

NO CALCULATOR! All by hand!

1. Find the area enclosed by $f(x) = x^2 + 2$ and $g(x) = 2x + 5$.

$$x^2 + 2 = 2x + 5$$

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

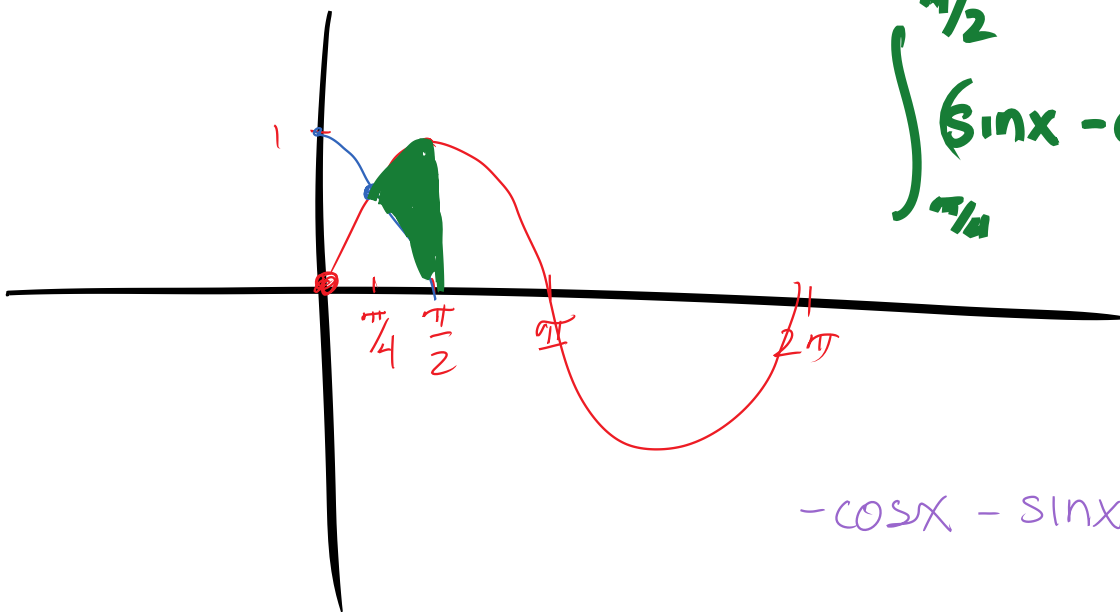
$$x = 3 \quad x = -1$$

$$\int_{-1}^3 [(2x+5) - (x^2+2)] dx$$

$$\int_{-1}^3 (2x+5-x^2-2) dx$$

$$\int_{-1}^3 (-x^2 + 2x + 3) dx$$

$$-\frac{x^3}{3} + x^2 + 3x \Big|_{-1}^3 = \boxed{10\frac{2}{3}}$$

2. Find the area between $y = \sin x$ and $y = \cos x$ over the interval $[\frac{\pi}{4}, \frac{\pi}{2}]$.

$$\int_{\pi/4}^{\pi/2} (\sin x - \cos x) dx$$

$$-\cos x - \sin x \Big|_{\pi/4}^{\pi/2}$$

$$[0 - (1)] - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)$$

$$-1 - (-\sqrt{2}) = \boxed{-1 + \sqrt{2}}$$

3. Find the area between $x = y^2 + 4y - 22$ and $x = 3y + 8$.

$$y^2 + 4y - 22 = 3y + 8$$

$$y^2 + y - 30 = 0$$

$$(y + 6)(y - 5) = 0$$

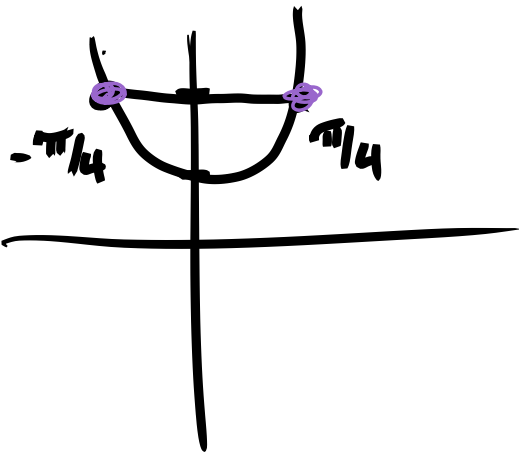
$$y = -6 \quad y = 5$$

$$\int_{-6}^5 [(3y + 8) - (y^2 + 4y - 22)] dy$$

$$\int_{-6}^5 (3y + 8 - y^2 - 4y + 22) dy$$

$$\int_{-6}^5 (-y^2 - y + 30) dy = -\frac{y^3}{3} - \frac{y^2}{2} + 30y \Big|_{-6}^5 = 221.8\bar{3}$$

4. Find the area of the region bounded by the line $y = 2$ and the graph of $y = \sec^2 x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.



$$2 \int_0^{\pi/4} (2 - \sec^2 x) dx$$

$$2 [2x - \tan x]_0^{\pi/4}$$

$$2 \left(\frac{\pi}{2} - 1 \right) \quad \boxed{\pi - 2}$$