7.4 Day 1

Friday, February 22, 2019 8:19 AM

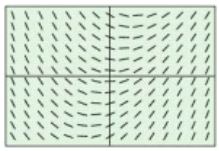
AP Problems from HW (page 357)

1. Multiple Choice Which of the following differential equations would produce the slope field shown below?

(A) $\frac{dy}{dx} = y - 3x$ (B) $\frac{dy}{dx} = y - \frac{x}{3}$

(C) $\frac{dy}{dx} = y + \frac{x}{3}$ (D) $\frac{dy}{dx} = x + \frac{y}{3}$

(E) $\frac{dy}{dx} = x - \frac{y}{3}$



2. Multiple Choice If the substitution $\sqrt{x} = \sin y$ is made in the integrand of $\int_0^{1/2} \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$, the resulting integral is

(A) $\int_0^{1/2} \sin^2 y\,dy$ (B) $2\int_0^{1/2} \frac{\sin^2 y}{\cos y}\,dy$

(C) $2\int_0^{\pi/4} \sin^2 y\,dy$ (D) $\int_0^{\pi/4} \sin^2 y\,dy$

(E) $2\int_0^{\pi/6} \sin^2 y\,dy$

$x = 1/2$

$\sqrt{1/2} = \sin y$

$\frac{1}{\sqrt{2}} = \sin y$

$y = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$y = \pi/4$

3. Multiple Choice $\int x\,e^{2x}\,dx =$

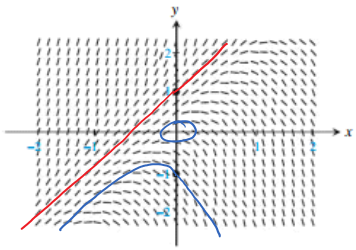
(A) $\frac{x\,e^{2x}}{2} - \frac{e^{2x}}{4} + C$ (B) $\frac{x\,e^{2x}}{2} - \frac{e^{2x}}{2} + C$

(C) $\frac{x\,e^{2x}}{2} + \frac{e^{2x}}{4} + C$ (D) $\frac{x\,e^{2x}}{2} + \frac{e^{2x}}{2} + C$

(E) $\frac{x^2\,e^{2x}}{4} + C$

4. Free Response Consider the differential equation $dy/dx = 2y - 4x$.

(a) The slope field for the differential equation is shown below. Sketch the solution curve that passes through the point $(0, 1)$ and sketch the solution curve that goes through the point $(0, -1)$.



$\frac{dy}{dx} = 2y - 4x$

$2 = 2y - 4x \Rightarrow \frac{4x}{2} + \frac{2}{2} = \frac{2y}{2}$

$y = 2x + 1$

$y = 2x + b$

$\frac{dy}{dx} = 2$

(b) There is a value of b for which $y = 2x + b$ is a solution to the differential equation. Find this value of b. Justify your answer.

(c) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. It appears from the slope field that g has a local maximum at the point $(0, 0)$. Using the differential equation, prove analytically that this is so.

$(0,0) \quad \frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 2\frac{dy}{dx} - 4 \qquad \frac{d^2y}{dx^2} = -4$

$\int_0^{\pi/4} \frac{\sin y}{\sqrt{\cos^2 y}} \cdot 2\sin y \cos y\,dy \cdot \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$

$\sqrt{x} = \sin y$

$x = \sin^2 y \quad (\sin y)^2$

$1\,dx = 2\sin y \cos y$

$= \frac{\sin y}{\sqrt{1-x}}$

$\int_0^{\pi/4} \frac{\sin y}{\cos y} \cdot 2\sin y \cos y\,dy$

$= \frac{\sin y}{\sqrt{1-x}} \cdot 2\sin y \cos y$

$2\int_0^{\pi/4} \sin^2 y\,dy$

$= \frac{\sin y}{\sqrt{1-\sin^2 y}}$

$\cos^2 y + \sin^2 y = 1$

$\cos^2 y = 1 - \sin^2 y$