\frac{27\pi}{16}\right)$

55. $-2i = 2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$, so the roots are

$\sqrt[6]{2}\left(\cos\frac{2k\pi + 3\pi/2}{6} + i\sin\frac{2k\pi + 3\pi/2}{6}\right)$

$= \sqrt[6]{2}\left(\cos\frac{\pi(4k+3)}{12} + i\sin\frac{\pi(4k+3)}{12}\right),$

$k = 0, 1, 2, 3, 4, 5:$

$\frac{1+i}{\sqrt[6]{4}}, \sqrt[6]{2}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right),$

$\sqrt[6]{2}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right), \sqrt[6]{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right),$

$\sqrt[6]{2}\left(\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12}\right), \sqrt[6]{2}\left(\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}\right)$

56. $32 = 32(\cos 0 + i\sin 0)$, so the roots are

$\sqrt[5]{32}\left(\cos\frac{2k\pi + 0}{5} + i\sin\frac{2k\pi + 0}{5}\right)$

$= 2\left(\cos\frac{2k\pi}{5} + i\sin\frac{2k\pi}{5}\right), k = 0, 1, 2, 3, 4:$

$2(\cos 0 + i\sin 0) = 2, 2\left(\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right),$

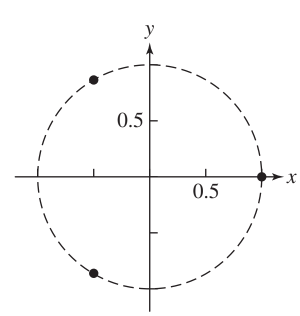
$2\left(\cos\frac{4\pi}{5} + i\sin\frac{4\pi}{5}\right), 2\left(\cos\frac{6\pi}{5} + i\sin\frac{6\pi}{5}\right),$

$2\left(\cos\frac{8\pi}{5} + i\sin\frac{8\pi}{5}\right)$

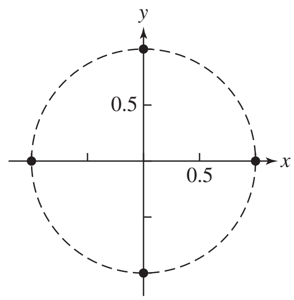
For #57–60, the nth roots of unity are

$\cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n},\ k = 0, 1, 2, \ldots, n-1.$

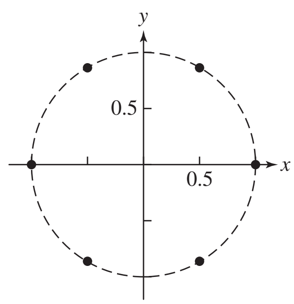
57. $1, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$



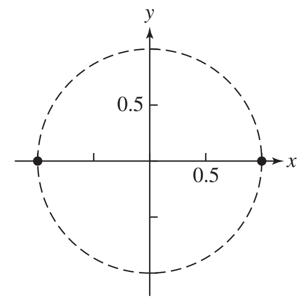
58. $\pm 1, \pm i$



59. $\pm 1, \frac{1}{2} \pm \frac{\sqrt{3}}{2}i, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$



60. $1, -1$



61. $z = (1 + \sqrt{3}i)^3 = \left[2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]^3$
$= 8(\cos\pi + i\sin\pi) = -8$; the cube roots are -2 and $1 \pm \sqrt{3}i$.

62. $z = (-2 - 2i)^4 = \left[2\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)\right]^4$
$= 64(\cos 5\pi + i\sin 5\pi) = -64$; the fourth roots are $2 \pm 2i$ and $-2 \pm 2i$.

63. $\frac{z_1}{z_2} = \frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} = \frac{r_1}{r_2} \cdot \frac{\cos\theta_1 + i\sin\theta_1}{\cos\theta_2 + i\sin\theta_2} \cdot \frac{\cos\theta_2 - i\sin\theta_2}{\cos\theta_2 - i\sin\theta_2}$

$= \frac{r_1}{r_2} \cdot \frac{\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 + i(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2)}{(\cos\theta_2)^2 + (\sin\theta_2)^2}$

$= \frac{r_1}{r_2} \cdot [\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 + i(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2)]$.

Now use the angle difference formulas:
$\cos(\theta_1 - \theta_2) = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2$ and $\sin(\theta_1 - \theta_2) = \sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2$.

So $\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$.

64. The n nth roots are given by $\sqrt[n]{r}\left(\cos\frac{2k\pi + \theta}{n} + i\sin\frac{2k\pi + \theta}{n}\right)$, $k = 0, 1, 2, \ldots n - 1$:

$\sqrt[n]{r}\left(\cos\frac{\theta}{n} + i\sin\frac{\theta}{n}\right), \sqrt[n]{r}\left(\cos\frac{\theta + 2\pi}{n} + i\sin\frac{\theta + 2\pi}{n}\right),$

$\sqrt[n]{r}\left(\cos\frac{\theta + 4\pi}{n} + i\sin\frac{\theta + 4\pi}{n}\right), \ldots, \sqrt[n]{r}\left(\cos\frac{\theta + 2(n-1)\pi}{n} + i\sin\frac{\theta + 2(n-1)\pi}{n}\right).$

The angles between successive values in this list differ by $\frac{2\pi}{n}$ radians, while the first and last roots differ by $2\pi - \frac{2\pi}{n}$, which also makes the angle between them $\frac{2\pi}{n}$. Also, the modulus of each root is $\sqrt[n]{r}$, placing it at that distance from the origin, on the circle with radius $\sqrt[n]{r}$.

65. False. If $z = r(\cos\theta + i\sin\theta)$, then it is also true that $z = r[\cos(\theta + 2n\pi) + i\sin(\theta + 2n\pi)]$ for any integer n.
For example, here are two trigonometric forms for $1 + i$: $\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right), \sqrt{2}\left(\cos\frac{9\pi}{4} + i\sin\frac{9\pi}{4}\right)$.

66. True. $i^3 = i^2 \cdot i = -i$, so i is a cube root of $-i$.

67. $2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = 2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -1 + \sqrt{3}i$

The answer is B.

68. Any complex number has n distinct nth roots, so $1 + i$ has five 5th roots. The answer is E.

69. $\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\cdot\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

$= (\sqrt{2}\cdot\sqrt{2})\left[\cos\left(\frac{\pi}{4} + \frac{7\pi}{4}\right) + i\sin\left(\frac{\pi}{4} + \frac{7\pi}{4}\right)\right]$

$= 2(\cos 2\pi + i\sin 2\pi)$

$= 2$

The answer is A.

70. $(\sqrt{i})^4 = [(\sqrt{i})^2]^2 = i^2 = -1 \neq 1$. The answer is E.

71. (a) $a + bi = r(\cos\theta + i\sin\theta)$, where $r = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}\left(\frac{b}{a}\right)$. Then $a + (-bi) = r'(\cos\theta' + i\sin\theta')$, where $r' = \sqrt{a^2 + (-b)^2}$ and $\theta' = \tan^{-1}\left(\frac{-b}{a}\right)$. Since $r' = \sqrt{a^2 + b^2} = r$ and $\theta' = \tan^{-1}\left(\frac{-b}{a}\right) = -\tan^{-1}\left(\frac{b}{a}\right) = -\theta$, we have $a - bi = r(\cos(-\theta) + i\sin(-\theta))$

(b) $z\cdot\bar{z} = r[\cos\theta + i\sin\theta]\cdot r[\cos(-\theta) + i\sin(-\theta)]$

$= r^2[\cos\theta\cos(-\theta) + i(\sin(-\theta))(\cos\theta) + i(\sin\theta)(\cos(-\theta)) - (\sin\theta)(\sin(-\theta))]$

Since $\sin\theta$ is an odd function (i.e., $\sin(-\theta) = -\sin(\theta)$) and $\cos\theta$ is an even function (i.e., $\cos(-\theta) = \cos\theta$), we have

$z\cdot\bar{z} = r^2[\cos^2\theta - i(\sin\theta)(\cos\theta) + i(\sin\theta)(\cos\theta) + \sin^2\theta]$

$= r^2[\cos^2\theta + \sin^2\theta]$

$= r^2$

(c) $\frac{z}{\bar{z}} = \frac{r[\cos\theta + i\sin\theta]}{r[\cos(-\theta) + i\sin(-\theta)]} = \cos[\theta - (-\theta)] + i\sin[\theta - (-\theta)]$

$= \cos(2\theta) + i\sin(2\theta)$

(d) $-z = -(a + bi) = -a + (-bi) = r(\cos\theta + i\sin\theta)$, where $r = \sqrt{(-a^2) + (-b)^2}$ and $\theta = \tan^{-1}\left(\frac{-b}{-a}\right) = \tan^{-1}\left(\frac{b}{a}\right)$

Recall, however, that $(-a, -b)$ is in the quadrant directly opposite the quadrant that holds (a, b) (i.e., if (a, b) is in Quadrant I, $(-a, -b)$ is in Quadrant III, and if (a, b) is in Quadrant II, $(-a, -b)$ is in Quadrant IV). Thus,

$-z = \sqrt{a^2 + b^2}(\cos(\theta + \pi) + i\sin(\theta + \pi)) = r(\cos(\theta + \pi) + i\sin(\theta + \pi))$.

72. (a) $|z| = \sqrt{(r\cos\theta)^2 + (r\sin\theta)^2}$

$= \sqrt{r^2\cos^2\theta + r^2\sin^2\theta}$

$= |r|\sqrt{\cos^2\theta + \sin^2\theta}$.

$= |r|$

(b) $|z_1\cdot z_2| = |[r_1\cos\theta_1 + (r_1\sin\theta_1)\text{i}]\cdot[\text{r}_2\cos\theta_2 + (r_2\sin\theta_2)i]|$

$= |r_1r_2\cos\theta_1\cos\theta_2 + (r_1r_2\cos\theta_1\sin\theta_2)i + (r_1r_2\sin\theta_1\cos\theta_2)i + (r_1r_2\sin\theta_1 u_2)i^2|$

$= |r_1r_2[\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 + (\cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2)i]|$

$= |r_1r_2[\cos(\theta_1 + \theta_2) + (\sin(\theta_1 + \theta_2))i]|$

$= \sqrt{(r_1r_2)^2[\cos^2(\theta_1 + \theta_2) + \sin^2(\theta_1 + \theta_2)]}$

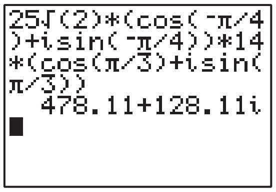
$= \sqrt{(r_1r_2)^2}$

$= |r_1|\cdot|r_2|$

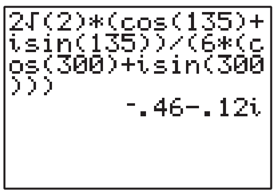
$= |z_1|\cdot|z_1|$ by **(a)**

73. Set the calculator for rounding to 2 decimal places. In **(b)**, use Degree mode.

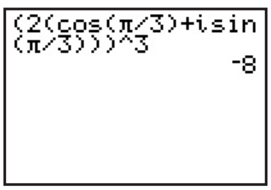
(a)



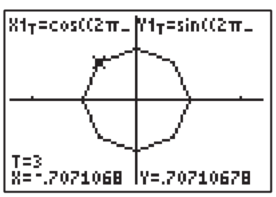
(b)



(c)



74. (a)



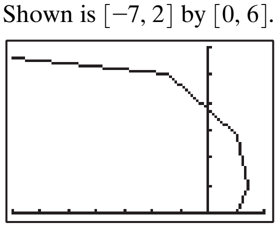
$[-2.4, 2.4]$ by $[-1.6, 1.6]$

(b) Yes. $6\pi/8$, $10\pi/8$, $14\pi/8$

(c) For the fifth and seventh roots of unity, all of the roots except the complex number 1 generate the corresponding roots of unity. For the sixth roots of unity, only $2\pi/6$ and $10\pi/6$ generate the sixth roots of unity.

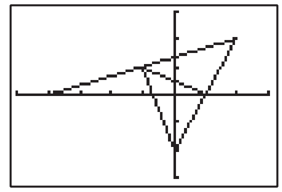
(d) $2\pi k/n$ generates the nth roots of unity if and only if k and n have no common factors other than 1.

75. Using $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right) \approx 0.62$, we have $\sqrt{2} + i \approx \sqrt{3}\,(\cos 0.62 + i \sin 0.62)$, so graph $x(t) = (\sqrt{3})^t \cos(0.62t)$ and $y(t) = (\sqrt{3})^t \sin(0.62t)$.
Use Tmin $= 0$, Tmax $= 4$, Tstep $= 1$.
Shown is $[-7, 2]$ by $[0, 6]$.



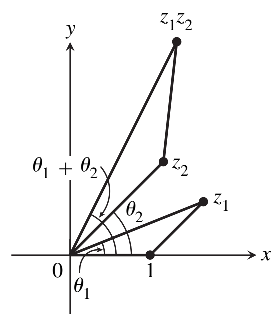
$[-7, 2]$ by $[0, 6]$

76. $-1 + i = \sqrt{2}\left(\cos\frac{3\pi}{4} + i \sin\frac{3\pi}{4}\right)$, so graph
$x(t) = (\sqrt{2})^t \cos(0.75\pi t)$ and $y(t) = (\sqrt{2})^t \sin(0.75\pi t)$
Use Tmin $= 0$, Tmax $= 4$, Tstep $= 1$.
Shown is $[-5, 3]$ by $[-3, 3]$.



$[-5, 3]$ by $[-3, 3]$

77. Suppose that $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$. Each triangle's angle at the origin has the same measure: For the smaller triangle, this angle has measure θ_1; for the larger triangle, the side from the origin out to z_2 makes an angle of θ_2 with the x-axis, while the side from the origin to z_1z_2 makes an angle of $\theta_1 + \theta_2$, so that the angle between is θ_1 as well.
The corresponding side lengths for the sides adjacent to these angles have the same ratio: the two longest sides have lengths $|z_1| = r_1$ (for the smaller) and $|z_1z_2| = r_1r_2$, for a ratio of r_2. For the other two sides, the lengths are 1 and r_2, again giving a ratio of r_2. Finally, the law of sines can be used to show that the remaining side have the same ratio.



78. Construct an angle with vertex at z_2, and one ray from z_2 to 0, congruent to the angle formed by 0, 1, and z_1, with vertex 1. Also, be sure that this new angle is oriented in the appropriate direction: e.g., if z_1 is located "counterclockwise" from 1 then the points on this new ray should also be located counterclockwise from z_2. Now similarly construct an angle with vertex at 0, and one ray from 0 to z_2, congruent to the angle formed by 1, 0, and z_1, with vertex 0. The intersection of the two newly constructed rays is z_1z_2.

79. The solutions are the cube roots of 1:
$\cos\left(\frac{2\pi k}{3}\right) + i\sin\left(\frac{2\pi k}{3}\right)$, $k = 0, 1, 2$ or
$1, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

80. The solutions are the fourth roots of 1:
$\cos\left(\frac{\pi k}{2}\right) + i\sin\left(\frac{\pi k}{2}\right)$, $k = 0, 1, 2, 3$ or $-1, 1, -i, i$

81. The solutions are the cube roots of -1:
$\cos\left(\frac{\pi + 2\pi k}{3}\right) + i\sin\left(\frac{\pi + 2\pi k}{3}\right)$, $k = 0, 1, 2$ or
$-1, \frac{1}{2} + \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i$

82. The solutions are the fourth roots of −1: $\cos\left(\frac{\pi + 2\pi k}{4}\right) + i\sin\left(\frac{\pi + 2\pi k}{4}\right)$, $k = 0, 1, 2, 3$ or $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i, -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

83. The solutions are the fifth roots of −1: $\cos\left(\frac{\pi + 2\pi k}{5}\right) + i\sin\left(\frac{\pi + 2\pi k}{5}\right)$, $k = 0, 1, 2, 3, 4$, or $-1, \approx 0.81 + 0.59i, 0.81 - 0.59i, -0.31 + 0.95i, -0.31 - 0.95i$

84. The solutions are the fifth roots of 1: $\cos\left(\frac{2\pi k}{5}\right) + i\sin\left(\frac{2\pi k}{5}\right)$, $k = 0, 1, 2, 3, 4$ or $1, \approx 0.31 + 0.95i, 0.31 - 0.95i, -0.81 + 0.59i, -0.81 - 0.59i$

■ Chapter 6 Review

1. $\mathbf{u} - \mathbf{v} = \langle 2 - 4, -1 - 2\rangle = \langle -2, -3\rangle$

2. $2\mathbf{u} - 3\mathbf{w} = \langle 4 - 3, -2 + 9\rangle = \langle 1, 7\rangle$

3. $|\mathbf{u} + \mathbf{v}| = \sqrt{(2 + 4)^2 + (-1 + 2)^2} = \sqrt{37}$

4. $|\mathbf{w} - 2\mathbf{u}| = \sqrt{(1 - 4)^2 + (-3 + 2)^2} = \sqrt{10}$

5. $\mathbf{u} \cdot \mathbf{v} = 8 - 2 = 6$

6. $\mathbf{u} \cdot \mathbf{w} = 2 + 3 = 5$

7. $3\overrightarrow{AB} = 3\langle 3 - 2, 1 - (-1)\rangle = \langle 3, 6\rangle$; $|3\overrightarrow{AB}| = \sqrt{3^2 + 6^2} = \sqrt{45} = 3\sqrt{5}$

8. $\overrightarrow{AB} + \overrightarrow{CD} = \langle 3 - 2, 1 - (-1)\rangle + \langle 1 - (-4), -5 - 2\rangle = \langle 6, -5\rangle$; $|\overrightarrow{AB} + \overrightarrow{CD}| = \sqrt{6^2 + 5^2} = \sqrt{61}$

9. $\overrightarrow{AC} + \overrightarrow{BD} = \langle -4 - 2, 2 - (-1)\rangle + \langle 1 - 3, -5 - 1\rangle = \langle -8, -3\rangle$; $|\overrightarrow{AC} + \overrightarrow{BD}| = \sqrt{8^2 + 3^2} = \sqrt{73}$

10. $\overrightarrow{CD} - \overrightarrow{AB} = \langle 1 - (-4), -5 - 2\rangle - \langle 3 - 2, 1 - (-1)\rangle = \langle 4, -9\rangle$; $|\overrightarrow{CD} + \overrightarrow{AB}| = \sqrt{4^2 + 9^2} = \sqrt{97}$

11. (a) $\frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{\langle -2, 1\rangle}{\sqrt{(-2)^2 + 1^2}} = \frac{\langle -2, 1\rangle}{\sqrt{5}} = \left\langle -\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right\rangle$

(b) $-3 \cdot \frac{\overrightarrow{AB}}{|\overrightarrow{BA}|} = -3\left\langle \frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right\rangle = \left\langle \frac{6}{\sqrt{5}}, -\frac{3}{\sqrt{5}}\right\rangle$

12. **(a)** $\frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{\langle 2, 0\rangle}{\sqrt{2^2 + 0^2}} = \frac{\langle 2, 0\rangle}{2} = \langle 1, 0\rangle$

(b) $-3 \cdot \frac{\overrightarrow{AB}}{|\overrightarrow{BA}|} = -3\,\langle 1, 0\rangle = \langle -3, 0\rangle$

For #13 and 14, the direction angle θ of $\langle a, b\rangle$ has $\tan\theta = \frac{b}{a}$; start with $\tan^{-1}\left(\frac{b}{a}\right)$, and add (or subtract) 180° if the angle is not in the correct quadrant. The angle between two vectors is the absolute value of the difference between their angles; if this difference is greater than 180°, subtract it from 360°.

13. **(a)** $\theta_u = \tan^{-1}\left(\frac{3}{4}\right) \approx 0.64$, $\theta_v = \tan^{-1}\left(\frac{5}{2}\right) \approx 1.19$

(b) $\theta_v - \theta_u \approx 0.55$

14. **(a)** $\theta_u = \pi + \tan^{-1}(-2) = \cos^{-1}\left(-\frac{1}{\sqrt{5}}\right) \approx 2.03$, $\theta_v = \tan^{-1}\left(\frac{2}{3}\right) \approx 0.59$

(b) $\theta_u - \theta_v \approx 1.45$

15. $(-2.5\cos 25°, -2.5\sin 25°) \approx (-2.27, -1.06)$

16. $(-3.1\cos 135°, -3.1\sin 135°) = (1.55\sqrt{2}, -1.55\sqrt{2})$

17. $(2\cos(-\pi/4), 2\sin(-\pi/4)) = (\sqrt{2}, -\sqrt{2})$

18. $(3.6\cos(3\pi/4), 3.6\sin(3\pi/4)) = (-1.8\sqrt{2}, 1.8\sqrt{2})$

19. $\left(1, -\frac{2\pi}{3} + (2n + 1)\pi\right)$ and $\left(-1, -\frac{2\pi}{3} + 2n\pi\right)$, n an integer.

20. $\left(2, \frac{5\pi}{6} + (2n + 1)\pi\right)$ and $\left(-2, \frac{5\pi}{6} + 2n\pi\right)$, n an integer.

21. **(a)** $\left(-\sqrt{13}, \pi + \tan^{-1}\left(-\frac{3}{2}\right)\right) \approx (-\sqrt{13}, 2.16)$ or $\left(\sqrt{13}, 2\pi + \tan^{-1}\left(-\frac{3}{2}\right)\right) \approx (\sqrt{13}, 5.30)$

(b) $\left(\sqrt{13}, \tan^{-1}\left(-\frac{3}{2}\right)\right) \approx (\sqrt{13}, -0.98)$ or $\left(-\sqrt{13}, \pi + \tan^{-1}\left(-\frac{3}{2}\right)\right) \approx (-\sqrt{13}, 2.16)$

(c) The answers from **(a)**, and also $\left(-\sqrt{13}, 3\pi + \tan^{-1}\left(-\frac{3}{2}\right)\right) \approx (-\sqrt{13}, 8.44)$ or $\left(\sqrt{13}, 4\pi + \tan^{-1}\left(-\frac{3}{2}\right)\right) \approx (\sqrt{13}, 11.58)$

22. **(a)** $(-10, 0)$ or $(10, \pi)$ or $(-10, 2\pi)$

(b) $(10, -\pi)$ or $(-10, 0)$ or $(10, \pi)$

(c) The answers from **(a)**, and also $(10, 3\pi)$ or $(-10, 4\pi)$

23. **(a)** $(5, 0)$ or $(-5, \pi)$ or $(5, 2\pi)$

(b) $(-5, -\pi)$ or $(5, 0)$ or $(-5, \pi)$

(c) The answers from **(a)**, and also $(-5, 3\pi)$ or $(5, 4\pi)$

24. **(a)** $\left(-2, \frac{\pi}{2}\right)$ or $\left(2, \frac{3\pi}{2}\right)$

(b) $\left(2, -\frac{\pi}{2}\right)$ or $\left(-2, \frac{\pi}{2}\right)$

(c) The answers from **(a)**, and also $\left(-2, \frac{5\pi}{2}\right)$ or $\left(2, \frac{7\pi}{2}\right)$

25. $t = -\frac{1}{5}x + \frac{3}{5}$, so $y = 4 + 3\left(-\frac{1}{5}x + \frac{3}{5}\right) = -\frac{3}{5}x + \frac{29}{5}$: Line through $\left(0, \frac{29}{5}\right)$ with slope $m = -\frac{3}{5}$

26. $t = x - 4$, so $y = -8 - 5(x - 4) = -5x + 12$, $1 \le x \le 9$: segment from $(1, 7)$ to $(9, -33)$.

27. $t = y + 1$, so $x = 2(y + 1)^2 + 3$: Parabola that opens to right with vertex at $(3, -1)$.

28. $x^2 + y^2 = (3\cos t)^2 + (3\sin t)^2 =$ $9\cos^2 t + 9\sin^2 t = 9$, so $x^2 + y^2 = 9$: Circle of radius 3 centered at $(0, 0)$.

29. $x + 1 = e^{2t}$, $t = \dfrac{\ln(x+1)}{2}$, so $y = e^{\frac{1}{2}\ln(x+1)} = e^{\ln}$ $= \sqrt{x+1}$: square root function starting at $(-1, 0)$

30. $t = \sqrt[3]{x}$, so $y = \ln\left(\sqrt[3]{x}\right) = \ln x^{1/3} = \dfrac{1}{3}\ln x$: the logarithmic function, with asymptote at $x = 0$

31. $m = \dfrac{4-(-2)}{3-(-1)} = \dfrac{6}{4} = \dfrac{3}{2}$, so $\Delta x = 2$ when $\Delta y = 3$. One possibility for the parametrization of the line is: $x = 2t + 3$, $y = 3t + 4$.

32. $m = \dfrac{1-3}{5-(-2)} - \dfrac{-2}{7}$, so $\Delta x = 7$ when $\Delta y = -2$. One possibility for the parametrization of the segment is: $x = 7t + 5$, $y = -2t + 1$, $-1 \le t \le 0$. Another possibility is $x = 7t - 2$, $y = -2t + 3$, $0 \le t \le 1$.

33. $a = -3$, $b = 4$, $|z_1| = \sqrt{3^2 + 4^2} = 5$

34. $z_1 = 5\left\{\cos\left[\cos^{-1}\left(-\dfrac{3}{5}\right)\right] + i\sin\left[\cos^{-1}\left(-\dfrac{3}{5}\right)\right]\right\}$ $\approx 5[\cos(2.21) + i\sin(2.21)]$

35. $6(\cos 30° + i\sin 30°) = 6\left(\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i\right) = 3\sqrt{3} + 3i$

36. $3(\cos 150° + i\sin 150°) = 3\left(-\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i\right)$ $= -1.5\sqrt{3} + 1.5i$

37. $2.5\left(\cos\dfrac{4\pi}{3} + i\sin\dfrac{4\pi}{3}\right) = 2.5\left(-\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i\right)$ $= -1.25 - 1.25\sqrt{3}\,i$

38. $4(\cos 2.5 + i\sin 2.5) \approx -3.20 + 2.39i$

39. $3 - 3i = 3\sqrt{2}\left(\cos\dfrac{7\pi}{4} + i\sin\dfrac{7\pi}{4}\right)$. Other representations would use angles $\dfrac{7\pi}{4} + 2n\pi$, n an integer.

40. $-1 + i\sqrt{2} = \sqrt{3}\left\{\cos\left[\cos^{-1}\left(-\dfrac{1}{\sqrt{3}}\right)\right] + i\sin\left[\cos^{-1}\left(-\dfrac{1}{\sqrt{3}}\right)\right]\right\} \approx \sqrt{3}[\cos(2.19) + i\sin(2.19)]$. Other representations would use angles $2.19 + 2n\pi$, n an integer.

41. $3 - 5i = \sqrt{34}\left\{\cos\left[\tan^{-1}\left(-\dfrac{5}{3}\right)\right] + i\sin\left[\tan^{-1}\left(-\dfrac{5}{3}\right)\right]\right\}$ $\approx \sqrt{34}[\cos(-1.03) + i\sin(-1.03)]$ $\approx \sqrt{34}[\cos(5.25) + i\sin(5.25)]$

Other representations would use angles $\approx 5.25 + 2n\pi$, n an integer.

42. $-2 - 2i = 2\sqrt{2}\left(\cos\dfrac{5\pi}{4} + i\sin\dfrac{5\pi}{4}\right)$. Other representations would use angles $\dfrac{5\pi}{4} + 2n\pi$, n an integer.

43. $z_1 z_2 = (3)(4)[\cos(30° + 60°) + i\sin(30° + 60°)]$ $= 12(\cos 90° + i\sin 90°)$

$z_1/z_2 = \dfrac{3}{4}[\cos(30° - 60°) + i\sin(30° - 60°)]$ $= \dfrac{3}{4}[\cos(-30°) + i\sin(-30°)]$ $= \dfrac{3}{4}(\cos 330° + i\sin 330°)$

44. $z_1 z_2 = (5)(-2)[\cos(20° + 45°) + i\sin(20° + 45°)]$ $= -10(\cos 65° + i\sin 65°)$ $= 10(\cos 245° + i\sin 245°)$

$z_1/z_2 = \dfrac{5}{-2}[\cos(20° - 45°) + i\sin(20° - 45°)]$ $= -2.5[\cos(-25°) + i\sin(-25°)]$ $= 2.5(\cos 155° + i\sin 155°)$

45. (a) $\left[3\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)\right]^5 = 3^5\left(\cos\dfrac{5\pi}{4} + i\sin\dfrac{5\pi}{4}\right)$ $= 243\left(\cos\dfrac{5\pi}{4} + i\sin\dfrac{5\pi}{4}\right)$

(b) $-\dfrac{243\sqrt{2}}{2} - \dfrac{243\sqrt{2}}{2}i$

46. (a) $\left[2\left(\cos\dfrac{\pi}{12} + i\sin\dfrac{\pi}{12}\right)\right]^8 = 2^8\left(\cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3}\right)$ $= 256\left(\cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3}\right)$

(b) $-128 + 128\sqrt{3}\,i$

47. (a) $\left[5\left(\cos\dfrac{5\pi}{3} + i\sin\dfrac{5\pi}{3}\right)\right]^3 = 5^3(\cos 5\pi + i\sin 5\pi)$ $= 125(\cos\pi + i\sin\pi)$

(b) $-125 + 0i = -125$

48. (a) $\left[7\left(\cos\dfrac{\pi}{24} + i\sin\dfrac{\pi}{24}\right)\right]^6 = 7^6\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)$ $= 117,649\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)$

(b) $\dfrac{117,649\sqrt{2}}{2} + \dfrac{117,649\sqrt{2}}{2}i$

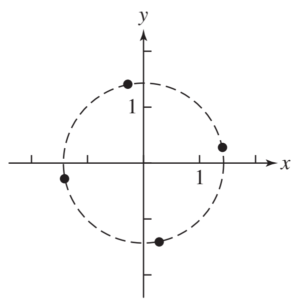
For #49–52, the nth roots of $r(\cos\theta + i\sin\theta)$ are $\sqrt[n]{r}\left(\cos\dfrac{2k\pi + \theta}{n} + i\sin\dfrac{2k\pi + \theta}{n}\right)$, $k = 0, 1, 2, \ldots, n - 1$.

49. $3 + 3i = 3\sqrt{2}\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)$, so the roots are

$$\sqrt[4]{3\sqrt{2}}\left(\cos\frac{2k\pi + \pi/4}{4} + i\sin\frac{2k\pi + \pi/4}{4}\right)$$
$$= \sqrt[8]{18}\left(\cos\frac{\pi(8k+1)}{16} + i\sin\frac{\pi(8k+1)}{16}\right),$$

$k = 0, 1, 2, 3$:

$$\sqrt[8]{18}\left(\cos\frac{\pi}{16} + i\sin\frac{\pi}{16}\right),$$
$$\sqrt[8]{18}\left(\cos\frac{9\pi}{16} + i\sin\frac{9\pi}{16}\right),$$
$$\sqrt[8]{18}\left(\cos\frac{17\pi}{16} + i\sin\frac{17\pi}{16}\right),$$
$$\sqrt[8]{18}\left(\cos\frac{25\pi}{16} + i\sin\frac{25\pi}{16}\right)$$



50. $8 = 8(\cos 0 + i \sin 0)$, so the roots are

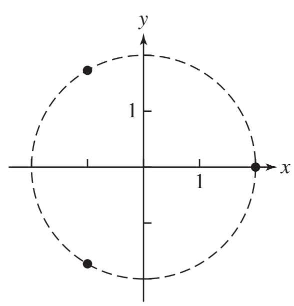
$$\sqrt[3]{8}\left(\cos \frac{2k\pi + 0}{3} + i \sin \frac{2k\pi + 0}{3}\right)$$

$$= 2\left(\cos \frac{2k\pi}{3} + i \sin \frac{2k\pi}{3}\right), k = 0, 1, 2:$$

$2(\cos 0 + i \sin 0) = 2$,

$2\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$,

$2\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$



51. $1 = \cos 0 + i \sin 0$, so the roots are

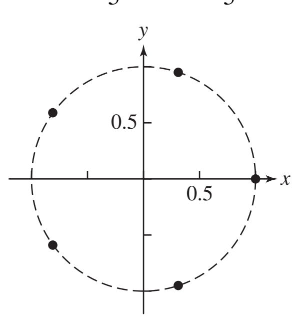
$$\cos \frac{2k\pi + 0}{5} + i \sin \frac{2k\pi + 0}{5} = \cos \frac{2k\pi}{5} + i \sin \frac{2k\pi}{5},$$

$k = 0, 1, 2, 3, 4$:

$\cos 0 + i \sin 0 = 1$, $\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}$,

$\cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5}$, $\cos \frac{6\pi}{5} + i \sin \frac{6\pi}{5}$

$\cos \frac{8\pi}{5} + i \sin \frac{8\pi}{5}$



52. $-1 = 1(\cos \pi + i \sin \pi)$, so the roots are

$$\cos \frac{2k\pi + \pi}{6} + i \sin \frac{2k\pi + \pi}{6}$$

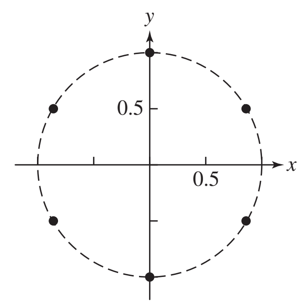
$$= \cos \frac{\pi(2k + 1)}{6} + i \sin \frac{\pi(2k + 1)}{6},$$

$k = 0, 1, 2, 3, 4, 5$:

$\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}$, $\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} = i$,

$\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}$, $\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}$,

$\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} = -i$, $\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}$



53. Graph **(b)**

54. Not shown

55. Graph **(a)**

56. Not shown

57. Not shown

58. Graph **(d)**

59. Graph **(c)**

60. Not shown

61. $x^2 + y^2 = 4$ — a circle with center $(0, 0)$ and radius 2

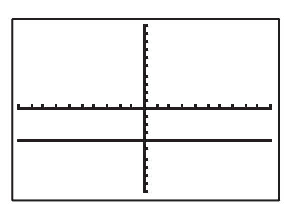
62. $r^2 + 2r \sin \theta = x^2 + y^2 + 2y = 0$. Completing the square: $x^2 + (y^2 + 1)^2 = 1$ — a circle of radius 1 with center $(0, -1)$

63. $r^2 + 3r \cos \theta + 2r \sin \theta = x^2 + y^2 + 3x + 2y = 0$. Completing the square: $\left(x + \frac{3}{2}\right)^2 + (y + 1)^2 = \frac{13}{4}$

— a circle of radius $\frac{\sqrt{13}}{2}$ with center $\left(-\frac{3}{2}, -1\right)$

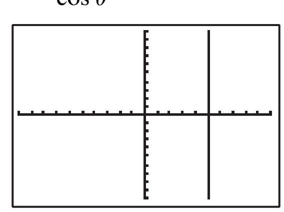
64. $1 - \frac{3}{r \cos \theta} = 1 - \frac{3}{x} = 0 \Rightarrow x - 3 = 0, x = 3$ — a vertical line through $(3, 0)$

65. $r = \frac{-4}{\sin \theta} = -4 \csc \theta$



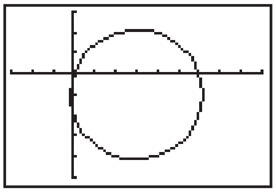
$[-10, 10]$ by $[-10, 10]$

66. $r = \frac{5}{\cos \theta} = 5 \sec \theta$



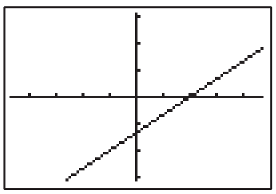
$[-10, 10]$ by $[-10, 10]$

67. $(r\cos\theta - 3)^2 + (r\sin\theta + 1)^2 = 10$, so $r^2(\cos^2\theta + \sin^2\theta) + r(-6\cos\theta + 2\sin\theta) + 10 = 10$, or $r = 6\cos\theta - 2\sin\theta$



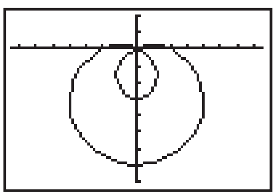
[–3, 9] by [–5, 3]

68. $2r\cos\theta - 3r\sin\theta = 4$, $r = \dfrac{4}{2\cos\theta - 3\sin\theta}$



[−4.7, 4.7] by [−3.1, 3.1]

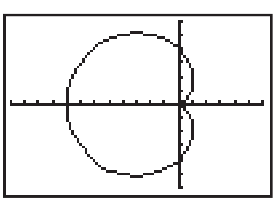
69.



[–7.5, 7.5] by [–8, 2]

Domain: $(-\infty, \infty)$
Range: $[-3, 7]$
Symmetric about the y-axis
Continuous
Bounded
Maximum $|r|$ value: 7
No asymptotes

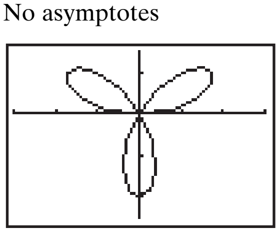
70.



[–12, 6] by [–6, 6]

Domain: $(-\infty, \infty)$
Range: $[0, 8]$
Symmetric about the x-axis
Continuous
Bounded
Maximum $|r|$ value: 8
No asymptotes

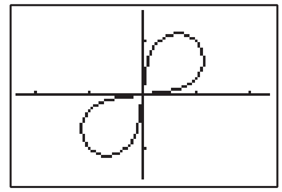
71.



[–3, 3] by [–2.5, 1.5]

Domain: $(-\infty, \infty)$
Range: $[-2, 2]$
Symmetric about the y-axis
Continuous
Bounded
Maximum $|r|$ value: 2
No asymptotes

72.



[−2.35, 2.35] by [−1.55, 1.55]

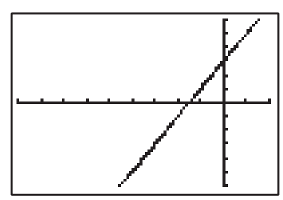
Domain: $\left[0, \frac{\pi}{2}\right] \cup \left[\pi, \frac{3\pi}{2}\right]$
Range: $[0, \sqrt{2}]$
Symmetric about the origin
Bounded
Maximum $|r|$ value: $\sqrt{2}$
No asymptotes

73. **(a)** $r = a\sec\theta \Rightarrow \dfrac{r}{\sec\theta} = a \Rightarrow r\cos\theta = a \Rightarrow x = a.$

(b) $r = b\csc\theta \Rightarrow \dfrac{r}{\csc\theta} = b \Rightarrow r\sin\theta = b \Rightarrow y = b.$

(c) $y = mx + b \Rightarrow r\sin\theta = mr\cos\theta + b \Rightarrow$
$r(\sin\theta - m\cos\theta) = b \Rightarrow r = \dfrac{b}{\sin\theta - m\cos\theta}.$
The domain of r is any value of θ for which $\sin\theta \neq m\cos\theta \Rightarrow \tan\theta \neq m \Rightarrow \theta \neq \arctan(m)$.

(d)



[−9, 2] by [−6, 6]

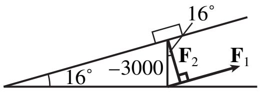
74. **(a)** $\mathbf{v} = 540\langle\sin 80°, \cos 80°\rangle \approx \langle 531.80, 93.77\rangle$

(b) The wind vector is $\mathbf{w} = 55\langle\sin 100°, \cos 100°\rangle \approx \langle 54.16, -9.55\rangle$. Actual velocity vector: $\mathbf{v} + \mathbf{w} \approx \langle 585.96, 84.22\rangle$. Actual speed: $\|\mathbf{v} + \mathbf{w}\| \approx \sqrt{585.96^2 + 84.22^2} \approx 591.98$ mph. Actual bearing: $\tan^{-1}\left(\dfrac{585.96}{84.22}\right) \approx 81.82°$.

75. **(a)** $\mathbf{v} = 480\langle\sin 285°, \cos 285°\rangle \approx \langle -463.64, 124.23\rangle$

(b) The wind vector is $\mathbf{w} = 30\langle\sin 265°, \cos 265°\rangle \approx \langle -29.89, -2.61\rangle$. Actual velocity vector: $\mathbf{v} + \mathbf{w} \approx \langle -493.53, 121.62\rangle$. Actual speed: $\|\mathbf{v} + \mathbf{w}\| \approx \sqrt{493.53^2 + 121.62^2} \approx 508.29$ mph. Actual bearing: $360° + \tan^{-1}\left(\dfrac{-493.53}{121.62}\right) \approx 283.84°$.

76. $\mathbf{F} = \langle 120\cos 20°, 120\sin 20°\rangle + \langle 300\cos(-5°), 300\sin(-5°)\rangle \approx \langle 411.62, 14.90\rangle$, so $\|\mathbf{F}\| \approx \sqrt{411.62^2 + 14.90^2} \approx 411.89$ lb and $\theta = \tan^{-1}\left(\dfrac{14.90}{411.62}\right) \approx 2.07°$.

77. 

$\mathbf{F}_1$ Force to keep car from going downhill

$\mathbf{F}_2$ Force perpendicular to the street

(a) $\mathbf{F}_1 = -3000 \sin 16° \approx -826.91$, so the force required to keep the car from rolling down the hill is approximately 826.91 pounds.

(b) $\mathbf{F}_2 = -3000 \cos 16° \approx -2883.79$, so the force perpendicular to the street is approximately 2883.79 pounds.

78. $\mathbf{F} = 36 \cdot \dfrac{\langle 3, 5\rangle}{\sqrt{3^2 + 5^2}} = \dfrac{\langle 108, 180\rangle}{\sqrt{34}}$

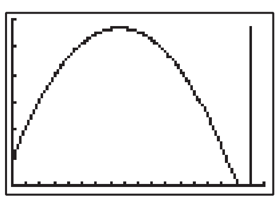
Since $\overrightarrow{AB} = \langle 10, 0\rangle$, $\mathbf{F} \cdot \overrightarrow{AB} = (10)\left(\dfrac{108}{\sqrt{34}}\right) + 0$

$= \dfrac{1080}{\sqrt{34}} \approx 185.22$ foot-pounds.

79. (a) $h = -16t^2 + v_0 t + s_0 = -16t^2 + 245t + 200$

(b) Graph and trace: $x = 17$ and $y = -16t^2 + 245t + 200$ with $0 \le t \le 16.1$ (upper limit may vary) on [0, 18] by [0, 1200]. This graph will appear as a vertical line from about (17, 0) to about (17, 1138). Tracing shows how the arrow begins at a height of 200 ft, rises to over 1000 ft, then falls back to the ground.

(c) Graph $x = t$ and $y = -16t^2 + 245t + 200$ with $0 \le t \le 16.1$ (upper limit may vary).



[0, 18] by [0, 1200]

(d) When $t = 4$, $h = 924$ ft.

(e) When $t \approx 7.66$, the arrow is at its peak: about 1138 ft.

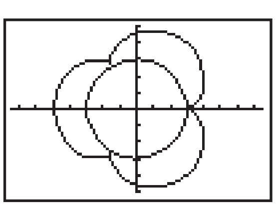
(f) The arrow hits the ground ($h = 0$) after about 16.09 sec.

80. $x = 35 \cos\left(\dfrac{\pi}{10} t\right)$, $y = 50 + 35 \sin\left(\dfrac{\pi}{10} t\right)$, assuming the wheel turns counterclockwise.

81. $x = 40 \sin\left(\dfrac{2\pi}{15} t\right)$, $y = 50 - 40 \cos\left(\dfrac{2\pi}{15} t\right)$, assuming the wheel turns counterclockwise.

82. $x = -40 \sin\left(\dfrac{\pi}{9} t\right)$, $y = 50 + 40 \cos\left(\dfrac{\pi}{9} t\right)$, assuming the wheel turns counterclockwise.

83. (a)



[–7.5, 7.5] by [–5, 5]

(b) All 4's should be changed to 5's.

84. $x = (66 \cos 5°)t$ and $y = -16t^2 + (66 \sin 5°)t + 4$. $y = 0$ when $t \approx 0.71$ sec (and also when $t \approx -0.352$, but that is not appropriate in this problem). When $t \approx 0.71$ sec, $x \approx 46.75$ ft.

85. $x = (66 \cos 12°)t$ and $y = -16t^2 + (66 \sin 12°)t + 3.5$. $y = 0$ when $t \approx 1.06$ sec (and also when $t \approx -0.206$, but that is not appropriate in this problem). When $t \approx 1.06$ sec, $x \approx 68.65$ ft.

86. $x = (70 \cos 45°)t$ and $y = -16t^2 + (70 \sin 45°)t$. The ball traveled 40 yd (120 ft) horizontally after about 2.42 sec, at which point it is about 25.96 ft above the ground, so it clears the crossbar.

87. If we assume that the initial height is 0 ft, then $x = (85 \cos 56°)t$ and $y = -16t^2 + (85 \sin 56°)t$. [If the assumed initial height is something other than 0 ft, add that amount to y.]

(a) Find graphically: The maximum y value is about 77.59 ft (after about 2.20 seconds).

(b) $y = 0$ when $t \approx 4.404$ sec

88. $x = (v_0 \cos 30°)t$ and $y = -16t^2 + (v_0 \sin 30°)t + 2.5$. v_0 must be (at least) just over 125 ft/sec. This can be found graphically (by trial and error), or algebraically: the ball is 400 ft from the plate (i.e., $x = 400$) when $t = \dfrac{400}{v_0 \cos 30°} = \dfrac{800/\sqrt{3}}{v_0}$. Substitute this value of t in the parametric equation for y. Then solve to see what value of v_0 makes y equal to 15 ft.

$$-16\left(\frac{800}{\sqrt{3}v_0}\right)^2 + v_0 \sin 30°\left(\frac{800}{\sqrt{3}v_0}\right) + 2.5 = 15$$

$$\frac{-16(640{,}000)}{3v_0^2} + \frac{400}{\sqrt{3}} = 12.5$$

$$\frac{-16(640{,}000)}{3} + \frac{400v_0^2}{\sqrt{3}} = 12.5v_0^2$$

$$\frac{-16(640{,}000)}{3} = v_0^2\left(12.5 - \frac{400}{\sqrt{3}}\right)$$

$$\frac{-16(640{,}000)}{3\left(12.5 - \frac{400}{\sqrt{3}}\right)} = v_0^2$$

$$\pm 125 \approx v_0$$

The negative root doesn't apply to this problem, so the initial velocity needed is just over 125 ft/sec.

89. Kathy's position: $x_1 = 60 \cos\left(\dfrac{\pi}{6} t\right)$ and

$y_1 = 60 + 60 \sin\left(\dfrac{\pi}{6} t\right)$

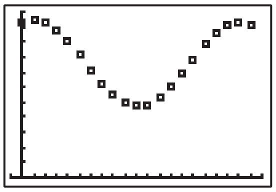
Ball's position:
$x_2 = -80 + (100 \cos 70°)t$ and
$y_2 = -16t^2 + (100 \sin 70°)t$

Find (graphically) the minimum of $d(t) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. It occurs when $t \approx 2.64$ sec; the minimum distance is about 17.65 ft.

90. $x = (20 \cos 50°)t$ and $y = -16t^2 + (20 \sin 50°)t + 5$. $y = 0$ when $t = 1.215$ sec (and also when $t = -0.257$, but that is not appropriate in this problem). When $t = 1.215$ sec, $x = 15.62$ ft. The dart falls several feet short of the target.

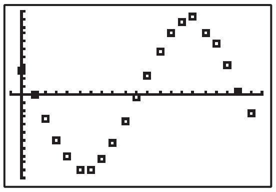
Chapter 6 Project

Answers are based on the sample data shown in the table.

1. 

[−0.1, 2.1] by [0, 1.1]

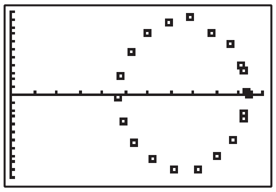
2. Sinusoidal regression produces $y = 0.28 \sin(3.46x + 1.20) + 0.75$ or, with a phase shift of 2π, $y = 0.28 \sin(3.46x - 5.09) + 0.75$ $= 0.28 \sin(3.46(x - 1.47)) + 0.75$.

3. 

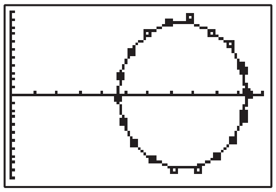
[−0.1, 2.1] by [−1.1, 1.1]

The curve $y = 0.9688 \cos(3.46(x - 1.47))$ closely fits the data.

4. The distance and velocity both vary sinusoidally, with the same period but a phase shift of 90° — like the x- and y-coordinates of a point moving around a circle. A scatter plot of distance versus time should have the shape of a circle (or ellipse).



[0, 1.1] by [−1.1, 1.1]

5. 

[0, 1.1] by [−1.1, 1.1]