AP Calculus AB Name______________________

8.1 Review (Day 3)

t	0	2	5	6
$v(t)$	-7	-2	5	11

1. The table above describes the velocity of a particle at various times.
   a. Using LRAM with three unequal subintervals,
      i. approximate the displacement of the particle on [0,6].

      $2 \cdot -7 + 3 \cdot -2 + 1 \cdot 5 = -15$

      ii. approximate the distance travelled of the particle on [0,6].

      $2 \cdot 7 + 3 \cdot 2 + 1 \cdot 5 = 25$

      iii. approximate the average velocity on [0,6].

      $\frac{-15}{6}$

      iv. approximate the average speed on [0,6].

      $\frac{25}{6}$

   b. Estimate the average acceleration on [0,6].

   Average acceleration $= \frac{11 - -7}{6-0} = 3$

   c. Give an estimate for $a(5.5)$.

   $a(5.5) = v'(5.5) \approx \frac{11-5}{6-5} = 6$

2.

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	−10	−8	−4	7

The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.

(a) Estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{20}^{40} v(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t)\,dt$.

(a) $a(36) = v'(36) \approx \frac{v(40) - v(32)}{40 - 32} = \frac{11}{8}$ meters/sec^2

(b) $\int_{20}^{40} v(t)\,dt$ is the particle's change in position in meters from time $t = 20$ seconds to time $t = 40$ seconds.

$$\int_{20}^{40} v(t)\,dt \approx \frac{v(20)+v(25)}{2} \cdot 5 + \frac{v(25)+v(32)}{2} \cdot 7 + \frac{v(32)+v(40)}{2} \cdot 8$$

$= -75$ meters

A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.

(b) Find the average rate at which water is draining from the tank between time $t = 0$ and time $t = 8$ hours.

(c) Find $r'(3)$. Using correct units, explain the meaning of that value in the context of this problem.

(d) Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 9000 liters.

(a) $\lim\limits_{t \to 5^-} r(t) = \lim\limits_{t \to 5^-} \left(\dfrac{600t}{t+3} \right) = 375 = r(5)$

$\lim\limits_{t \to 5^+} r(t) = \lim\limits_{t \to 5^+} \left(1000e^{-0.2t} \right) = 367.879$

Because the left-hand and right-hand limits are not equal, r is not continuous at $t = 5$.

(b) $\dfrac{1}{8}\displaystyle\int_0^8 r(t)\,dt = \dfrac{1}{8}\left(\int_0^5 \dfrac{600t}{t+3}\,dt + \int_5^8 1000e^{-0.2t}\,dt \right)$

$= 258.052$ or 258.053

(c) $r'(3) = 50$

The rate at which water is draining out of the tank at time $t = 3$ hours is increasing at 50 liters/hour^2.

(d) $12{,}000 - \displaystyle\int_0^A r(t)\,dt = 9000$

Amy loves fruit! She has 17 pieces of fruit on her counter. Amy buys fruit at a rate of $B(t)$ fruit per week, but she consumes fruit at a rate of $C(t)$ fruit per week. Let time begin at week 0.

a. What is the meaning of $B(3)$ and $C(4)$?

B(3) is the rate at which Amy buys fruit the 3rd week.

C(4) is the rate at which Amy consumes fruit the 4th week.

b. What is the meaning of $\int_1^3 B(t)dt$?

The number of fruit Amy has bought between weeks 1 and 3.

c. What is the meaning of $\int_3^6 C(t)dt$?

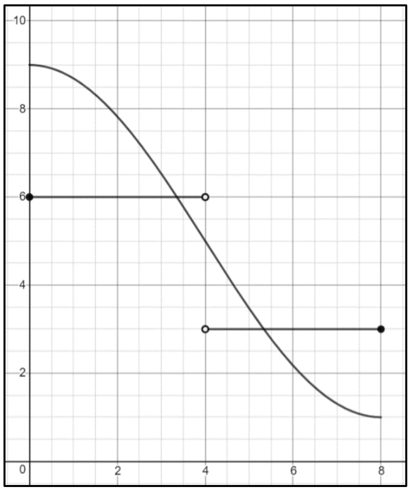
The number of fruit Amy has eaten between weeks 3 and 6.

d. What is the meaning of $17 + \int_0^4 [B(t) - C(t)]dt$?

The number of fruit on the counter after 4 weeks.

Challenging Problem!

The amount of chocolate milk in a storage container, in thousands of gallons, is modeled by a continuous function on the time interval $0 \le t \le 8$, were t is measured in hours. In this model:



- The rate at which chocolate milk flows into the storage container is $f(t) = 4\cos\left(\frac{\pi}{8}t\right) + 5$ thousands of gallons per hour for $0 \le t \le 8$.
- The rate at which chocolate milk flows out of the storage container is $g(t) = \begin{cases} 6, & 0 \le t < 4 \\ 3, & 4 < t \le 8 \end{cases}$ thousands of gallons per hour.
- The graphs of f and g intersect at t = 3.357 and t = 5.333 hours.
- At time t = 0, there are 4,000 gallons of chocolate milk in the storage container.

a. Calculate the average rate of change of f on [0, 8] and describe its meaning.

$f(0) = 9$

$f(8) = 1$

$\frac{9-1}{(0-8)} = -1$

The average RoC is −1 thousands of gallons per hr². The rate at which chocolate milk flows in decreases by 1000 gallons per hr².

b. On $0 \le t \le 8$, find the time interval(s) when the amount of chocolate milk in the storage container is decreasing. Justify your answer.

$A(t) = 4 + \int_0^t f(x)\,dx - \int_0^t g(x)\,dx$

$A'(t) = f(t) - g(t) = 0$

+ − + −

0 3.357 4 5.333 8 $A'(t)$

$A(t)$ decreases on $[3.357, 4) \cup [5.333, 8]$ because $A'(t) = f(t) - g(t) < 0$ on $(3.357, 4) \cup (5.333, 8)$

c. On $0 \le t \le 8$, at what time t is the amount of chocolate milk in the storage container the greatest? The least? To the nearest thousands of gallons, compute the amount of chocolate milk in the storage container at those times. Justify your answer.

t	$A(t)$
0	4
4	10.1859
8	8
3.357	10.5059
5.333	11.4879

$A(4) = 4 + \int_0^4 f(t)\,dt - \int_0^4 6\,dt$

$A(8) = 4 + \int_0^8 f(t)\,dt - \int_0^4 6\,dt - \int_4^8 3\,dt$

$A(3.357) = 4 + \int_0^{3.357} f(t)\,dt - \int_0^{3.357} 6\,dt$

$A(5.333) = 4 + \int_0^{5.333} f(t)\,dt - \int_0^4 6\,dt - \int_4^{5.333} 3\,dt$

The least amount is 4000 gal at $t=0$ because this is a left endpt w/ $A'(t) = f(t) - g(t) > 0$ on $(0, 3.357)$.

The greatest amount is 11.4879 thousands of gallons at $t = 5.333$ because $A'(t) = f(t) - g(t)$ changes from + to −.

d. Write, but do not solve, and equation whose solution finds all times t when the storage container holds exactly 3,200 gallons of chocolate milk.

$$4 + \int_0^t f(x)\,dx - \int_0^t g(x)\,dx = 3.2$$