Section 9.10/Review
pg 673: 5, 8, 30, 33, 39
AP Review: 2, 3

5. $f(x) = \frac{1}{x}$ $c = 1$

$f' = \frac{-1}{x^2}$ $f'' = \frac{2}{x^3}$ $f''' = \frac{-6}{x^4}$ $f^{(4)} = \frac{24}{x^5}$

$\frac{1}{x} \approx 1 - (x-1)' + \frac{2(x-1)^2}{2!} - \frac{6(x-1)^3}{3!} + \frac{24(x-1)^4}{4!} + \dots$

$\approx 1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4 + \dots$

$\sum_{n=0}^{\infty} (-1)^n (n-1)^n$

8. $f(x) = e^x$ $c = 1$

$f' = e^x$ $f'' = e^x$ $f''' = e^x$

$= e + e(x-1) + \frac{e(x-1)^2}{2!} + \frac{e(x-1)^3}{3!} + \dots$

$\sum_{n=0}^{\infty} \frac{e(x-1)^n}{n!}$

30. $f(x) = \ln(1+x^2)$

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n}}{n}$

$\ln x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \dots + \frac{(-1)^{n-1}(x-1)^n}{n} + \dots$

think about.....

$[(1+x^2) - 1] = x^2$

$\ln(1+x^2) = (x^2) - \frac{(x^2)^2}{2} + \frac{(x^2)^3}{3} - \frac{(x^2)^4}{4} + \dots +$

$= x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \dots + \frac{(-1)^{n+1}(x^{2n})}{n}$

33. $f(x) = \cos(4x)$

$\sum_{n=0}^{\infty} \frac{(-1)^n (4x)^{2n}}{(2n)!}$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \dots + \frac{(-1)^n x^{2n}}{(2n)!}$

$\cos(4x) = 1 - \frac{(4x)^2}{2!} + \frac{(4x)^4}{4!} - \frac{(4x)^6}{6!} + \frac{(4x)^8}{8!}$

39. $\cos^2 x$ Recall double angle formula!

$\cos 2x = \cos^2 x - \sin^2 x$
$\cos 2x = \cos^2 x - (1 - \cos^2 x)$
$\cos 2x = 2\cos^2 x - 1$

$1 + \cos 2x = 2\cos^2 x$
$\frac{1}{2}(1 + \cos(2x)) = \cos^2 x$

so... $\cos^2 x = \frac{1}{2}[1 + \cos(2x)]$

$= \frac{1}{2}\left[1 + \left[1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \dots \frac{(-1)^n (2x)^{2n}}{(2n)!}\right]\right]$

so $\frac{1}{2}\left[1 + \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n}}{(2n)!}\right]$

AP Review

#2

a. $\lim_{n \to \infty} \left| \frac{(-1)^n (x-1)^{n+1}}{n+1} \cdot \frac{n}{(-1)^{n-1}(x-1)^n} \right| = \lim_{n \to \infty} \left| \frac{(-1)(x-1)n}{n+1} \right| = |x-1|$

$-1 < x-1 < 1$
$0 < x < 2$

check endpts $x = 0$ $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(-1)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^{2n-1}}{n}$

$= \sum \frac{-1}{n}$ diverges

conv. abs $(0, 2)$

converges $(0, 2]$

conditionally @ $x = 2$

$x = 2$ $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(1)^n}{n}$ converges by alt. series converges $(0,2]$

conditionally @ $x=2$

$x=2$ $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(1)^n}{n}$ converges by alt. series

b. $f(x) = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \dots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(x-1)^n}{n}$

$f'(x) = g(x) = 1 - (x-1) + (x-1)^2 - (x-1)^3 = \sum_{n=0}^{\infty} (-1)^n (x-1)^n$

c. $\frac{1}{x}$

d. $h(x) = \underbrace{f(x^3+1)}_{\text{chain rule}}$ so $(x^3+1)-1 = x^3$

$h'(x) = f'(x^3+1) \cdot 3x^2$

plug in for x

use answer from part b & the fact $\underbrace{(x^3+1)}-1 = x^3$

then $h'(x) = 3x^2 \left[1 - x^3 + (x^3)^2 - (x^3)^3 + \dots = \sum_{n=1}^{\infty} (-1)^{n-1} (x^3)^n \right]$

$= 3 \left[x^2 - x^5 + x^8 - x^{11} + \dots = \sum_{n=0}^{\infty} (-1)^n x^2 \cdot x^{3n} \right]$

$= 3 \sum (-1)^n x^{3n+2}$

3.a $f(x) = \cos(x^2) = 1 + \frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \dots + \sum_{n=0}^{\infty} \frac{(-1)^n (x^2)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n (x^{4n})}{(2n)!}$

b. $\lim_{n\to\infty} \left| \frac{(-1)^{n+1} x^{4n+4}}{(2n+2)!} \cdot \frac{(2n)!}{(-1)^n x^{4n}} \right| = \lim_{n\to\infty} \left| \frac{(-1)\, x^4}{(2n+2)(2n+1)} \right| = 0 \qquad R = \infty$

c. $\cos(1) \approx 13/24$ Error $\leq \left| \frac{1(x^2)^6}{6!} \right| = \frac{1}{6!} = \frac{1}{720} \leq \frac{1}{500}$