9.8 Day 3 Worksheet Name_____

1. Given that $f(x)=\sum_{n=0}^{\infty}2^n x^n$. $1+2x+4x^2+8x^3+\ldots$

a) Find a power series for $f'(x)$.

$f'(x)=0+1\cdot 2+2\cdot 4x+3\cdot 2\cdot 4x^2$

$f'(x)=\sum_{n=1}^{\infty} n\cdot 2^n x^{n-1}$ or $\sum_{n=0}^{\infty} 2^n\cdot n x^{n-1}$

$2^0\cdot 0=0 \quad n=0$

$2^1\cdot 1x^0=2 \quad n=1$

$2^2\cdot 2x^1=8x \quad n=2$

b) Find a power series for $\int f(x)dx$.

$C+x+x^2+\frac{4}{3}x^3+\frac{8}{4}x^4+\ldots$

$C+\sum_{n=0}^{\infty}\frac{2^n x^{n+1}}{n+1}$

2. Given that $f(x)=\sum_{n=0}^{\infty}\left(-\frac{1}{2}\right)^n(x-3)^n$. $=1+\left(-\frac{1}{2}\right)(x-3)+\left(\frac{1}{4}\right)(x-3)^2-\left(\frac{1}{8}\right)(x-3)^3+\ldots$

$f'=-\frac{1}{2}+\frac{1}{2}(x-3)-\frac{3}{8}(x-3)^2+\ldots$

$F=C+(x-3)-\frac{1}{4}(x-3)^2+\frac{1}{12}(x-3)^3+\ldots$

a) Find a power series for $f'(x)$.

$\sum_{n=0}^{\infty}\left(-\frac{1}{2}\right)^n\cdot n(x-3)^{n-1}$

$0+\frac{-1}{2}+\left(\frac{1}{4}\right)\cdot 2(x-3)-\frac{1}{8}(3)(x-3)^2$:)

b) Find a power series for $\int f(x)dx$.

$C+\sum_{n=0}^{\infty}\left(\frac{-1}{2}\right)^n\frac{(x-3)^{n+1}}{n+1}$

3. Given that $f(x)=\frac{1}{1+x^2}\approx 1-x^2+x^4-x^6$, find the 7th order MacLaurin Polynomial for $g(x)=\tan^{-1}x$.

$g(x)=C+x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}$

$g(0)=\tan^{-1}(0)=0$

$g(x)=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}$

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(20) Converges: $x = 5$

Conv. abs: $x = 5$

Conv. cond: NEVER

(21) Converges: $(-5, 13]$

Conv. abs: $(-5, 13)$

Conv. cond: $x = 13$

(27) Conv: $(-\frac{1}{2}, \frac{1}{2})$

Conv. abs: $(-\frac{1}{2}, \frac{1}{2})$

Conv. cond: NEVER

(37) Conv: $(-h, h)$

Conv. abs.: $(-h, h)$

Conv. cond: NEVER