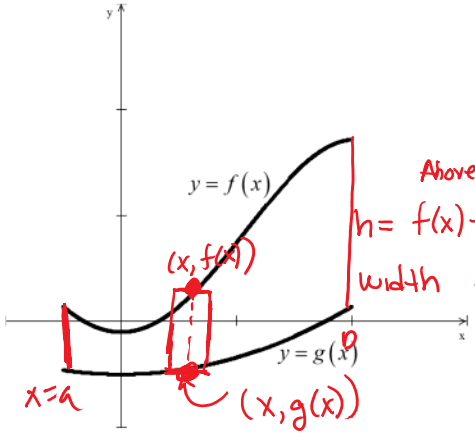
8.2 Day 1 (Wednesday 1/8)

Tuesday, January 7, 2020 10:41 AM

AP Calculus BC
8.2 Day 1
Areas in the Plane

Name_____

Area Between Curves:



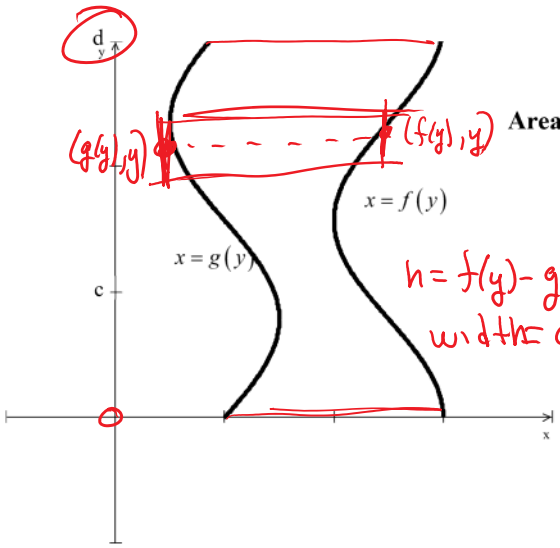
$$\text{Area} = \int_{x=a}^{x=b} (f(x)-g(x))\,dx$$

Above - Below

$h = f(x)-g(x)$

width $\Delta x = dx$

Area of one rectangle $(f(x)-g(x)) \cdot dx$



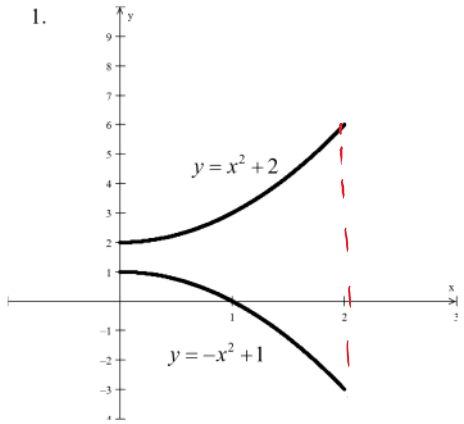
$$\text{Area} = \int_{y=c}^{y=d} (f(y)-g(y))\,dy$$

$h = f(y)-g(y)$

width $= dy$

Find the area between the curves analytically.

1.

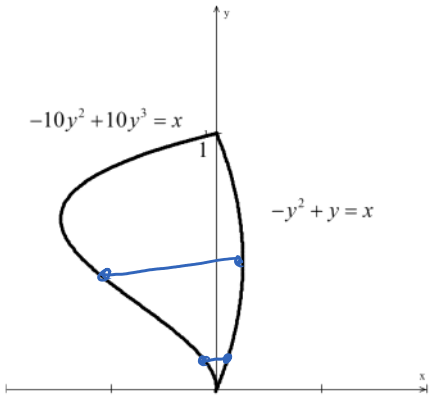


$$\text{Area} = \int_0^2 \left[(x^2+2) - (-x^2+1)\right] dx$$

$$= \int_0^2 (2x^2+1)\, dx$$

$$= \frac{2}{3}x^3 + x \Big|_0^2 = \frac{16}{3} + 2 - 0 = 22/3$$

2.



$$A = \int_0^1 \left[(-y^2+y) - (-10y^2+10y^3)\right] dy$$

$$= \int_0^1 (-10y^3 + 9y^2 + y)\, dy$$

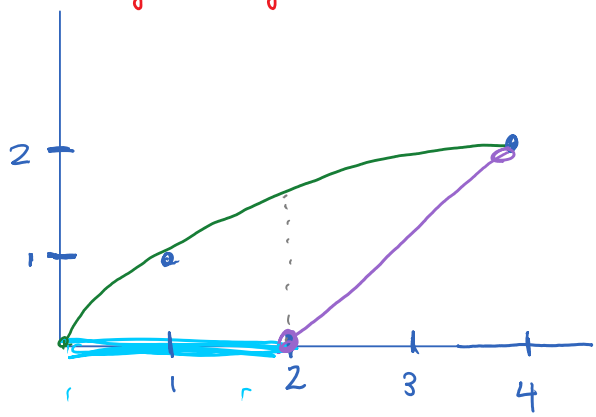
$$-\frac{10}{4}y^4 + 3y^3 + \frac{y^2}{2} \Big|_0^1 = -\frac{10}{4} + 3 + \frac{1}{2} = 1$$

Examples without given graphs:

Find the area of the region enclosed by lines and curves.

a. $y = \sqrt{x}$ $y = x - 2$ $y = 0$

$x = y^2$ $x = y + 2$



w/ respect to x

$$\int_0^2 \sqrt{x}\,dx + \int_2^4 (\sqrt{x} - (x-2))\,dx$$

w/ respect to y

$$\int_0^2 (y + 2 - y^2)\,dy$$

b. $x = y(2 - y)$ $x = -6y$

$y(2-y) = -6y$

$2y - y^2 = -6y$

$y^2 - 8y = 0$

$y = 0$ $y = 8$

check

$y = 1$

$x = 1(2-1) = 2$ $x = -6(1) = -6$

$$\int_0^8 (y(2-y) - (-6y))\,dy$$