9.8 Day 3

Tuesday, March 3, 2020 10:02 AM

Calc BC 9.8 Day 3 (1)

Term by Term Diff & Integration

If $f(x) = \sum_{n=0}^{\infty} a_n(x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + a_3(x-c)^3 + \dots$

then...

(1) $f'(x) = a_1 + 2a_2(x-c) + 3a_3(x-c)^2 + \dots \sum_{n=1}^{\infty} n \cdot a_n(x-c)^{n-1}$

could we write this $\sum_{n=0}^{\infty} n \cdot a_n(x-c)^{n-1}$

(2) $\int f(x)\,dx = C + a_0(x-c) + a_1\frac{(x-c)^2}{2} + \frac{a_2(x-c)^3}{3} + \frac{a_3(x-c)^4}{4} + \dots$

$$= C + \sum_{n=0}^{\infty} \frac{a_n(x-c)^{n+1}}{n+1}$$

Ex! Suppose $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n} = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots$

a. find the power series for $f'(x)$.

$f'(x) = \sum \frac{n \cdot x^{n-1}}{n} = \sum_{n=1}^{\infty} x^{n-1}$

let's check to confirm...

$f'(x) = 1 + x + x^2 + x^3 + \dots \qquad \sum_{n=1}^{\infty} x^{n-1} = 1 + x + x^2 + x^3 + \dots$:)

(b) Find the power series for $\int f(x)\,dx$

$F(x) \stackrel{?}{=} C + \sum_{n=1}^{\infty} \frac{x^{n+1}}{n(n+1)}$

let's confirm

$f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n} = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots$

then $F(x) = C + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{12} + \frac{x^5}{20}$ —

Can you write a power series $\int f(x)\,dx$

$\sum_{n=2}^{\infty} \frac{x^n}{n(n-1)}$ => getting ready for MC questions

works for Taylor & MacLaurin Polys

Ex Given that $\sqrt{1+2x} \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{2}$,

find the 2nd order MacLaurin Polynomial

for $g(x) = \frac{1}{\sqrt{1+2x}}$

so what is $\frac{d}{dx}(1+2x)^{1/2} = \frac{1}{2}(1+2x)^{-1/2} \cdot 2$ oh...

so $g(x) = 1 + x + \frac{3}{2}x^2 \ldots$ ← derivative