AP Review Questions for Quiz 9.7-9.8 Name________________________

BC2000 #3 No Calculator whole problem

The Taylor series about $x = 5$ for a certain function f converges to $f(x)$ for all x in the interval of convergence. The nth derivative of f at $x = 5$ is given by $f^{(n)}(5) = \dfrac{(-1)^n n!}{2^n(n+2)}$, and $f(5) = \dfrac{1}{2}$.

(a) Write the third-degree Taylor polynomial for f about $x = 5$.

(b) Find the radius of convergence of the Taylor series for f about $x = 5$.

(c) Show that the sixth-degree Taylor polynomial for f about $x = 5$ approximates $f(6)$ with error less than $\dfrac{1}{1000}$.

a)

$n=1$ $f'(5) = \dfrac{(-1)^1(1)!}{2^1(1+2)} = \dfrac{-1}{2(3)} = -\dfrac{1}{6}$ $n=2$ $f''(5) = \dfrac{(-1)^2(2)!}{2^2(2+2)} = \dfrac{2}{16} =$

$n=3$ $f'''(5) = \dfrac{-1(3)!}{2^3(5} = \dfrac{-6}{8(5)} = -\dfrac{6}{40}$

$P_3(x) = \frac{1}{2} - \frac{1}{6}(x-5) + \frac{2}{16 \cdot 2!}(x-5)^2 - \frac{6}{40 \cdot 3!}(x-5)^2 = \frac{1}{2} - \frac{1}{6}(x-5) + \frac{1}{16}(x-5)^2 - \frac{1}{40}(x-5)^3$

b) $a_n = \dfrac{f^{(n)}(5)}{n!} = \dfrac{(-1)^n}{2^n(n+2)}$

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+1}(x-5)^{n+1}}{2^{n+1}(n+3)} \cdot \frac{2^n(n+2)}{(-1)^n(x-5)^n}\right| = \lim_{n\to\infty}\left|\frac{(-1)(x-5)(n+2)}{2(n+3)}\right|$$

$\left|\frac{x-5}{2}\right| < 1$ $-1 < \frac{x-5}{2} < 1$ $-2 < x-5 < 2$

$3 < x < 7$

interval of convergence $= 2$

More on back...

c (over)

c) $\frac{f^{(7)}(6)}{n!} = \left| \frac{(-1)^7}{2^7(7+2)} \right| < \frac{1}{1000}$

$\frac{1}{2^7(9)} < \frac{1}{1000}$

The taylor series w/ abs value terms decreasing to 0

BC2002 #6 No Calculator

The Maclaurin series for the function f is given by

$$f(x)=\sum_{n=0}^{\infty}\frac{(2x)^{n+1}}{n+1}=2x+\frac{4x^2}{2}+\frac{8x^3}{3}+\frac{16x^4}{4}+\ldots+\frac{(2x)^{n+1}}{n+1}+\ldots$$

only

on its interval of convergence.

(a) Find the interval of convergence of the Maclaurin series for f. Justify your answer.

(b) Find the first four terms and the general term for the Maclaurin series for $f'(x)$.

Don't / do general term

(c) Use the Maclaurin series you found in part (b) to find the value of $f'\left(-\frac{1}{3}\right)$.

a) $\lim_{n\to\infty}\left|\frac{(2x)^{n+2}}{n+2}\cdot\frac{n+1}{(2x)^{n+1}}\right| = \lim_{n\to\infty}\left|\frac{2x(n+1)}{(n+2}\right| =$

$|2x|<1$ $\quad -1<2x<1$

$\boxed{-\frac{1}{2}<x<1/2}$

Check $x=1/2$ $\quad \sum_{n=0}^{\infty}\frac{(1)^{n+1}}{n+1}$ diverges b/c harmonic series

check $x=-1/2$ $\quad \sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{n+1}$ converges b/c alt. series test

converges abs $\left(-\frac{1}{2},\frac{1}{2}\right)$

converges cond $x=-\frac{1}{2}$

converges $[-1/2, 1/2)$

b) $f'(x)=2+4x+8x^2+16x^3$ $\qquad \sum_{n=1}(2x)^n$

no c