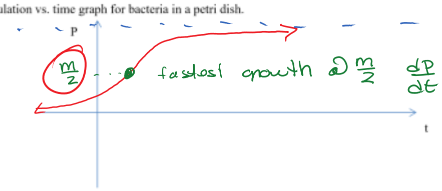
7.5 Day 2 (Monday 12/9)

Sunday, December 8, 2019 8:09 AM

Section 7.5B Notes
Logistic Growth Modeling

Sketch a population vs. time graph for bacteria in a petri dish.

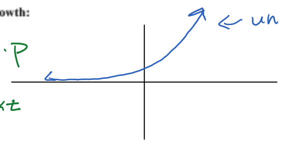


$\frac{dP}{dt}$ is the greatest

m ⇒ max carrying cap. (Limit to growth)

Difference between exponential and logistic growth differential equations:

Exp. Growth:



← unlimited growth

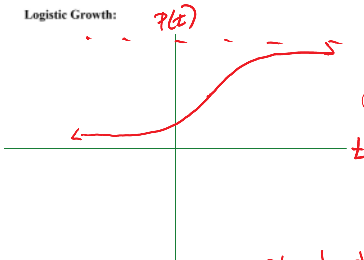
$\frac{dP}{dt} = k \cdot P$

$P = P_0 e^{kt}$

$\frac{dP}{dt} \to \infty$

$P \to \infty$

Logistic Growth:



$\frac{dP}{dt} = kP(m-P)$

$t \to \infty \quad P \to m$

$\frac{dP}{dt} \to 0$

standard form for a Logistic

$P = \frac{m}{1+Ce^{-kmt}}$

$m = 4000$

$km = .001(4000) = 4$

$P = \frac{4000}{1+Ce^{-4t}}$

$200 = \frac{4000}{1+C}$

$C = 19$

$P = \frac{4000}{1+19e^{-4t}}$

Ex: A population of groundhogs is modeled by $\frac{dP}{dt} = .001P(4000-P)$ with an initial population of 200 ground hogs. Find the logistic function that models the population in terms of time. (i.e. solve the differential equation!)

(k, m − P)

$\int \frac{1}{P(4000-P)}\,dP = \int 0.001\,dt$

$\frac{1}{P(4000-P)} = \frac{A}{P} + \frac{B}{4000-P}$

$1 = A(4000-P) + BP$

when $P=0$: $1 = 4000A$, $A = \frac{1}{4000}$

when $P=4000$: $1 = 4000B$, $B = \frac{1}{4000}$

$\frac{1}{4000}\int\left[\frac{1}{P} + \frac{1}{4000-P}\right]dP = .001t + C$

$\frac{1}{4000}\left[\ln|P| + \ln|4000-P|\cdot -1\right] = .001t + C$

$\frac{-1}{4000}\left(-\ln|P| + \ln|4000-P|\right) = .001t + C$

$\frac{-1}{4000}\ln\frac{4000-P}{P} = .001t + C$

$\ln\left(\frac{4000-P}{P}\right) = -4t + C$

$\frac{4000-P}{P} = e^{-4t+C}$

$\frac{4000}{P} - 1 = e^{-4t} \cdot e^{C}$

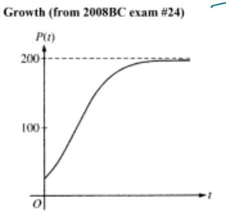
$\frac{4000}{P} - 1 = Ce^{-4t}$

$\frac{4000}{200} - 1 = C$

$\frac{4000}{P} - 1 = 19e^{-4t}$

$\left(\frac{4000}{P}\right)^{-1} = \left(19e^{-4t} + 1\right)^{-1}$

AP Question involving Logistic Growth (from 2008BC exam #24)



Which of the following differential equations for a population P could model the logistic growth shown in the figure above?

(A) $\frac{dP}{dt} = 0.2P - 0.001P^2$

(B) $\frac{dP}{dt} = 0.1P - 0.001P^2$

(C) $\frac{dP}{dt} = 0.2P^2 - 0.001P$

(D) $\frac{dP}{dt} = 0.1P^2 - 0.001P$

(E) $\frac{dP}{dt} = 0.1P^2 + 0.001P$

$\frac{dP}{dt} = kP(m-P) = kmP - kP^2$

$\frac{dy}{dt} = ky(m-y)$

P

$\frac{4000}{200} - 1 = C$

$19 = C$

$\left(\frac{4000}{P}\right)^{-1} = \left(19e^{-4t} + 1\right)^{-1}$

$\frac{P}{4000} = \frac{1}{19e^{-4t} + 1}$

$$P(t) = \frac{4000}{1 + 19e^{-4t}}$$