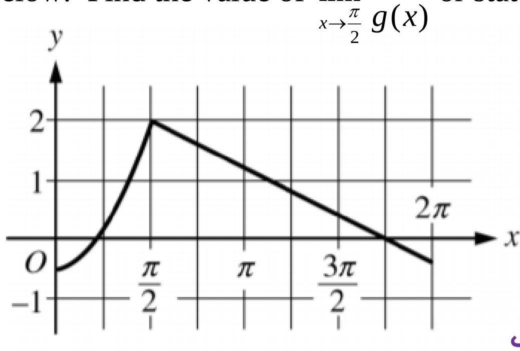
AP Calculus AB **Name:**
9.2 day 2

1. Let $f(x) = e^x \cos x$. Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is below. Find the value of $\lim\limits_{x \to \frac{\pi}{2}} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.



Graph of g'

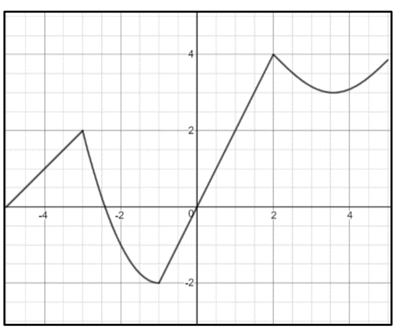
$f\left(\frac{\pi}{2}\right) = e^{\frac{\pi}{2}} \cdot \cos\frac{\pi}{2} = 0$ ✓

$f'(x) = e^x \cdot -\sin x + \cos x \cdot e^x$

$f'\left(\frac{\pi}{2}\right) = e^{\pi/2} \cdot -\sin\frac{\pi}{2} + \cos\frac{\pi}{2} \cdot e^{\frac{\pi}{2}} = -e^{\pi/2}$

$\lim\limits_{x \to \frac{\pi}{2}} \frac{f(x)}{g(x)} = \lim\limits_{x \to \frac{\pi}{2}} \frac{f'(x)}{g'(x)} = \frac{-e^{\pi/2}}{2}$

2. Let $h(x) = 4\sin(8x)$. Let m be a function graphed below. Find the value of $\lim\limits_{x \to 0} \frac{h(x)}{m(x)}$ or state that it does not exist. Justify your answer.



$h(0) = 4\sin 8 \cdot 0 = 0$ ✓

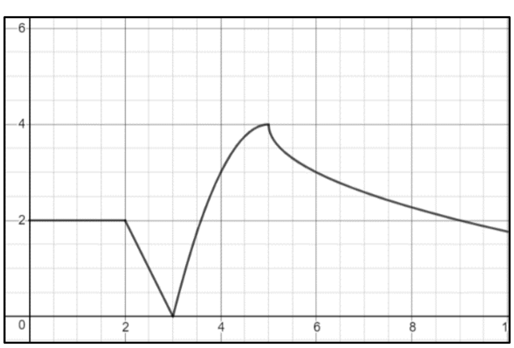
$m(0) = 0$ ✓

$h'(x) = 4\cos 8x \cdot 8$

$h'(0) = 32$

$\lim\limits_{x \to 0} \frac{h(x)}{m(x)} = \lim\limits_{x \to 0} \frac{h'(x)}{m'(x)} = \frac{32}{2} = 16$

3. Let $z(x) = \cos\left(\frac{\pi}{2}x\right)$. Let p be a differentiable function such that $p(1) = 0$ and whose derivative, p', is graphed below. Find the value of $\lim\limits_{x \to 1} \frac{p(x)}{z(x)}$ or state that it does not exist. Justify your answer.

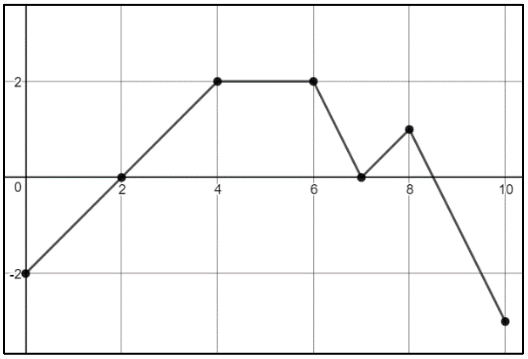


$z(1) = \cos\frac{\pi}{2} = 0$ ✓

$z'(x) = -\frac{\pi}{2}\sin\frac{\pi}{2}x$

$\lim\limits_{x \to 1} \frac{p(x)}{z(x)} = \lim\limits_{x \to 1} \frac{p'(x)}{z'(x)} = \frac{2}{-\frac{\pi}{2}} = -\frac{4}{\pi}$

4. Let $f(x)=\int_4^x g(t)dt$ with g graphed below. Find $\lim_{x\to 0}\frac{f(x)}{x^2+4x}$.

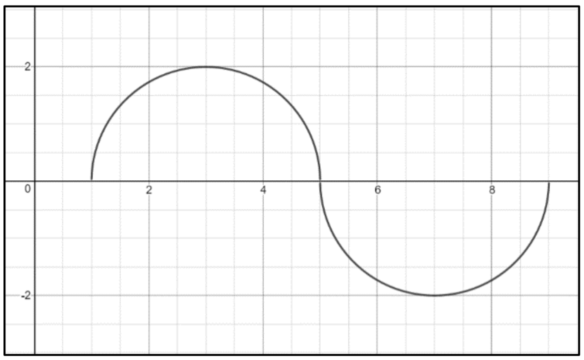


$f(0)=\int_4^0 g(t)\,dt=0$ ✓

$0^2+4\cdot 0=0$ ✓

$\lim_{x\to 0}\frac{f(x)}{x^2+4x}=\lim_{x\to 0}\frac{g(x)}{2x+4}$ $\frac{-2}{4}=\frac{-1}{2}$

5. Let $h(x)=\int_7^x m(t)dt$ with m comprised of semicircles and graphed below. Find $\lim_{x\to 3}\frac{\ln(x-2)}{h(x)}$.



$\ln(3-2)=\ln 1=0$ ✓

$h(3)=\int_7^3 m(t)\,dt=0$ ✓

$\lim_{x\to 3}\frac{\ln(x-2)}{h(x)}=\lim_{x\to 3}\frac{\frac{1}{x-2}}{h'(x)}=\frac{1}{2}$

6. Given thrice differentiable functions f and g with values in the table below, find the following limits.

x	f	f'	f''	g	g'	g''
0	1	3	5	7	9	11
1	0	0	-4	0	0	6
2	6	0	-2	5	0	7
3	4	2	-3	-7	5	2

a. $\lim_{x\to 3}\frac{f(x)}{g(x)}$

$=-\frac{4}{7}$

b. $\lim_{x\to 1}\frac{f(x)}{g(x)}$

$=\lim_{x\to 1}\frac{f'(x)}{g'(x)}$

$=\lim_{x\to 1}\frac{f''(x)}{g''(x)}$

$=\frac{-2}{3}$

c. $\lim_{x\to 2}\frac{3f'(x)+2}{6g'(x)-4}$

$=\frac{-1}{2}$

d. $\lim_{x\to 1}\frac{f(g(x))}{g(f(x))}$

$=\frac{1}{7}$

e. $\lim_{x\to 3}\frac{\int_3^x g'(t)dt}{\sin \pi x}$

$=\lim_{x\to 3}\frac{g'(x)}{\pi\cos\pi x}$

$=\frac{-5}{\pi}$

f. $\lim_{x\to 3}\frac{f(x)-4}{g'(x)-5}$

$=\lim_{x\to 3}\frac{f'(x)}{g''(x)}=1$

7. Evaluate $\lim_{x\to 2}\frac{2x^2-8}{x-2}$ $= \lim_{x\to 2}\frac{4x}{1}$

A. 0 B. 2 C. 4 D. 8

8. Evaluate $\lim_{x\to 1}\frac{x-1}{1-\sqrt{x}}$ $= \lim_{x\to 1}\frac{1}{-\frac{1}{2}x^{-1/2}} = -2$

A. -2 B. $-\frac{1}{2}$ C. $\frac{1}{2}$ D. 2

9. Evaluate $\lim_{x\to\infty}\frac{\sin x}{x}$

A. 0 B. 1 C. 2π D. ∞

10. Evaluate $\lim_{x\to 0}\frac{4e^x-\sin x-4}{x^2+4x}$ $= \lim_{x\to 0}\frac{4e^x-\cos x}{2x+4} = \frac{3}{4}$

A. 0 B. $\frac{3}{4}$ C. 1 D. nonexistent

11. Evaluate $\lim_{x\to 2}\frac{\int_2^x e^{\frac{t}{2}}dt}{x^3-8}$ $= \lim_{x\to 2}\frac{e^{x/2}}{3x^2} = \frac{e}{12}$

A. 0 B. $\frac{e}{24}$ C. $\frac{e}{12}$ D. e

12. Evaluate $\lim_{x\to e}\frac{(x^{20}-3x)-(e^{20}-3e)}{x-e}$ $f'(x) = 20x^{19}-3$

A. 0 B. $20e^{19}-3$ C. $20e^{19}-3e$ D. nonexistent

13. Evaluate $\lim_{x\to 3}\frac{\tan(x-3)}{3e^{x-3}-x}$ $= \lim_{x\to 3}\frac{\sec^2(x-3)}{3e^{x-3}-1} = \frac{1}{2}$

A. 0 B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. nonexistent

In problems 14-18, justify your answer.

14. Evaluate $\lim_{x\to 0}\frac{x^2-4x}{2x-1}$ $= 0$

15. Evaluate $\lim_{x\to\infty}\frac{x^3}{e^x}$ $= \lim_{x\to\infty}\frac{3x^2}{e^x} = \lim_{x\to\infty}\frac{6x}{e^x} = \lim_{x\to\infty}\frac{6}{e^x} = 0$

$\infty^3 = \infty$ ✓
$e^\infty = \infty$

$3\infty^2 = \infty$ ✓
$e^\infty = \infty$

$6\infty = \infty$ ✓
$e^\infty = \infty$

16. Evaluate $\lim_{x\to\infty}\frac{\ln(100x)}{x}$ $= \lim_{x\to\infty}\frac{\frac{100}{100x}}{1} = 0$

$\ln 100\cdot\infty = \infty$ ✓
$\infty = \infty$

17. Evaluate $\lim_{x\to\infty}\frac{x}{\sqrt{e^x}}$ $= \lim_{x\to\infty}\frac{1}{\frac{1}{2}(e^x)^{-1/2}\cdot e^x} = \lim_{x\to\infty}\frac{2}{\sqrt{e^x}} = 0$

$\infty = \infty$ ✓
$\sqrt{e^\infty} = \infty$

18. a. Evaluate $\lim_{x\to 1^+}\frac{\cos\pi x}{\ln x}$ $= -\frac{1}{0^+} = -\infty$

b. Evaluate $\lim_{x\to 1^-}\frac{\cos\pi x}{\ln x}$ $= \frac{-1}{0^-} = \infty$

c. Evaluate $\lim_{x\to 1}\frac{\cos\pi x}{\ln x}$ DNE because the $\lim_{x\to 1^+} \neq \lim_{x\to 1^-}$