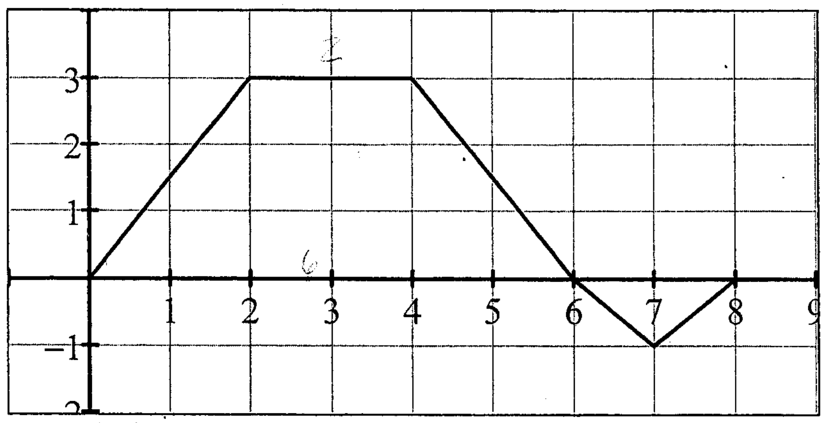
Some 7.1 and 7.2 AP Problems

1997 MC#8,9



v(t)

A bug begins to crawl up a vertical wire at time $t=0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

At what value of t does the bug ~~change direction~~?

A) 2 B) 4 C) 6 D) 7 E) 8

What is the total distance ~~the bug~~ traveled from $t=0$ to $t=8$?

A) 14 B) 13 C) 11 D) 8 E) 6

$\frac{1}{2}(3)(8) = 12 + 1$

1997 MC#16

The area of the region enclosed by the graph of $y = x^2 + 1$ and the line $y = 5$ is

A) $\frac{14}{3}$ B) $\frac{16}{3}$ C) $\frac{28}{3}$ D) $\frac{32}{3}$ E) 8π

$\int_0^2 (5-x^2-1)\,dx$

$2\int_0^2 (4-x^2)\,dx = 2\left(-\frac{x^3}{3} + 4x\right)\Big|_0^2$

1997 MC#83

What is the area of the region in the first quadrant enclosed by the graphs of $y = \cos x$, $y = x$, and the y-axis?

A) 0.127 B) 0.385 C) 0.400 D) 0.600 E) 0.947

1997 MC #87

At time $t \ge 0$, the acceleration of a particle moving on the x-axis is $a(t) = t + \sin t$. At $t = 0$, the velocity of the particle is -2. For what value of t will the velocity of the particle be zero?

A) 1.02 B) 1.48 C) 1.85 D) 2.81 E) 3.14

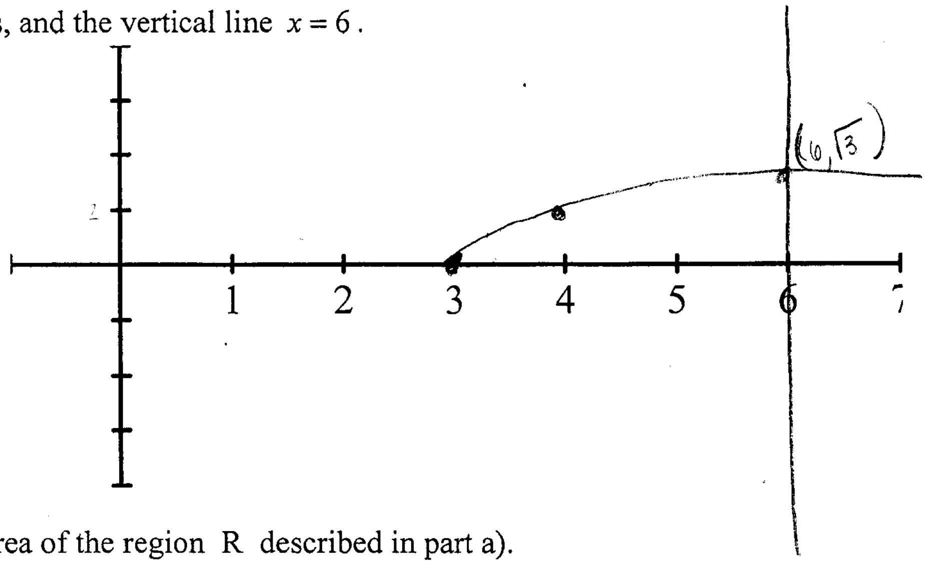
$v(t) = \frac{t^2}{2} - \cos t - 1$

must find

997 FR#3

et f be the function given by $f(x)=\sqrt{x-3}$.

a) On the axes provided below, sketch the graph of f and shade the region R enclosed by the graph of f, the x-axis, and the vertical line $x=6$.



b) Find the area of the region R described in part a).

$$\int_3^6 (x-3)^{1/2}\,dx = \frac{2}{3}(x-3)^{3/2}\Big|_3^6$$

$$\frac{2}{3}(3)^{3/2}$$

$$\frac{2}{3}\sqrt{27} = \frac{2}{3} \quad = \frac{6}{3}\sqrt{3} = 2\sqrt{3}$$

c) Rather than using the line $x=6$ as in part a), consider the line $x=w$, where w can be any number greater than 3. Let $A(w)$ be the area of the region enclosed by the graph of f, the x-axis, and the vertical line $x=w$. Write an integral expression for $A(w)$.

$$\int_3^w (x-3)^{1/2}\,dx$$

d) Let $A(w)$ be described in part c). Find the rate of change of A with respect to w when $w=6$.

$$\frac{d}{dx}\int_3^w (x-3)^{1/2}\,dx \quad (w-3)^{1/2}$$

$$(6-3)^{1/2} = (3)^{1/2}$$

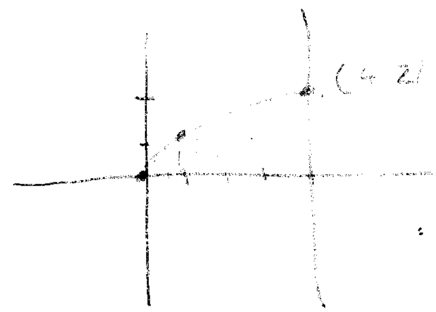
998 FR#1a,b

Let R be the region bounded by the x-axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.

a) Find the area of region R.

$$\int_0^4 (\sqrt{x})\,dx$$

$$= \frac{2}{3}x^{3/2}\Big|_0^4 = 16/3$$



b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.

$$\int_0^h \sqrt{x}\,dx = \int_h^4 \sqrt{x}\,dx \qquad \frac{2}{3}x^{3/2}\Big|_0^h = \frac{2}{3}x^{3/2}\Big|_h^4$$

$$\frac{2}{3}h^{3/2} = \frac{2}{3}4^{3/2} - \frac{2}{3}h^{3/2}$$

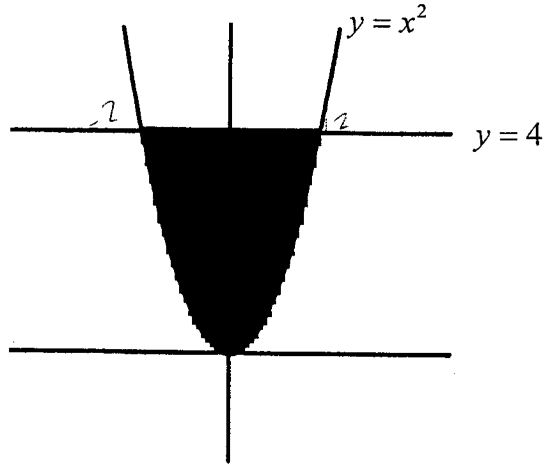
$$\frac{2}{3}h^{3/2} = \frac{16}{3} - \frac{2}{3}h^{3/2}$$

$$\frac{4}{3}h^{3/2} = 16/3 \qquad h^{3/2} = 4$$

$$h = 4^{2/3}$$

$$\boxed{h = \sqrt[3]{16}}$$

999 FR #2a



The shaded region, R, is bounded by the graph of $y = x^2$ and the line $y = 4$, as shown in the figure above.

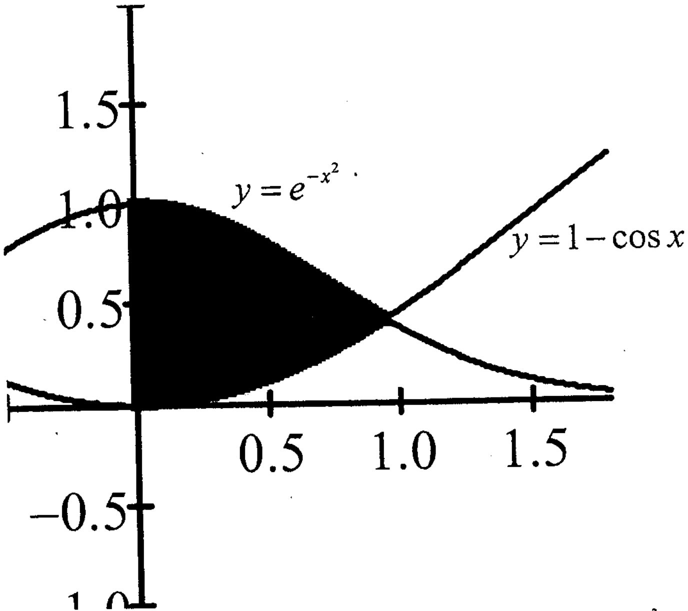
a) Find the area of R

$$2\int_0^2 (4 - x^2)\,dx = 2\,4x - \frac{x^3}{3}\Big|_0^2 = 8 - \frac{8}{3}$$

$$= 2\left(\frac{24}{3} - \frac{8}{3}\right) = 2\left(\frac{16}{3}\right)$$

$$= 32/3$$

2000 FR #1a



Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

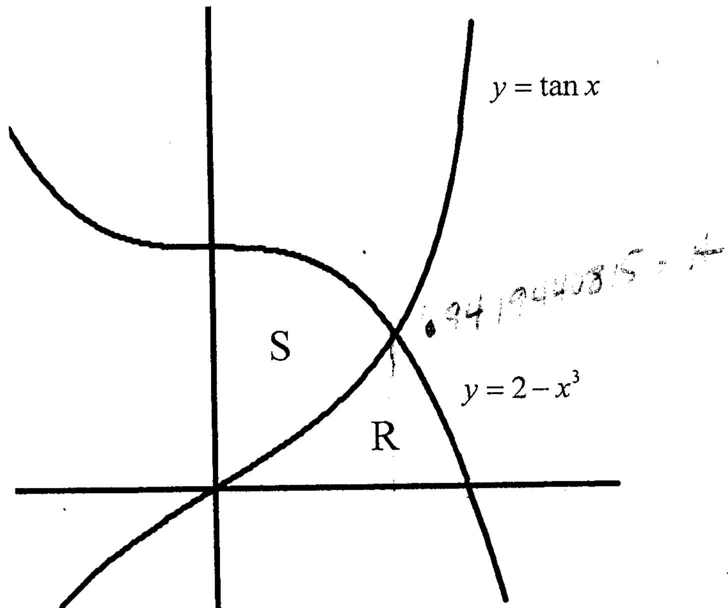
a) Find the area of the region R.

$e^{-x^2} = 1 - \cos x$ at $x = .941944 = A$

$$\int_0^A \left(e^{-x^2} - (1 - \cos x)\right) dx$$

$$\boxed{\approx 0.59}$$

.001 FR#1a,b



Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.

a) Find the area of R.

$x = \tan^{-1} y$? $\int_0^A \tan x\,dx + \int_A^{\sqrt[3]{2}} (2 - x^3)\,dx = .729$

b) Find the area of S.

$\int_0^A (2 - x^3 - \tan x)\,dx = 1.160$

2002 FR#1a

Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.

$\int_{1/2}^{1} (e^x - \ln x)\,dx = 1.222$