9.4

Saturday, December 28, 2019 11:18 AM

Pg. 616 #3-6, 8, 10-13, 15, 16, 22-30, 49-54

3. $\sum_{n=1}^{\infty} \frac{1}{2n-1}$ $a_n = \frac{1}{2n-1} = 1, \frac{1}{3}, \frac{1}{5} \ldots$ $b_n < a_n$

$b_n = \frac{1}{2n} = 1, \frac{1}{4}, \frac{1}{6} \ldots$

$a_n > 0$

$b_n > 0$

$\sum b_n \Rightarrow$ diverges $\therefore \sum a_n$ diverges

Diverges by comparison test

4. $\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$ $a_n = \frac{1}{3n^2+2}$ $\frac{1}{5}, \frac{1}{14}, \frac{1}{29}$ $b_n > a_n$

$b_n = \frac{1}{3n^2}$ $1, \frac{1}{4}, \frac{1}{9}$

$\sum \frac{1}{3n^2} \Rightarrow$ converges by p series $p=2>1$ $\therefore \sum \frac{1}{3n^2+2}$

Direct comparison test $\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$ converges

5. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$ $a_n = \frac{1}{\sqrt{n}-1}$ $b_n = \frac{1}{\sqrt{n}}$ $p = \frac{1}{2}$

$\underline{a_n > b_n}$ $\sum b_n \Rightarrow$ diverges by p-series diverges

$\therefore \sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$ diverges by Direct comparison test

6. $\sum_{n=0}^{\infty} \frac{4^n}{5^n+3}$ $a_n = \frac{4^n}{5^n+3}$ $b_n = \frac{4^n}{5^n} = \left(\frac{4}{5}\right)^n$

$b_n > a_n$ converges geo $|r|>1$

$\sum_{n=0}^{\infty} \frac{4^n}{5^n+3}$ converges by Direct comparison test

8. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$ $a_n = \frac{1}{\sqrt{n^3+1}}$ $b_n = \frac{1}{\sqrt{n^3}} = \frac{1}{n^{3/2}}$

$p>1$ p-series test converges

$b_n > a_n$

$n=1$

$b_n > a_n$ converges

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$ converges by Direct comparison test

10. $\sum_{n=1}^{\infty} \frac{1}{4\sqrt[3]{n}-1}$ $a_n = \frac{1}{4\sqrt[3]{n}-1}$ $b_n = \frac{1}{4\sqrt[3]{n}}$

$\frac{1}{4n^{1/3}}$ $p < 1$ p-series test diverges

$a_n > b_n$

$\sum_{n=1}^{\infty} \frac{1}{4\sqrt[3]{n}-1}$ Diverges By Direct comparison test

11. $\sum_{n=0}^{\infty} e^{-n^2} = \sum_{n=0}^{\infty} \frac{1}{e^{n^2}}$ $a_n = \frac{1}{e^{n^2}}$ $b_n = \frac{1}{e^n} = \left(\frac{1}{e}\right)^n$

n

$1, \frac{1}{e}, \frac{1}{e^4}, \frac{1}{e^9}$ $1, \frac{1}{e}, \frac{1}{e^2}, \frac{1}{e^3}$

$b_n > a_n$ $\sum b_n \rightarrow$ converges $|r| = \left|\frac{1}{e}\right| < 1$ by Geometric

$\sum_{n=0}^{\infty} e^{-n^2}$ converges by direct comparison test

12. $\sum_{n=1}^{\infty} \frac{3^n}{2^n-1}$ $a_n = \frac{3^n}{2^n-1}$ $b_n = \left(\frac{3}{2}\right)^n$ $\sum b_n$ diverges $|r| = \left|\frac{3}{2}\right| > 1$ by Geometric

$b_n < a_n$

By direct comparison test $\sum_{n=1}^{\infty} \frac{3^n}{2^n-1}$

13. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ $a_n = \frac{n}{n^2+1}$ $b_n = \frac{n}{n^2} = \frac{1}{n}$

b_n $p \le 1$ Diverges p-series test

$\lim_{n\to\infty} \frac{\frac{n}{n^2+1}}{1/n} = \lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$

$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ Diverges $\Rightarrow$ limit comparison test

15. $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}$ $a_n = \frac{1}{\sqrt{n^2+1}}$ $b_n = \frac{1}{\sqrt{n^2}} = \frac{1}{|n|}$

$p \le 1$ p-series test Diverges

15. $\sum_{n=0} \sqrt{n^2+1}$ $\quad a_n = \sqrt{n^2+1}$ $\quad b_n = \sqrt{n^2}$ $\quad |n|$ $\quad p \le 1$ p-series Diverges

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{|n|}{\sqrt{n^2+1}} = 1$$

$\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}$ Diverges By limit comparison test

16. $\sum_{n=1}^{\infty} \frac{2^n+1}{5^n+1}$ $\quad a_n = \frac{2^n+1}{5^n+1}$ $\quad b_n = \frac{2^n}{5^n} = \left(\frac{2}{5}\right)^n$ Geometric bn converges $r = 2/5 < 1$

converge

$$\lim_{n\to\infty} \frac{2^n+1}{5^n+1} \cdot \frac{5^n}{2^n}$$

$$\lim_{n\to\infty} \frac{2^n+1}{5^n+1} \cdot \lim_{n\to\infty} \frac{5^n}{2^n}$$

L Hop

$$\lim_{n\to\infty} \frac{2^n \ln 2}{5^n \cdot \ln 5} \cdot \lim_{n\to\infty} \frac{5^n \cdot \ln 5}{2^n \ln 2} = 1$$

by limit comparison test $\sum_{n=1}^{\infty} \frac{2^n+1}{5^n+1}$ Converges

22) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$ $\quad b_n = \frac{1}{n}$ $\quad \sum bn$ Diverge $p \le 1$ harmonic series

$$\lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{\left(\frac{1}{n}\right)} \to \frac{0}{0}$$

$$\text{l'Hop } \lim_{n\to\infty} \frac{\cos\left(\frac{1}{n}\right) \cdot \cancel{-n^{-2}}}{\cancel{-n^{-2}}} = 1$$

$\boxed{\sum_{n=1}^{\infty} \text{ Diverges by limit comparison test}}$

23. C -> Diverges $\quad$ 24. b Diverges $\quad$ 25. f -> converges

26. g converges $\quad$ 27. a. Diverges $\quad$ 28. d converges

$$\lim_{n\to\infty} \frac{n^3}{n^3 g} = 1 \qquad \frac{1}{n^3} \quad p > 1$$

29. e or f $\quad \lim_{b\to\infty} \int_1^b \frac{x}{(x^2+1)^2}\,dx$ $\quad u = x^2+1 \quad u(b) = b^2+1 \quad u(1) = 4$ $\quad du = 2x\,dx$

$$\frac{1}{2}\lim_{b\to\infty} \int_4^{b^2+1} u^{-2}\,du = \lim_{b\to\infty} -\frac{1}{2} u^{-1}\Big|_4^{b^2+1} = \lim_{b\to\infty} \frac{-1}{2(b^2+1)} + \frac{1}{2\cdot 4}$$

converges

$\frac{1}{2}\lim_{b\to\infty}\int_4^{b^2+1} u^{-2}\,du = \lim_{b\to\infty} -\frac{1}{2}u^{-1}\Big|_4 = \lim_{b\to\infty} \frac{-1}{2(b^2+1)} + \frac{1}{2\cdot 4}$ converges

or

$a_n = \frac{x}{(x^2+1)^2}$ $\quad b_n = \frac{x}{x^4} = \frac{1}{x^3}$ $\quad b_n > a_n$

$\frac{1}{4}, \frac{2}{25}$ $\quad 1, \frac{1}{8}$ $\quad b_n$ converges b/c $p > 1$

30. $\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$ f $\quad a_n = \frac{3}{n(n+3)}$ $\quad b_n = \frac{3}{n^2}$ $\quad b_n > a_n$

$\frac{3}{4}, \frac{3}{10}, \frac{3}{18}$ $\quad 3, \frac{3}{4}, \frac{3}{9}$ $\quad b_n$ converges b/c $p > 2$

49. False (look @ # 30)

50. True 51. True 52. False 53. True 54 False

I can add a divergent & convergent series together