

Evaluate each limit expression. L'Hospital's rule can be used for many of these problems, but remember:

- Ignore the conditions of L'Hospital's rule at your own peril.
- If L'Hospital's Rule fails to lead to a limit, nothing is learned about the original limit.
- More efficient strategies may exist for some problems even if L'Hospital's rule is effective.

1. $\lim_{t \rightarrow \pi} \frac{\tan 3t}{\pi - t} \rightarrow \frac{0}{0}$ LR applies

$$\lim_{t \rightarrow \pi} \frac{3 \sec^2 3t}{-1} = \frac{3(\sec 3\pi)^2}{-1} = \frac{3}{-1} = \boxed{3}$$

2. $\lim_{p \rightarrow 0} \frac{\cos p}{p^2} = \frac{1}{0^2}$

NO LR

$$\boxed{\infty}$$

3. $\lim_{x \rightarrow 0} \frac{\sin 5x}{x} \rightarrow \frac{0}{0}$

LR or $\frac{\sin ax}{ax}$

$$= \boxed{5}$$

4. $\lim_{x \rightarrow \infty} \frac{5x^2 - 3x}{7x^2 + 1} \rightarrow \frac{\infty}{\infty}$

LR or End behavior

$$= \boxed{5/7}$$

5. $\lim_{b \rightarrow 3} \frac{b-3}{b^2-3} = \frac{3-3}{9-3} = \frac{0}{6}$

= 0

$$\boxed{\text{NO LR}}$$

6. $\lim_{x \rightarrow \infty} \frac{2x + \sin x}{3x} =$

LR or separate

$$\boxed{2/3}$$

7. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{2x - \pi}{\cos x} \rightarrow \frac{0}{0}$

LR

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{2}{-\sin x} = \frac{2}{-\sin \frac{\pi}{2}} = \frac{2}{-1} = \boxed{-2}$$

8. $\lim_{x \rightarrow 1} \frac{x^3 - 1}{4x^3 - x - 3} \rightarrow \frac{0}{0}$

LR

$$\lim_{x \rightarrow 1} \frac{3x^2}{12x^2 - 1} = \frac{3}{12 - 1} = \boxed{\frac{3}{11}}$$

9. $\lim_{m \rightarrow -3} \frac{m+3}{m^2-9} \rightarrow \frac{0}{0}$

LR or Factor

$$\boxed{-1/6}$$

10. $\lim_{y \rightarrow 2} \frac{y^2 - y - 2}{y + 1} = \frac{0}{3} = 0$

NO LR

11. $\lim_{x \rightarrow 0} \frac{e^x - (1-x)}{x} \rightarrow \frac{0}{0}$

LR

$$\lim_{x \rightarrow 0} \frac{e^x - 1 + x}{x}$$

$$\lim_{x \rightarrow 0} \frac{e^x + 1}{1} = \frac{2}{1} = \boxed{2}$$

12. $\lim_{x \rightarrow 1} \frac{\ln x^2}{x^2 - 1} \rightarrow \frac{0}{0}$

LR

$$\lim_{x \rightarrow 1} \frac{\frac{1}{x^2} \cdot 2x}{2x} = \boxed{1}$$

13. $\lim_{x \rightarrow \infty} \frac{x^2}{e^x} \rightarrow \frac{\infty}{\infty}$

LR TWICE

$\lim_{x \rightarrow \infty} \frac{2x}{e^x} = \frac{\infty}{\infty}$

$\lim_{x \rightarrow \infty} \frac{2}{e^x} = \boxed{0}$

16. $\lim_{y \rightarrow \infty} \frac{\ln y}{y^2} \rightarrow \frac{\infty}{\infty}$

LR or end behavior

$\lim_{y \rightarrow \infty} \frac{\frac{1}{y}}{2y} = \lim_{y \rightarrow \infty} \frac{1}{2y^2} = \boxed{0}$

19. $\lim_{x \rightarrow \infty} \frac{x^2 + 2x + 3}{x - 1} \rightarrow \frac{\infty}{\infty}$

LR or end behavior

$\boxed{\infty}$

22. $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x - 3} \rightarrow \frac{0}{0}$

LR

$\lim_{x \rightarrow 3} \frac{\frac{1}{2}(x+1)^{-1/2} \cdot 1}{1} = \frac{1}{2\sqrt{4}} = \boxed{\frac{1}{4}}$

25. $\lim_{x \rightarrow 9} \frac{x - 9}{\sqrt{x} - 3} \rightarrow \frac{0}{0}$

LR or factor

$\lim_{x \rightarrow 9} \frac{1}{\frac{1}{2}x^{-1/2}} = 2\sqrt{9} = \boxed{6}$

14. $\lim_{x \rightarrow \infty} \frac{x^3}{x + 2} \rightarrow \frac{\infty}{\infty}$

LR or end behavior

$\lim_{x \rightarrow \infty} \frac{3x^2}{1} = \infty$

17. $\lim_{x \rightarrow 0} \frac{\arcsin x}{x} \rightarrow \frac{0}{0}$

LR

$\lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt{1-x^2}}}{1} = \frac{1}{\sqrt{1-0}} = \boxed{1}$

20. $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{e^x} = \frac{0}{1}$

$= \boxed{0}$

NO LR

23. $\lim_{x \rightarrow 0} \frac{\sqrt{x+3} - \sqrt{3}}{x} \rightarrow \frac{0}{0}$

LR

$\lim_{x \rightarrow 0} \frac{\frac{1}{2}(x+3)^{-1/2}}{1} = \frac{1}{2}(3)^{-1/2} = \boxed{\frac{1}{2\sqrt{3}}}$

26. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{1 + \cos 2x} \rightarrow \frac{0}{0}$

LR TWICE

$\lim_{x \rightarrow \frac{\pi}{2}} \frac{-\cos x}{-2\sin 2x} = \frac{0}{0}$
 $\lim_{x \rightarrow \frac{\pi}{2}} \frac{+\sin x}{-4\cos 2x} = \boxed{\frac{+1}{4}}$

15. $\lim_{a \rightarrow \infty} \frac{e^a}{a^2} \rightarrow \frac{\infty}{\infty}$

LR TWICE

$\lim_{a \rightarrow \infty} \frac{e^a}{2a}$

$\lim_{a \rightarrow \infty} \frac{e^a}{2} = \boxed{\infty}$

18. $\lim_{x \rightarrow \infty} \frac{3x^2 - 2x + 1}{2x^2 + 3} \rightarrow \frac{\infty}{\infty}$

LR or end behavior

$= \boxed{\frac{3}{2}}$

21. $\lim_{x \rightarrow 3} \frac{2(x-3)}{x^2 - 9} \rightarrow \frac{0}{0}$

LR

$\lim_{x \rightarrow 3} \frac{2}{2x} = \frac{2}{2(3)} = \boxed{\frac{1}{3}}$

24. $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} \rightarrow \frac{0}{0}$

LR

$\lim_{x \rightarrow 0} \frac{-\sin x}{1} = \boxed{0}$

27. $\lim_{h \rightarrow 2} \frac{h^2 - h - 2}{h - 2} \rightarrow \frac{0}{0}$

LR

$\lim_{h \rightarrow 2} \frac{2h - 1}{1} = \boxed{3}$