9.8 Day 1

Monday, January 13, 2020 6:24 PM

Pg 654 #1, 4-7, 10,41, 44,50, 51, 55a

1. $\sum_{n=0}^{\infty} nx^n = 0 + x + 2x^2 + 3x^3 + \ldots$ $C = 0$

4. $\sum_{n=0}^{\infty} \frac{(-1)^n (x-\pi)^{2n}}{(2n)!} = 1 - \frac{x-\pi}{2} + \frac{(x-\pi)^4}{4!} + \ldots$ $C = \pi$

5. $\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n+1} = 1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \ldots$ $C = 0$

$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)+1} \cdot \frac{(n+1)}{x^n} \right| = \lim_{n\to\infty} \left| \frac{\cancel{x^n} \cdot x \cdot (n+1)}{(n+2) \cdot \cancel{x^n}} \right| = \lim_{n\to\infty} |x|$ $|x| < 1$ $\boxed{R = 1}$

6. $\sum_{n=0}^{\infty} (3x)^n = 1 + 3x + 9x^2 + 27x^3 + \ldots$ $C = 0$

$\lim_{n\to\infty} \left| \frac{(3x)^{n+1}}{(3x)^n} \right| = \lim_{n\to\infty} |3x|$ $|3x| < 1$ $-1 < 3x < 1$ $-\frac{1}{3} < x < \frac{1}{3}$ $\boxed{R = 1/3}$

7. $\sum_{n=1}^{\infty} \frac{(4x)^n}{n^2} = 4x + \frac{16x^2}{4} + \frac{64x^3}{9} + \ldots$ $C=0$

$\lim_{n\to\infty} \left| \frac{(4x)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(4x)^n} \right| = \lim_{n\to\infty} |4x|$ $|4x| < 1$ $-\frac{1}{4} < \frac{4x}{4} < \frac{1}{4}$ $\boxed{R = 1/4}$

10. $\sum_{n=0}^{\infty} \frac{(2n)!\, x^{2n}}{n!}$ $\lim_{n\to\infty} \left| \frac{(2(n+1))!\, x^{2(n+1)}}{(n+1)!} \cdot \frac{n!}{(2n)!\, x^{2n}} \right| = \lim_{n\to\infty} \left| \frac{(2n+2)! \cdot n!}{(n+1)!\,(2n)!} \cdot \frac{\cancel{x^{2n}} \cdot x^2}{\cancel{x^{2n}}} \right|$

$\lim_{n\to\infty} \left| \frac{(2n+2)\cdot(2n+1)\,\cancel{(2n)!}}{\cancel{(2n)!}} \cdot \frac{1\,\cancel{n!}}{(n+1)\cdot\cancel{n!}} \cdot x^2 \right|$ $R = \infty$

41. $\sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=1}^{\infty} \frac{x^{n-1}}{(n-1)!}$ 44. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n-1}}{2n-1}$

55a $|x| < 2$ many possibilities

one is a Geometric $\sum_{n=1} \left(\frac{x}{2}\right)^n$ $\left|\frac{x}{2}\right| < 1$ $-1 < \frac{x}{2} < 1$

$-2 < x < 2$