9.3 (Thursday 2/6)

Wednesday, February 5, 2020 12:26 PM

Section 9.3 Notes
Integral and P-Series Tests

Warm up: Determine if the series will converge or diverge.

A) $\sum_{n=1}^{\infty} 8\left(\frac{5}{6}\right)^n$ Geometric $\frac{5}{6} < 1$

$\frac{\frac{40}{6}}{1-5/6} = 40$ ∴ converges

B) $\sum_{n=1}^{\infty}\frac{(n+1)!}{n!}$

$(n+1)!$
$(n+1)(n+1-1)!$
$(n+1)(n)!$

$\sum_{n=1}^{\infty} \frac{(n+1)\cancel{n!}}{\cancel{n!}}$

$\lim_{n\to\infty} n+1 \neq 0$

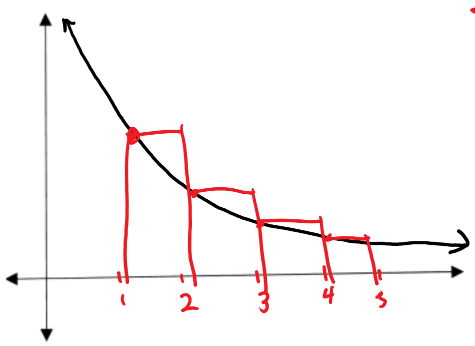
Series Diverges by nth Term test

Take a look at the following series:

$\sum_{n=1}^{\infty}\frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots\ldots$

This is called the Harmonic Series.

Why does it diverge??? Look @ $f(x) = \frac{1}{x}$ and the area under the curve



$\sum_{n=1}^{\infty}\frac{1}{n} = LRAM_{\infty} \geq \int_1^{\infty} f(x)\,dx \Rightarrow$ Diverge

$\lim_{b\to\infty}\int_1^b \frac{1}{x}\,dx = \lim_{b\to\infty} \ln x \Big|_1^b = \lim_{b\to\infty}\left[\ln b - \ln 1\right]$

$= \infty$

so $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges

<u>The Integral Test</u>

Suppose $a_n = f(x)$ where $f(x)$ is continuous, positive, and decreasing, then

$\sum_{n=N}^{\infty} a_n$ and $\int_N^{\infty} f(x)\,dx$ either both converge or both diverge.

Ex: Converge or diverge?

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

positive ✓
continuous ✓
decreasing ✓

$$\int_1^{\infty} \frac{1}{x^2}\,dx = \lim_{b\to\infty} \int_1^b x^{-2}\,dx = \lim_{b\to\infty} \left[-x^{-1}\right]_1^b$$

$$= \lim_{b\to\infty} \left[-\frac{1}{b} + \frac{1}{1}\right] = 1$$

so $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by $\int$ test.

Ex: Converge or diverge?

$$\sum_{n=1}^{\infty} \frac{4}{\sqrt[3]{n}}$$

cont. ✓
positive ✓
decreasing ✓

$$\int_1^{\infty} \frac{4}{\sqrt[3]{x}}\,dx$$

$$\Rightarrow \lim_{b\to\infty} \int_1^b \frac{4}{x^{1/3}}\,dx = \lim_{b\to\infty} \left[6x^{2/3}\right]_1^b = \lim_{b\to\infty} \left[6b^{2/3} - 6\right] = \infty$$

$\sum_{n=1}^{\infty} \frac{4}{\sqrt[3]{n}}$ diverges by $\int$ test.

Try: Converge or diverge?

$$\sum_{n=2}^{\infty} \frac{\ln(n)}{n}$$

cont. ✓
positive ✓
decreasing ✓

$$\int_2^{\infty} \frac{\ln x}{x}\,dx = \lim_{b\to\infty} \int_2^b \frac{\ln x}{x}\,dx$$

$u = \ln x$ $\quad u(b) = \ln b$

$du = \frac{1}{x}$ $\quad u(2) = \ln 2$

$$= \lim_{b\to\infty} \int_{\ln 2}^{\ln b} u\,du = \lim_{b\to\infty} \frac{u^2}{2}\Big|_{\ln 2}^{\ln b} = \lim_{b\to\infty} \left[\frac{\ln^2 b}{2} - \frac{\ln^2 2}{2}\right] = \infty$$

$\sum_{n=2}^{\infty} \frac{\ln n}{n}$ diverges by $\int$ test.

P-Series Test

$$\sum \frac{K}{x^p}$$

K = constant

$p > 1$, then Σ converges

$p \le 1$, then Σ diverges

Ex: Converge or diverge?

$$\sum_{n=1}^{\infty} \frac{3}{\sqrt{n}}$$

$p = \frac{1}{2} < 1$

Diverges by p-series test

Try: Converge or diverge?

$$\sum_{n=1}^{\infty} \frac{\pi}{n^4}$$ $p = 4 > 1$ Converges by p-series test

$$\sum_{n=1}^{\infty} \frac{1}{4n}$$ $p = 1 \le 1$ Diverges by p-series test