AP Calculus AB
8.3 Day 2 Worksheet

Name___________________________

Use the given information to find the volume of the solid.

1. The solid is created with cross-sections in the area created between $y = \sqrt{x}$ *and* $y = x$.

a. The cross sections are semi-circles which are perpendicular to the x-axis and have diameters that are in the xy plane.

$d = \sqrt{x} - x$

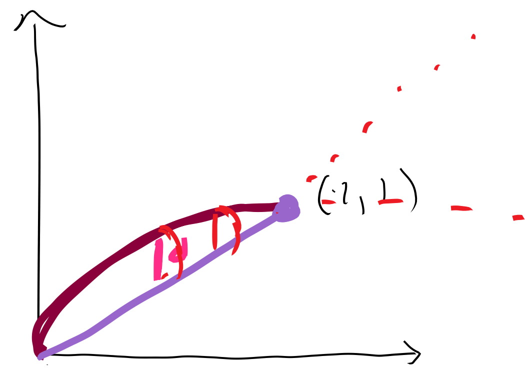
$A_{cs} = \frac{1}{2}\pi r^2$

$r = \frac{1}{2}(\sqrt{x} - x)$

$= \frac{1}{2}\pi\left(\frac{1}{2}(\sqrt{x} - x)\right)^2$

$A_{cs} = \frac{\pi}{8}(\sqrt{x} - x)^2$

$$\text{Volume} = \frac{\pi}{8}\int_{x=0}^{x=1} (\sqrt{x} - x)^2\,dx \approx \boxed{0.013}$$

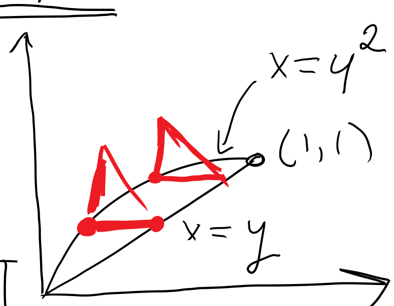


b. The cross sections are isosceles right triangles with bases perpendicular to the y-axis.

$A_{cs} = \frac{1}{2}b^2$

$b = y - y^2$

$$\text{Volume} = \int_{y=0}^{y=1} \frac{1}{2}(y - y^2)^2\,dy = \frac{1}{2}\int_0^1 (y - y^2)^2\,dy \approx \boxed{0.017}$$



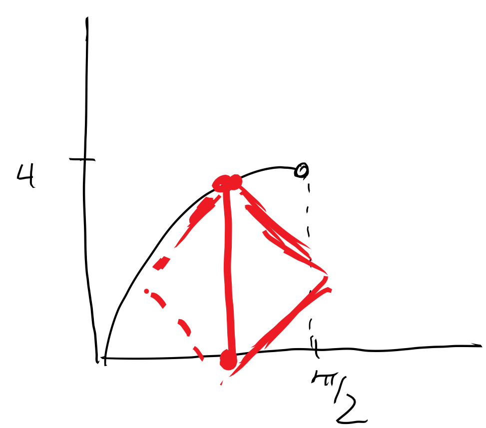
2. The solid is created with cross-sections in the area created between $y = 4\,sinx, x = \frac{\pi}{2}$, *and the* $x - axis$. The cross sections are squares with the diagonals perpendicular to the x-axis.

$d = 4\sin x$

$A_{cs} = \frac{d^2}{2}$

$= \frac{(4\sin x)^2}{2}$

$$\int_0^{\pi/2} 8\sin^2 x\,dx \approx \boxed{6.283}$$



3. The solid is created with cross-sections in the area between $x = y^4, x = 0, and$ y=16. The cross-sections are circular disks with diameters that are perpendicular to the y-axis.

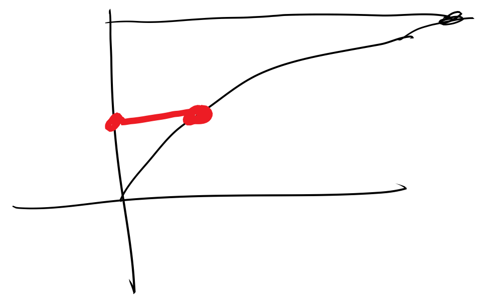
$d = y^4$

$r = \frac{1}{2} y^4$

$A = \pi r^2$

$A = \pi \left(\frac{1}{2} y^4\right)^2$

$A = \frac{\pi}{4} y^8$



$$\frac{\pi}{4}\int_0^{16} y^8\,dy = 5{,}996{,}905{,}646$$

Review for the 8.2 Quiz

Find the area between the curves and lines $y = -2x + 3$ *and* $x = y^2$ (without calculator and completely simplify)

$y - 3 = -2x$

$x = \frac{y-3}{-2}$

$\frac{y-3}{-2} = y^2$

$y - 3 = -2y^2$

$2y^2 + y - 3 = 0$

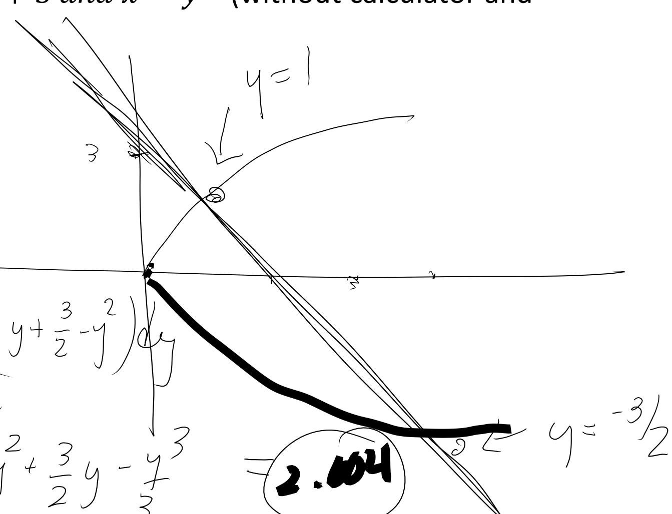
$(2y+3)(y-1) = 0$

$y = -\frac{3}{2} \quad y = 1$

$$\int_{-3/2}^{1} \left(\frac{y-3}{-2} - y^2\right) dy$$

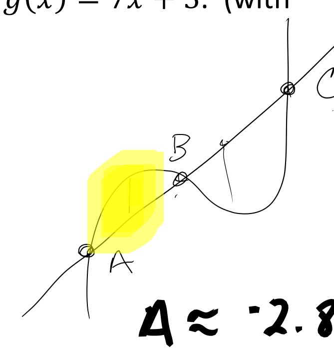
$$\int_{-3/2}^{1} \left(-\frac{1}{2}y + \frac{3}{2} - y^2\right) dy$$

$$-\frac{1}{4}y^2 + \frac{3}{2}y - \frac{y^3}{3} = 2.604$$



Find the area between the curves and lines $f(x) = x^3 - 2x + 1$ *and* $g(x) = 7x + 3$. (with calculator)

$$\int_A^B [f(x) - g(x)]\,dx + \int_B^C [g(x) - f(x)]\,dx$$



$A \approx -2.882$

$B \approx -0.223$

$C \approx 3.105$