7.3

Thursday, January 10, 2019 8:00 AM

Solve the following system of equations

Given:

System

$R1:\quad x-2y+z=7$

$R2:\quad 3x-5y+z=14$

$R3:\quad 2x-2y-z=3$

Augmented Matrix

$$\begin{bmatrix} 1 & -2 & 1 & 7 \\ 3 & -5 & 1 & 14 \\ 2 & -2 & -1 & 3 \end{bmatrix}$$

$R1:\quad x-2y+z=7$

$R2-3R1:\quad y-2z=-7$

$R3:\quad 2x-2y-z=3$

$$\begin{bmatrix} 1 & -2 & 1 & 7 \\ 0 & 1 & -2 & -7 \\ 2 & -2 & -1 & 3 \end{bmatrix}$$

$R1:\quad x-2y+z=7$

$R2-3R1:\quad y-2z=-7$

$R3-2R1:\quad 2y-3z=-11$

$$\begin{bmatrix} 1 & -2 & 1 & 7 \\ 0 & 1 & -2 & -7 \\ 0 & 2 & -3 & -11 \end{bmatrix}$$

$R1:\quad x-2y+z=7$

$R2-3R1:\quad y-2z=-7$

$R3-2R1-2(R2-3R1):\quad z=3$

$$\begin{bmatrix} 1 & -2 & 1 & 7 \\ 0 & 1 & -2 & -7 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

DEFINITION Row Echelon Form of a Matrix

A matrix is in **row echelon form** if the following conditions are satisfied.

1. Rows consisting entirely of 0's (if there are any) occur at the bottom of the matrix.
2. The first entry in any row with nonzero entries is 1.
3. The column subscript of the leading 1 entries increases as the row subscript increases.

Notes:

1. **REF** (Row Echelon Form) is also known as **Triangular** form.
2. **REF is not unique,** so you may get a different result using your calculator or compare to someone else.

Reduced Row Echelon Form (RREF)

$$\begin{aligned} R1: &\quad x-2y+z=7 \\ R2: &\quad y-2z=-7 \\ R3: &\quad z=3 \end{aligned} \qquad \begin{bmatrix} 1 & -2 & 1 & 7 \\ 0 & 1 & -2 & -7 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\begin{aligned} R1+2R2: &\quad x-3z=-7 \\ R2: &\quad y-2z=-7 \\ R3: &\quad z=3 \end{aligned} \qquad \begin{bmatrix} 1 & 0 & -3 & -7 \\ 0 & 1 & -2 & -7 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\begin{aligned} R1+2R2+3R3: &\quad x=2 \\ R2+2R3: &\quad y=-1 \\ R3: &\quad z=3 \end{aligned} \qquad \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

What Happens When There Are Infinite Solutions?

Solve The Following System Using RREF.

$$-3x+3y-z=-6$$
$$-9x+9y-3z=-18$$

$$\begin{bmatrix} -3 & 3 & -1 & -6 \\ -9 & 9 & -3 & -18 \end{bmatrix}$$

RREF

$$\begin{bmatrix} & & & \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

What Happens When There Are No Solutions?

Solve The Following System Using RREF.

$$5x+12y+4z=11$$
$$2x+5y+4z=-8$$
$$x+2y-4z=5$$

$$\begin{bmatrix} 5 & 12 & 4 & 11 \\ 2 & 5 & 4 & -8 \\ 1 & 2 & -4 & 5 \end{bmatrix}$$

RREF

$$\begin{bmatrix} & & & \\ & & & \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

7.3 Notes Day 1 Solving Systems of Equations Using Matrices

Solve the following system of equations

$3x - 2y = -2$

$x + y = 3$

$A \cdot A^{-1} = I$

$A^{-1} \cdot A = I$

$B \cdot A^{-1}$ $2\times1 \cdot 2\times2$!!

$A^{-1} \cdot B$

Solving Systems Using Inverse Matrices

Write the system of equations above using matrices below.

$$A = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

2x2 X Y 2x1

$A \cdot X = B$

$A \cdot A^{-1} \cdot X = A^{-1} \cdot B$

$I X = A^{-1} \cdot B$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} .8 \\ 2.2 \end{bmatrix}$

Solve The Following Systems Using Inverse Matrices.

What Happens When There are Infinit Solutions

$5x + 12y + 4z = 11$

$2.5x + 6y + 2z = 5.5$

$10x + 24y + 8z = 22$

What Happens When There Are No Solutions?

$5x + 12y + 4z = 11$

$2x + 5y + 4z = -8$

$x + 2y - 4z = 5$

Limitations of Using Inverse Matrices

Solve The Following System Using Inverse Matrices.

$$-3x+3y-z=-6$$
$$-9x+9y-3z=-18$$

$$A=\begin{bmatrix}5 & 12 & 4\\ 2 & 5 & 4\\ 1 & 2 & -4\end{bmatrix} \qquad B=\begin{bmatrix}11\\ -8\\ 5\end{bmatrix}$$