9.1 Day 2-3 Notes PERMUTATIONS

Permutation Counting Formula

Sometimes, we have more objects than we have "spots/positons" to fill. For instance, we may wish to consider how many ways 3 prize winners may be selected from a group of 11 entrants. In these instances, we are interested
in using *n* objects to fill *r* blanks, where n > r.

Permutations of n objects taken r at a time:

$$\frac{n!}{(n-r)!} = {}_nP_r$$

$$0 \le r \le n$$

Evaluate each of the following permutations.

1. Find the number of ways to arrange 5 objects chosen from a group of 8 objects.

$${}_8P_5 = \frac{8!}{(8-5)!} = \frac{8!}{3!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot \cancel{3 \cdot 2 \cdot 1}}{\cancel{3 \cdot 2 \cdot 1}}$$

$$= 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$$

$$= 6720$$

2. Sadly, only nine students entered Mr. V.'s annual Pi-Day Costume Contest. How many ways can he select three students to be the "Best Dressed", "First Runner-Up" and "Second Runner-Up"?

$${}_9P_3 = \frac{9!}{6!} = 9 \cdot 8 \cdot 7 = 504$$

3. A Precalculus classroom has 27 desks and 21 students. How many different seating charts are possible?

$${}_{27}P_{21} = \frac{27!}{6!} = 27 \cdot 26 \cdot 25 \cdot \ldots \cdot 7!$$

of letters

4. Using seven different Scrabble tiles, how many "words" or sequences can be made that use:

a. Three different letters?

$_7P_3 = \frac{7!}{4!} = 7 \cdot 6 \cdot 5$

b. Four different letters?

$_7P_4 = \frac{7!}{3!} = 7 \cdot 6 \cdot 5 \cdot 4$

c. Explain why the answer to part b. is four times the answer to part a.

4 more letters than can be used for the 4th letter in the "word"

5. Using seven different Scrabble tiles, how many "words" or sequences can be made that use:

a. Six different letters?

$_7P_6 = \frac{7!}{1!} = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$

$1! = 1$ AND $0! = 1$

b. Seven different letters?

$_7P_7 = \frac{7!}{0!} = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

c. Explain why the answer to parts a. and b. are the same.

There is only letter remaining for the 7th letter.

6. A filing system at a museum assigns each artifact a unique code consisting of two letters followed by three digits. How many codes are possible if neither letters nor digits may be repeated?

$\boxed{L_1 L_2 D_1 D_2 D_3}$

$_{26}P_2 \cdot {_{10}P_3}$

$26 \cdot 25 \cdot 10 \cdot 9 \cdot 8$

Bonus! How would your answer change if the letters and digits could appear in any order?

$_{26}P_2 \cdot {_{10}P_3} \cdot {_5P_5}$

$26 \cdot 25 \cdot 10 \cdot 9 \cdot 8 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

COMBINATIONS

Combination Counting Formula

In many applications we are only interested in the ways to *select* the *r* objects, regardless of the order in which we arrange them. These unordered selections are called **objects taken *r* at a time**.

no Repetition

Combinations of *n* objects taken *r* at a time:

$$\frac{n!}{(n-r)! \cdot r!} = {}_nC_r \qquad 0 \le r \le n$$

Evaluate each of the following combinations:

1. A student has 8 different colored pieces of paper and would like to select 3 of them for a class project. How many different color combinations are possible?

$$_8C_3 = \frac{8!}{5! \cdot 3!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1}$$

2. A standard deck of playing cards has 52 cards. How many 5-card poker hands can be dealt from the deck?

$$_{52}C_5 = \frac{52!}{47! \cdot 5!} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

3. For the annual MathRules party, Mr. V. is buying treats! At the store, he finds 7 varieties of soda and 10 varieties of snacks. How many combinations of 3 soda options and 4 snack options are possible?

$$_7C_3 \cdot {}_{10}C_4 = \frac{7!}{4! \cdot 3!} \cdot \frac{10!}{6! \cdot 4!}$$

4. Out of a group of 9 students, 4 of them are to serve as a committee to speak on behalf of the group.

no order or ranking

a. How many different committees may be formed?

$$_9C_4 = \frac{9!}{5! \cdot 4!}$$

b. After much discussion, it is determined that a committee of 5 representatives would be preferable. How many different 5 person committees may be formed from the group of 9 students?

$$_9C_5 = \frac{9!}{4! \cdot 5!}$$

a. How do your answers to parts *a* and *b* compare to one another? Explain why this makes sense:

Algebraically: Based on the formula for calculating combinations

$$\frac{9!}{5!\,4!} = \frac{9!}{4!\,5!}$$

Conceptually: Based on your understanding of combinations

Choosing 4 to participate

vs.

Choosing 4 to <u>NOT</u> participate

Challenge
You have 10 friends, but Pete and Sarah do not get along. In how many ways can you invite 6 of your friends to dinner so that Pete and Sarah are not both included?

PS 8

$_2C_1 \cdot {}_8C_5 + {}_2C_0 \cdot {}_8C_6$

or think this way
1 way to choose P
1 way to choose S

$1 \cdot {}_8C_5 + 1 \cdot {}_8C_5 + {}_8C_6$

Combination or Permutation??

1. A president, vice-president, and secretary are chosen from a 25-member garden club

   P

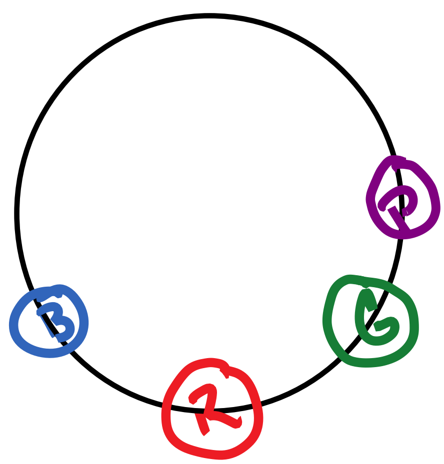
2. A cook chooses 5 potatoes from of a bag of 12 potatoes to make a potato salad

   C

3. A teacher makes a seating chart for 22 students in a classroom with 30 desks

   P

The figure below represents a bracelet with colored beads on it. How many ways can I arrange the beads?



Bracelet flat
not thinking or
mirrored image
$(n-1)!$
• circular table

mirrored image
$\frac{(n-1)!}{2}$

Would the number of arrangements change if there was a clasp on the bracelet?

$n!$