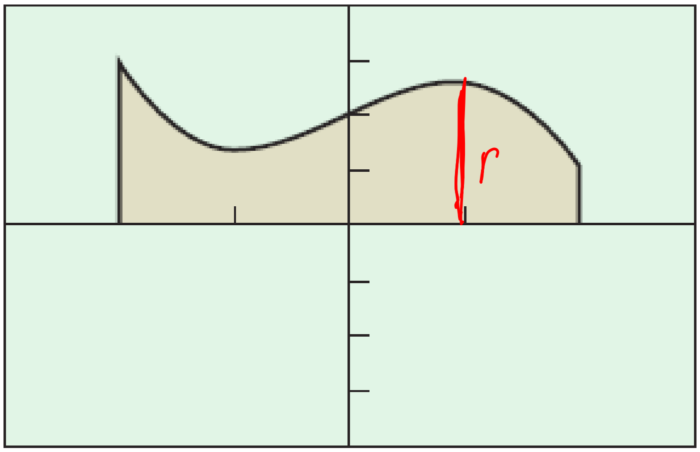
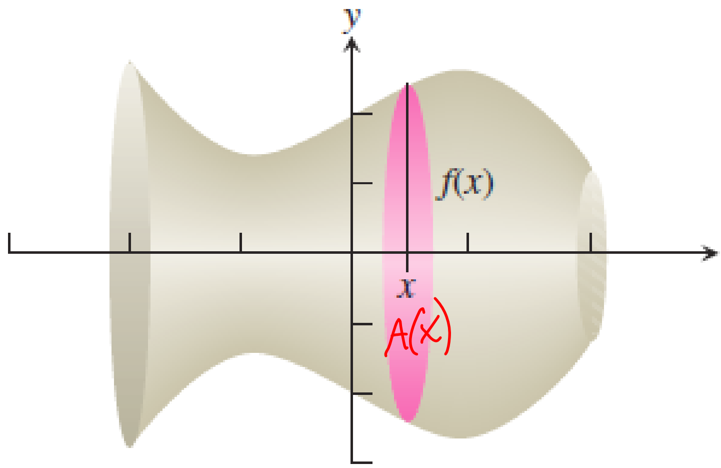
EXAMPLE 2 A Solid of Revolution

The region between the graph of $f(x) = 2 + x\cos x$ and the x-axis over the interval $[-2, 2]$ is revolved about the x-axis to generate a solid. Find the volume of the solid.





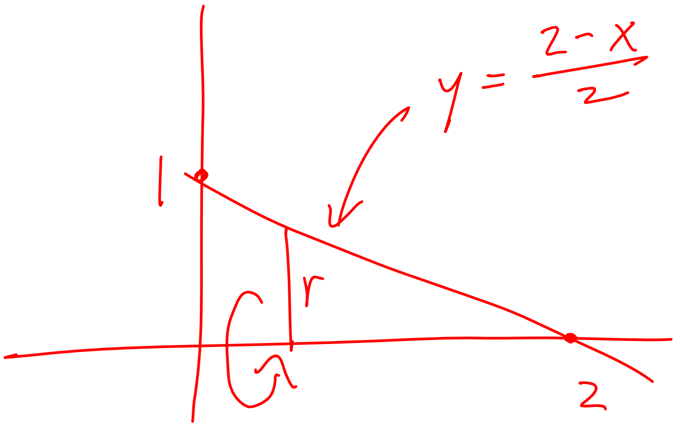
$r = f(x) = 2 + x\cos x$

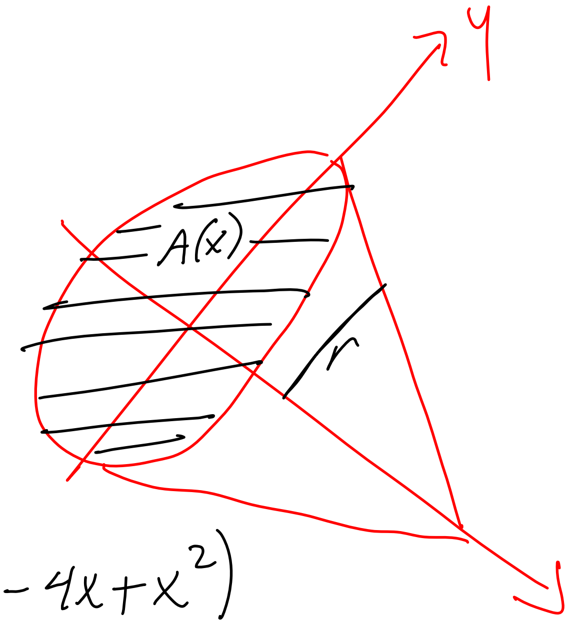
$A(x) = \pi r^2 = \pi(2 + x\cos x)^2$

$V = \pi \int_{-2}^{2} (2 + x\cos x)^2 \, dx \approx 52.43$ using fnInt

Draw both 2 dimensional and 3 dimensional pictures and find the volume of the solid generated by revolving the region bounded by $x+2y=2,\ x=0,\ y=0$ about the given axis.

a) Rotated about the x-axis



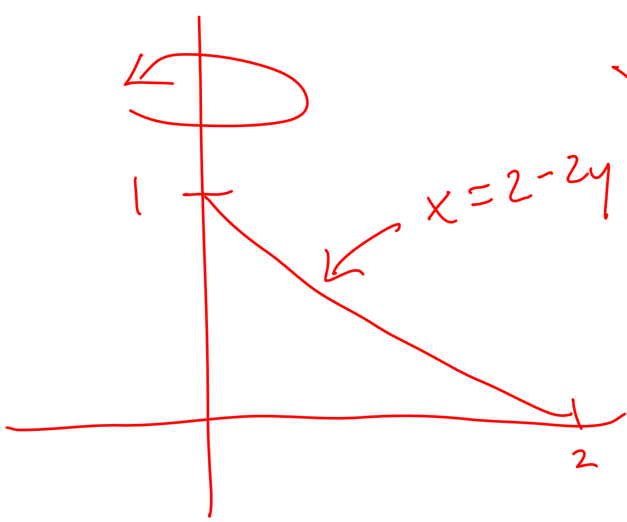


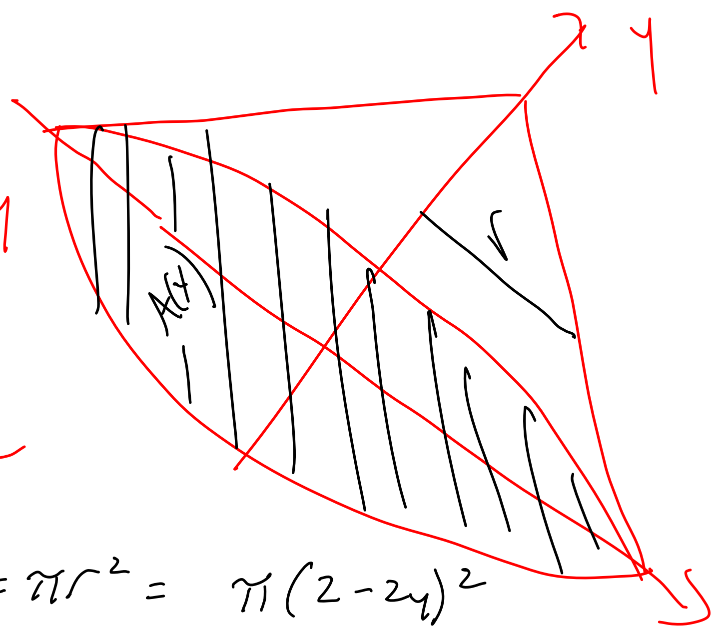
$$A(x) = \pi\left(\frac{2-x}{2}\right)^2 = \frac{\pi}{4}(4-4x+x^2)$$

$$V = \frac{\pi}{4}\int_0^2 (4-4x+x^2)\,dx = \frac{\pi}{4}\left[4x - 4\cdot\tfrac{1}{2}x^2 + \tfrac{1}{3}x^3\right]_0^2$$

$$= \frac{\pi}{4}\left[8 - 8 + \tfrac{1}{3}\cdot 8\right] = \frac{8\pi}{12} = \frac{2\pi}{3}$$

b) Rotated about the y-axis





$$r = 2-2y \qquad A(y) = \pi r^2 = \pi(2-2y)^2$$

$$= \pi(4-8y+4y^2)$$

$$V = \pi\int_0^1 (4-8y+4y^2)\,dy = \pi\left[4y - 8\cdot\tfrac{1}{2}y^2 + 4\cdot\tfrac{1}{3}y^3\right]_0^1 = \frac{4\pi}{3}$$