9.2 (Wednesday 2/5)

Tuesday, February 4, 2020 8:42 AM

## Section 9.2 – Series and Convergence

**What is a series?**

explicit rule

sum of $\rightarrow \sum_{k=1}^{\infty} a_k = a_1 + a_2 + a_3 + \dots + a_k + \dots$

what term # to start at

**Partial Sums –**

$S_1 = a_1$

$S_2 = a_1 + a_2$

$S_3 = a_1 + a_2 + a_3$

$S_7 = a_1 + a_2 + a_3 + \dots + a_7$

**Geometric Series**

$a$ = initial term $r$ = common ratio

$$a + a\cdot r + a\cdot r^2 + \dots + a\cdot r^{n-1} + \dots = \sum_{n=1}^{\infty} a\cdot r^{n-1}$$

**Think/Pair/Share**

Give an example of a geometric series that:

a) Converges $\sum_{n=1}^{\infty} \frac{1}{2^n}$ $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$

b) Diverges $\sum_{n=1}^{\infty} 2^n$

**Geometric Series Test**

A geometric series converges if $|r| < 1$

A geometric series diverges if $|r| \geq 1$

If it converges, then we can find the sum: $\boxed{S_\infty = \frac{a}{1-r}}$

Ex: Converge or diverge? If converges, then find $S_\infty$.

a) $\sum_{n=1}^{\infty} 2\left(\frac{3}{4}\right)^n$    $\frac{3}{4} < 1$ Geometric    $S_\infty = \frac{3/2}{1-3/4} = \frac{\frac{3}{2}}{1/4} = 6$

$a = 3/2$

$r = 3/4$

b) $\frac{\pi}{2} + \frac{\pi^2}{4} + \frac{\pi^3}{8} + \frac{\pi^4}{16} + \cdots$    $\frac{\pi}{2} \geq 1$    Diverges

$r = \frac{\pi}{2}$

Try: $\sum_{n=1}^{\infty} 3 \cdot \left(-\frac{1}{2}\right)^n$    $\left|-\frac{1}{2}\right| < 1$ converges

$a = -3/2$    $S_\infty = \frac{-3/2}{1+1/2} = -1$

$r = -1/2$

**Telescoping Series**

Ex: Find the sum. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$    $\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$

$A = 1$    $B = -1$

$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$

$= \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots$

$\lim_{n\to\infty} \left(1 - \frac{1}{n+1}\right) = 1 - 0 = 1$

**nth Term Test for Divergence** * always try this test first!!

If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges

Ex: $\sum_{n=1}^{\infty} \frac{2n+1}{3n-1}$ $\lim_{n\to\infty} \frac{2n+1}{3n-1} = \frac{2}{3} \neq 0$ Diverges by $n^{th}$ term test for divergence

$a_n$

EBM: $\frac{2n}{3n} = \frac{2}{3}$

Harmonic series

Ex: $\sum_{n=1}^{\infty} \frac{1}{n}$ $\lim_{n\to\infty} \frac{1}{n} = 0$ ∴ Test is inconclusive

Try: $\sum_{n=0}^{\infty} \frac{2n+1}{5n-6}$ $\lim_{n\to\infty} \frac{2n+1}{5n-6} = \frac{2}{5} \neq 0$

Diverges by $n^{th}$ term test for divergence

You try….. Find the sum of Type equation here. $\sum_{n=2}^{\infty} \frac{-2}{(n+1)(n+2)}$

$-2 = A(n+2) + B(n+1)$

$A = -2 \qquad B = 2$

$\sum_{n=2}^{\infty} \frac{-2}{n+1} + \frac{2}{n+2}$

$\lim_{n\to\infty} \left(\frac{-2}{3} + \frac{2}{n+2}\right) = -\frac{2}{3} + 0$