9.1A Notes — Basic Combinatorics

Multiplication Principle of Counting

If a task consists of a sequence of choices in which there are *p* selections for the first choice, *q* selections for the second choice, *r* selections for the third choice, and so on, then the total number of selections possible is determined by:

$p \cdot q \cdot r$

Math Selfies

You are at a math party and would like to take a bunch of trig selfies to post on Facebook. How many selfies will you need to take if you wish to have a selfie that includes you and each of: 4 friends, 3 backgrounds, and a trigonometric function?

$4 \cdot 3 \cdot 6$

$\boxed{72}$

Airport Codes

Since the 1930s, airports have been assigned three-letter codes which are prominently displayed on baggage tags attached at airport check-in desks. The standard naming convention is to use the first three letters of the city name in which the airport is located. (Of course, there are many cases in which this doesn't hold true. For instance, cities that have two airports, cities in different states that have the same name, all Canadian airport codes begin with "Y", and so on ☺) How many three-letter airport codes are possible?

$26 \cdot 26 \cdot 26$

$\boxed{17{,}576}$

Call Me, Maybe…

Initial use of three-digit area codes in the United States and Canada began in 1947 in large cities. Without any restrictions, how many three-digit area codes are possible?

$10 \cdot 10 \cdot 10$

$\boxed{1000}$

Challenge! At first, area codes were all in the form *NYX*, where *N* is any integer 2 through 9, *Y* is 0 or 1, and *X* is any integer 1 through 9 (if *Y* is 0) or any integer 2 through 9 (if *Y* is 1). (In other words, 312 was okay, but 311 was not.) With these restrictions, how many area codes were possible?

$8 \cdot 1 \cdot 9 \qquad 8 \cdot 1 \cdot 8$

$72 + 64 = \boxed{136}$

Why the restrictions? The restriction on *N* saves 0 for calling the operator, and 1 for signaling a long-distance call. The restriction on the second digit, limiting it to 0 or 1, was designed to help telephone equipment recognize the difference between a three-digit "area code" (with 0 or 1 as the second digit) and the three-digit "exchange" prefix (which had avoided 0 or 1 for the second digit, because of restrictions in existing switching equipment).

In the 1990s, some of the restrictions were lifted. Since that time, area codes allow for the first number to be any integer 2 through 9, and the second and third numbers to be any integer 0 through 9. How many area codes are now possible?

$8 \cdot 10 \cdot 10$

$\boxed{800}$

License and Registration, Please

Currently, non-personalized Illinois license plates come in two forms:

a. 7-digits such that the first and last digits may be any integer 1 through 9, and the middle five digits may be any integer 0 through 9. How many of these plates are possible?

$9 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 9$

$\boxed{8{,}100{,}000}$

b. A letter (A, J, X, B, C, D, F, T, Y, G, H, K, L, N, P, R, S, or V) followed by 6-digits such that each digit may be any integer 0 through 9. How many of these plates are possible?

$18 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

$\boxed{18{,}000{,}000}$

Factorial Notation

The factorial of non-negative integer is written as ___$n!$___ and represents the product of all positive integers less than or equal to n.

Examples: $3! = 3 \cdot 2 \cdot 1$

$\frac{5!}{3!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 20$

Permutation

The act of permuting objects refers to counting the number of ways that a set of n objects can be arranged in order. Each such ordering is called a permutation of the set.

Say cheese

How many ways can a group of 5 people line up for a photograph?

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$\boxed{120}$

How many ways can a group of 5 people (a dad and his 4 kids) line up for a photograph if the dad must be in the middle?

$4 \cdot 3 \cdot 1 \cdot 2 \cdot 1$

$\boxed{24}$

Words, Words, Words

How many 6-letter “words” can be formed from the letters in the word FRIDAY?

$6! = \boxed{720}$

Distinguishable Permutations

What happens to the number of unique orderings of a set of objects if some of the objects are identical?

reduces the # of combinations & permutations

How many “words” can be formed from the letters in each of the following:

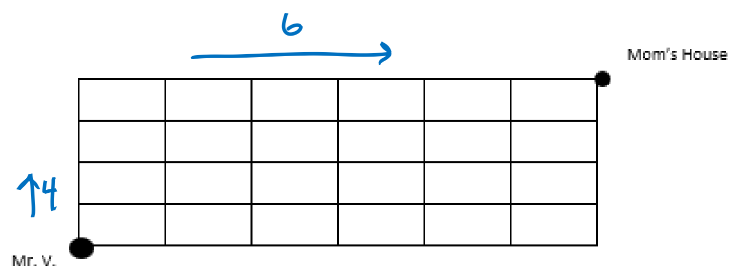
a. PIZZA b. BANANA c. SUCCESS d. Make up your own!

$\frac{5!}{2!} = \boxed{60}$ $\frac{6!}{3!\,2!} = \boxed{60}$ $\frac{7!}{2!\,3!} = \boxed{420}$

Happy Mother’s Day

Mr. V is travelling from home to visit his mother. If he travels only north and east, how many possible paths could he take?

$\frac{10!}{4!\,6!} = \boxed{210}$

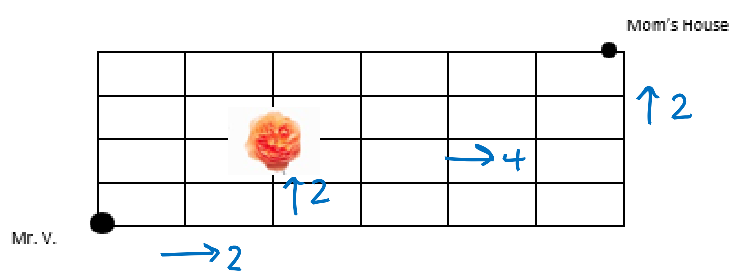


How many possible paths can Mr. V take if he must stop at the flower shop on the way?

TO! FROM

$\frac{4!}{2!\,2!} = 6$ $\frac{6!}{4!\,2!} = 15$

$6 \cdot 15 = \boxed{90}$



MORE PRACTICE

1. When getting dressed in the morning, Joe needed to choose a shirt and a pair of pants. He had a choice of a long-sleeve shirt, a short-sleeve shirt, or a dress shirt. He also had to choose between jeans, corduroy pants, khaki pants, and sweatpants. How many possible shirt-pant combinations did Joe have to choose from?

$3 \cdot 4$

$\boxed{12}$

2. Mr. Stuffed has 11 stuffed animals and likes to line up 3 of them on his nightstand to look at when he goes to bed. How many different ways can he do this?

order!

$11 \cdot 10 \cdot 9$

$\boxed{990}$

3. How many 5-digit numbers can be formed using the digits 2, 4, 6, 8, and 9 :

   a. Without repetition?

   $5! = \boxed{120}$

   b. With repetition?

   $5^5 = \boxed{3125}$

   c. Even numbers with repetition?
   4 evens

   $5^4 \cdot 4 = \boxed{2500}$

   d. Odd numbers with repetition?
   1 odd

   $5^4 \cdot 1 = \boxed{625}$

4. Four students hold a meeting in a room with nine chairs. How many different ways are possible for the students to be seated?

$9 \cdot 8 \cdot 7 \cdot 6 = \boxed{3024}$