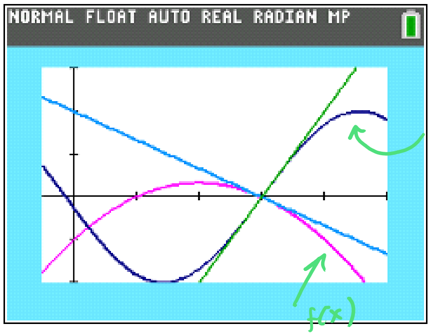
9.2 Day 1 (Friday 1/24)

Monday, January 20, 2020 9:06 AM

L' Hôpital Rule
L' Hospital Rule } l'Hop

Evaluate: $\lim_{x \to 3} \dfrac{\overbrace{-\frac{1}{3}(x-2)^2 + \frac{1}{3}}^{f(x)}}{\underbrace{2\sin(x-3)}_{g(x)}}$ $\to 0$ / $\to 0$

using table $\nearrow -\frac{1}{3}$



g(x)

f(x)

$f(x) = -\frac{1}{3}(x-2)^2 + \frac{1}{3}$

$f'(x) = -\frac{2}{3}(x-2) \cdot 1$

$g(x) = 2\sin(x-3)$

$g'(x) = 2\cos(x-3)$

$\lim_{x \to 3} f'(x) = -2/3$ $\quad \lim_{x \to 3} g'(x) = 2$

$\lim_{x \to 3} \dfrac{f'(x)}{g'(x)} = \dfrac{-2/3}{2} = -1/3$

l' Hôpital's Rule:

If $f(a) = g(a) = 0$, or if both are $\pm\infty$

then $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \lim_{x \to a} \dfrac{f'(x)}{g'(x)}$

★ use only for indeterminate forms $\frac{0}{0}$ or $\frac{\infty}{\infty}$

Ex1: $\lim_{x \to 0} \dfrac{e^x - 1}{5x}$ $\nearrow 0$ / $\searrow 0$ $\overset{L}{=} \lim_{x \to 0} \dfrac{e^x}{5} = \dfrac{1}{5}$ ☺

Ex2: $\lim_{x \to \infty} \dfrac{4x^2 + 12x}{3x^2 - 5}$ $\nearrow \infty$ / $\searrow \infty$ $\overset{L}{=} \lim_{x \to \infty} \dfrac{8x + 12}{6x}$ $\nearrow \infty$ / $\searrow \infty$ ☹

$$= \lim_{x \to \infty} \frac{8}{6} = \frac{4}{3}$$

Ex 3: $\lim_{x \to 3} \frac{\sqrt{1+x} - 2}{x - 3} \nearrow^{0}_{\searrow 0} \quad \lim_{x \to 3} \frac{\frac{1}{2}(1+x)^{-1/2}}{1} = \frac{1}{4}$

you try... $\lim_{x \to 0} \frac{3\sin x - 4x^2}{2\cos x - 2} \nearrow^{0}_{\searrow 0} = \lim_{x \to 0} \frac{3\cos x - 8x}{-2\sin x} \nearrow^{3}_{\searrow 0} = \frac{3}{0} = \text{dne}$

Try again $\lim_{x \to 0} \frac{\sqrt{1+x} - 1 - \frac{x}{2}}{x^2} \nearrow^{0}_{\searrow 0}$

$$\lim_{x \to 0} \frac{\frac{1}{2}(1+x)^{-1/2} - \frac{1}{2}}{2x} \nearrow^{0}_{\searrow 0} \quad \lim_{x \to 0} \frac{-\frac{1}{4}(1+x)^{-3/2}}{2} = -\frac{1}{8}$$

Ex: $\lim_{x \to 0^+} \frac{\sin x}{x^2} \nearrow^{0}_{\searrow 0} \quad \lim_{x \to 0^+} \frac{\cos x}{2x} \nearrow \frac{1}{\underset{+}{\text{small}}} = \infty$

Ex: $\lim_{h \to 0} \frac{\ln(e+h) - 1}{h} \nearrow^{0}_{\searrow 0} \quad \frac{\frac{1}{e+h}(1)}{1} = \frac{1}{e}$

or $f(x) = \ln x \qquad f'(e) = \frac{1}{e}$

$\lim \frac{f(a+h) - f(a)}{} = f'(a)$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = f'(a)$$