AP Calculus AB Name________________

8.1 Day 2

Besides being able to use the integral to find distance traveled and displacement, it can also be used to calculate net change and total accumulation of quantities. Whenever you want to find the cumulative effect of a varying rate of change, just integrate it!

Alex loves fruit loops! Alex eats fruit loops from a bowl at a rate modeled by $E(t)$ fruit loops/minute and Alex's mom pours more fruits loops into the bowl at a rate modeled by $P(t)$ fruit loops/minute. Let time begin at $t = 0$ and $E(t) \geq 0$ *and* $P(t) \geq 0$.

a. Write the meaning of $E(2)$ and $P(4)$.

The rate at which Alex eats fruit loops at $t=2$ minutes.

The rate at which Alex's mom adds fruit loops to the bowl at $t=4$ minutes

b. Write the meaning of $\int_1^4 E(t)dt$.

The number of fruit loops eaten between 1 and 4 minutes.

c. Write the meaning of $\int_3^7 P(t)dt$.

The number of fruit loops poured into the bowl between 3 and 7 minutes.

d. Write an integral expression that gives the total number of fruit loops in Alex's bowl at 5 minutes.

Beginning at time t = 0, Alex eats 20 fruit loops/minute and Alex's mom pours fruit loops into the bowl at a rate of 50 fruit loops/minute. If Alex's bowl has 15 fruit loops at time t = 0, how many fruit loops are in Alex's bowl after 8 minutes?

$$\int_0^8 20\, dt = 160$$

\# of fruit loops = $15 + 400 - 160 = 255$

$$\int_0^8 50\, dt = 400$$

From 1970 to 1980, the rate of the potato consumption in Ireland was $C(t) = 2.2 + 1.1^t$ millions of bushels per year, with t being years since the beginning of 1970.

a) How many bushels were consumed from the beginning of 1972 to the end of 1973?

$\int_2^4 C(t)\,dt \approx 7.066$ millions of bushels

b) How many bushels were consumed from the beginning of 1970 to the end of 1975?

$\int_0^6 C(t)\,dt \approx 21.295$ millions of bushels

c) What is the average rate of potato consumption from the beginning of 1970 to the end of 1975?

$\frac{1}{6}\int_0^6 C(t)\,dt = 3.549$ millions of bushels / year

A pump connected to a generator operates at a varying rate, depending on how much power is being drawn from the generator to operate other machinery. The rate (gallons per minute) at which the pump operates (denoted by r(t)) is recorded at 5 minute intervals for one hour as shown in the table below.

Time (min.)	**r(t) (gal/min)**
0	58
5	60
10	65
15	64
20	58
25	57
30	55
35	55
40	59
45	60
50	60
55	63
60	63

a. Approximate the number of gallons pumped over the given hour.

$5(58+60+65+64+58+57+55+55+59+60+60+63)$

3570 gallons

b. State the meaning of $\frac{1}{60}\int_0^{60} r(t)dt$

the average rate at which the pump operates over the 60 min time period.

Hawaii has some of the most beautiful beaches in the world! On a particular beach, it is estimated that there are 4,520 cubic feet of sand in the year 2000. For the next 10 years, erosion removes sand at a rate of $R(t)$ cubic feet per year, while the ocean washes sand onto the beach at a rate of $W(t)$ cubic feet per year.

Let $R(t) = 100t - 5$ and $W(t) = 300t(t-4)$. **Without a calculator**,

a. write and calculate an integral to find the number of cubic feet of sand that is removed from the beach after 10 years.

$$\int_0^{10} R(t)\,dt = \int_0^{10} 100t - 5\,dt = \frac{100t^2}{2} - 5t\Big|_0^{10} = 50t^2 - 5t\Big|_0^{10}$$

$$= 4{,}950 \text{ ft}^3$$

b. write and calculate an integral to find the number of cubic feet of sand that washes onto the beach after 10 years.

$$\int_0^{10} W(t)\,dt = \int_0^{10} 300t(t-4)\,dt = \int_0^{10} 300t^2 - 1200t\,dt$$

$$= \frac{300t^3}{3} - \frac{1200t^2}{2}\Big|_0^{10}$$

$$= 100t^3 - 600t^2 \qquad = 40{,}000 \text{ ft}^3$$

c. write and calculate an expression containing at least one integral to find the number of cubic feet of sand on the beach after 10 years.

$$4{,}520 + \int_0^{10} W(t)\,dt - \int_0^{10} R(t)\,dt = 39{,}570 \text{ ft}^3$$

A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S, where $S(t)$ is measured in millimeters and t is measured in days for $0 \le t \le 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2\sin(0.03t) + 1.5$.

(a) According to the model, what is the height of the water in the can at the end of the 60-day period?

$$\int_0^{60} S'(t)\,dt \approx 171.813 \text{ mm}$$

At a certain height, a tree trunk has a circular cross section. The radius $R(t)$ of that cross section grows at a rate modeled by the function

$$\frac{dR}{dt} = \frac{1}{16}\left(3 + \sin\left(t^2\right)\right) \text{ centimeters per year}$$

for $0 \le t \le 3$, where time t is measured in years. At time $t = 0$, the radius is 6 centimeters. The area of the cross section at time t is denoted by $A(t)$.

(a) Write an expression, involving an integral, for the radius $R(t)$ for $0 \le t \le 3$. Use your expression to find $R(3)$.

$$\int_0^3 R'(t)\,dt = R(3) - R(0) \qquad 6 + \int_0^3 R'(t)\,dt = R(3)$$

$$\boxed{R(3) = 6.611 \text{ cm}}$$

(b) Find the average rate of growth over the three year period of time.

$$\frac{1}{3}\int_0^3 R'(t)\,dt \approx .204 \text{ cm/year}$$