

$$5. f(x) = \frac{1}{3-x} \quad c=1$$

$$f(x) = \frac{1}{-1+3-(x-1)} = \frac{1}{2-(x-1)} \cdot \frac{1}{2} \quad (\cdot \frac{1}{2})$$

$$f(x) = \frac{\frac{1}{2}}{1-\frac{(x-1)}{2}} \quad a = \frac{1}{2} \quad r = \frac{x-1}{2}$$

$$\ast \text{ power series } \Rightarrow \frac{1}{2} + \frac{x-1}{4} + \frac{(x-1)^2}{8} + \dots + = \sum_{n=0}^{\infty} \frac{1}{2} \left(\frac{x-1}{2}\right)^n \text{ or } \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} (x-1)^n$$

$$\left|\frac{x-1}{2}\right| < 1 \quad -1 < \frac{x-1}{2} < 1 \quad -2 < x-1 < 2 \quad -1 < x < 3$$

$\ast$  interval of convergence =  $(-1, 3)$

$$11. f(x) = \frac{3}{3x+4}, \quad c=0$$

$$f(x) = \frac{3}{(4+3x)} \cdot \frac{1}{4} \quad f(x) = \frac{3/4}{1+\frac{3}{4}x} \quad a = 3/4 \quad r = \left(-\frac{3}{4}x\right)$$

$$\ast \text{ power series } \Rightarrow \frac{3}{4} + \frac{-9x^2}{16} + \frac{27x^3}{64} + \dots = \sum_{n=0}^{\infty} \frac{3}{4} \left(-\frac{3}{4}x\right)^n$$

$$\left|-\frac{3}{4}x\right| < 1 \quad -1 < \frac{3}{4}x < 1 \quad -4 < 3x < 4 \quad -\frac{4}{3} < x < \frac{4}{3}$$

$\ast$  interval of convergence  $(-4/3, 4/3)$

$$15. f(x) = \frac{2}{1-x^2} \quad a=2 \quad r=x^2$$

$$\ast \text{ power series } 2 + 2x^2 + 2x^4 + 2x^6 + \dots = \sum_{n=0}^{\infty} 2x^{2n}$$

$$|x^2| < 1 \quad -1 < x < 1$$

\* interval of convergence  $(-1, 1)$

16.  $f(x) = \frac{5}{5+x^2}$   $f(x) = \frac{1}{1+\frac{x^2}{5}}$   $a_1 = 1$   $r = \left(\frac{-x^2}{5}\right)$

\* power series  $1 - \frac{x^2}{5} + \frac{x^4}{25} + \frac{x^6}{125} + \dots = \sum_{n=0}^{\infty} \left(\frac{-x^2}{5}\right)^n$

$$\left| \frac{-x^2}{5} \right| < 1 \quad \frac{x^2}{5} < 1 \quad x^2 < 5$$
$$-\sqrt{5} < x < \sqrt{5}$$

\* interval of convergence  $(-\sqrt{5}, \sqrt{5})$

For #19 & 21  $\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$

19.  $f(x) = -\frac{1}{(x+1)^2} = \frac{d}{dx} \left( \frac{1}{x+1} \right)$

$$\frac{d}{dx} \sum_{n=0}^{\infty} (-1)^n n x^{n-1} \quad \sum_{n=1}^{\infty} (-1)^{n+1} (n+1) x^n$$

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+2} (n+2) x^{n+1}}{(-1)^{n+1} (n+1) x^n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+2} (n+2) x}{(-1)^{n+1} (n+1)} \right|$$

$$|x| < 1 \quad -1 < x < 1$$

interval of convergence  $(-1, 1)$

21.  $f(x) = \ln(x+1) = \int \frac{1}{x+1} dx$

$$c + \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$$

Find C since the center is @  $x=0$

$$C = \ln(0+1) \quad C = 0$$

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+1} x^{n+2}}{n+2} \cdot \frac{n+1}{(-1)^n x^{n+1}} \right| = \lim_{n \rightarrow \infty} \left| \frac{-1 x (n+1)}{(n+2)} \right|$$

$$|x| < 1 \quad -1 < x < 1$$

check endpoints

$$x = -1 \quad \sum_{n=0}^{\infty} \frac{(-1)^n (-1)^{n+1}}{n+1} = \sum_{n=0}^{\infty} \frac{-1}{n+1} \quad \text{diverges}$$

$$x = 1 \quad \sum_{n=0}^{\infty} \frac{(-1)^n (1)^{n+1}}{n+1}$$

alt series

$$\frac{1}{n+1} \geq 0 \quad \checkmark$$

$$\frac{1}{n+1} \geq \frac{1}{n+2} \quad \checkmark$$

$$\lim_{n \rightarrow \infty} \frac{1}{n+1} = 0 \quad \checkmark$$

\* conv. cond @  $x=1$

converges  $[-1, 1]$

converges abs  $(-1, 1)$