Algebra or U-Sub Worksheet Name KEY

Evaluate each indefinite integral by using U-Substitution, Algebra, or basic integral rules.

1. $\int \sec(2x)\tan(2x)\,dx$

$u = 2x$
$du = 2\,dx$
$\frac{1}{2}du = dx$

$\frac{1}{2}\int \sec u \tan u\, du$

$= \frac{1}{2}\sec u + C = \boxed{\frac{1}{2}\sec(2x) + C}$

2. $\int \frac{1}{\sqrt{1-x^2}}\,dx$ $= \boxed{\sin^{-1}x + C}$

3. $\int \frac{1}{(1-x)^2}\,dx$

$u = 1-x$
$du = -dx$
$-du = dx$

$-\int u^{-2}\,du = u^{-1} + C$

$= \boxed{\frac{1}{1-x} + C}$

4. $\int 3(\sin x)^{-2}\,dx$ $= 3\int \frac{1}{\sin^2 x}\,dx$

$= 3\int \csc^2 x\,dx$

$= \boxed{-3\cot x + C}$

5. $\int \frac{(\ln x)^6}{x}\,dx$

$u = \ln x$
$du = \frac{1}{x}dx$

$\int u^6\,du$

$= \frac{1}{7}u^7 + C = \boxed{\frac{1}{7}(\ln x)^7 + C}$

6. $\int \cot^7\left(\frac{x}{2}\right)\csc^2\left(\frac{x}{2}\right)dx$

$u = \cot\left(\frac{x}{2}\right)$
$du = -\frac{1}{2}\csc^2\left(\frac{x}{2}\right)dx$
$-2\,du = \csc^2\left(\frac{x}{2}\right)dx$

$-2\int u^7\,du$

$= -\frac{1}{4}u^8 + C$

$= \boxed{-\frac{1}{4}\cot^8\left(\frac{x}{2}\right) + C}$

7. $\int \frac{x^3+4x^2-7x+1}{x+1}\,dx$

$$\begin{array}{r|rrrr} -1 & 1 & 4 & -7 & 1 \\ & & -1 & -3 & 10 \\ \hline & 1 & 3 & -10 & 11 \end{array}$$

$\int \left(x^2+3x-10+\frac{11}{x+1}\right)dx$

$= \boxed{\frac{1}{3}x^3 + \frac{3}{2}x^2 - 10x + 11\ln|x+1| + C}$

8. $\int \frac{4x}{3x^2+2}\,dx$

$u = 3x^2+2$
$du = 6x\,dx$
$\frac{2}{3}du = 4x\,dx$

$= \frac{2}{3}\int \frac{1}{u}\,du$

$= \frac{2}{3}\ln|u| + C$

$= \boxed{\frac{2}{3}\ln(3x^2+2) + C}$

Evaluate each definite integral.

9. $\int_{-\frac{\pi}{4}}^{0} \tan x \sec^2 x\,dx$

$u = \tan x$ $\quad u(0) = 0$
$du = \sec^2 x\,dx$ $\quad u(-\frac{\pi}{4}) = -1$

$= \int_{-1}^{0} u\,du = \frac{1}{2}u^2\Big]_{-1}^{0}$

$= \frac{1}{2}(0) - \frac{1}{2}(-1)^2$

$= \boxed{-\frac{1}{2}}$

10. $\int_0^1 \sqrt{t^5+2t}\,(5t^4+2)\,dt$

$u = t^5+2t$
$du = (5t^4+2)dt$
$u(1) = 3$
$u(0) = 0$

$\int_0^3 u^{\frac{1}{2}}\,du$

$= \frac{2}{3}u^{3/2}\Big]_0^3$ $\quad \sqrt{27}$ $\quad 3\sqrt{3}$

$= \frac{2}{3}\cdot 3^{3/2} - 0 = \frac{2}{3}\cdot 3\sqrt{3} = \boxed{2\sqrt{3}}$