7.5 Day 2

Tuesday, December 3, 2019 6:57 PM

Section 7.5
p. 376: 23, 24, 27, 31, 34, 40, 44

$\frac{dP}{dt} = kP(m-P) \qquad P = \frac{m}{1+Ce^{-kmt}}$

23) $\frac{dP}{dt} = 0.006P(200-P) \qquad m = 200 \qquad k = .006 \qquad P(0) = 8$

a. Carry capacity $m = 200$

b. P when $\frac{dP}{dt}$ is the greatest $\frac{m}{2} = 100$ individuals

c. $\frac{dP}{dt} = .006(100)(200-100)$ 60 individuals per year

24. $\frac{dP}{dt} = .0008P(700-P) \qquad k = .0008 \qquad m = 700$

a. 700 individuals
b. 350 individuals
c. $\frac{dP}{dt} = .0008(350)(700-350)$ 98 individuals per year

27. $\frac{dP}{dt} = 0.006P(200-P)$ and $P = 8$ when $t = 0$

opt 1 hard ★ 2 options for this problem

$\int \frac{1}{P(200-P)}\, dP = \int .006\, dt$

$\frac{1}{P(200-P)} = \frac{A}{P} + \frac{B}{200-P}$

$1 = A(200-P) + BP$

when $P = 0$ $1 = 200A$ $A = \frac{1}{200}$ & $P = 200$ $1 = 200B$ $B = \frac{1}{200}$

$\frac{1}{200}\int \frac{1}{P}\, dP + \frac{1}{200}\int \frac{1}{200-P}\, dP = .006t$

$\frac{1}{200}\ln|P| + \frac{1}{200}\ln|200-P| = .006t + C$

$-.006t - C = \frac{1}{200}\ln|200-P| - \frac{1}{200}\ln P$

$-.006t - C = \frac{1}{200}\ln\frac{200-P}{P}$

$e^{-1.2t} \cdot e^{C} = \frac{200}{P} - 1$

$C \cdot 1 = \frac{200}{8} - 1$

$C = 25 - 1$

$-.006t - C = \frac{1}{200}\ln\frac{200-P}{P}$

$-1.2t - C = \ln\frac{200-P}{P}$

$C = 25-1$

$C = 24$

$24e^{-1.2t} = \frac{200}{P} - 1$

$1 + 24e^{-1.2t} = \frac{200}{P}$

$$\boxed{P = \frac{200}{1+24e^{-1.2t}}}$$

however.... now that is a lot of work

~~opt 2~~ not bad

$\frac{dP}{dt} = 0.006P(200-P)$ $\quad P=8$ when $t=0$

$KP(m-P)$

$m = 200 \qquad 200(.006) = 1.2 = mK$

$P = \frac{M}{1+Ae^{-mkt}} \qquad P = \frac{200}{1+Ae^{-1.2t}}$

now... $8 = \frac{200}{1+A} \qquad 1+A = \frac{200}{8}$

$1 + A = 25 \qquad A = 24$

$$\boxed{P = \frac{200}{1+24e^{-1.2t}}}$$

31. $P(t) = \frac{1000}{1+e^{4.8-0.7t}} = \frac{1000}{1+e^{4.8}\cdot e^{-0.7t}}$

$-km = -0.7$

$-1000K = -.7$

$$\boxed{\begin{array}{l}K = .0007\\ m = 1000\end{array}}$$

34. $\frac{dP}{dt} = \overset{K}{0.0004}\overset{P}{P}(\overset{m}{250}-P)$

$m = 250$

$K \cdot m = 0.1$

$P(t) = \frac{250}{1+Ae^{-0.1t}}$

let $t =$ time in yrs since 1970

$28 = \frac{250}{1+A}$

$A \approx 7.9286$

$1+A$

$$P(t) = \frac{250}{1+7.9286\,e^{-0.1t}}$$

40. True $\quad M=100 \quad$ meaning $\lim_{t\to\infty} P(t) = 100$

$\lim_{t\to-\infty} P(t) = 0$

$\therefore$ $P(0)$ must be between 0 & 100

44. we know $\frac{dP}{dt} = kP(M-P)$

to connect to this problem $\frac{dy}{dx} = k\underline{y}(M-\underline{y})$ B

this eliminates choices A, D, & E