7.4 Day 3

Friday, February 17, 2017 8:23 AM

Opener 7.4 Name_______________________________

In many countries throughout the world, a weird fungus that causes ants to turn into zombies has broken out. Suppose that after 20 days of the outbreak of the fungus, 200 ants in a colony are infected with the fungus. 10 days after that, 1000 ants are infected with the fungus. If there are 1 million ants in this particular colony, how long will it take until all of the ants are infected with the zombie fungus if this exponential growth continues at the same rate?

x^3

$(20, 200)$ $\quad (30, 1000)$ $\quad y = y_0 e^{kt}$

$$\frac{1000 = y_0 e^{30k}}{200 = y_0 e^{20k}}$$

$5 = e^{10k}$

$\ln 5 = \ln e^{10k}$

$\ln 5 = 10k \ln e$

$k = \frac{\ln 5}{10}$

$y = y_0 e^{\frac{t \ln 5}{10}}$

$200 = y_0 e^{\frac{20 \ln 5}{10}}$

$y_0 = 8$

$y = 8e^{\frac{t \ln 5}{10}}$

$1000000 = 8e^{\frac{t \ln 5}{10}}$

$t \approx 72.92$ days

Newton's Law of Cooling

If T is the temperature of an object at time t, and T_s is the s

t = time

T = Temp

$$T - T_s = (T_0 - T_s)e^{-kt}$$

(t, T)

T_s = surrounding temp

T_0 = initial temp

A hard-boiled egg at 98 degrees Celsius is put in a pan with 18 degrees Celsius water to cool down. After 5 minutes, the egg's temperature is found to be 38 degrees Celsius. How much longer will it take for the egg to reach 20 degrees Celsius (so we can eat it!)?

$T_s = 18$ $\quad T_0 = 98$ $\quad (5, 38)$ $\quad K = \frac{\ln .25}{-5} \approx .277$

$(t, 20)$

$38 - 18 = (98 - 18)e^{-5k}$

$20 = 80e^{-5k}$

$\frac{1}{4} = e^{-5k}$

$\ln \frac{1}{4} = -5k$

$T - 18 = 80e^{-.277t}$

$20 - 18 = 80e^{-.277t}$

$2 = 80e^{-.277t}$

$\frac{2}{80} = e^{-.277t}$

$\frac{\ln\left(\frac{2}{80}\right)}{-.277} = t$

(over)

$$\frac{t \approx 13.317 \text{ mins} \\ -5}{8.317 \text{ mins}}$$

Solve each differential equation.

a. $\frac{dy}{dx} = y\sqrt{x}$

$\int \frac{1}{y}\,dy = \int x^{1/2}\,dx$

$\ln y = \frac{2}{3}x^{3/2} + C$

$y = e^{2/3 x^{3/2} + C}$

$y = e^{2/3 x^{3/2}} \cdot e^{C}$ $\qquad e^{C} = A$

$\boxed{y = Ae^{2/3 x^{3/2}}}$

b. $\frac{dy}{dx} = \frac{4\sqrt{y} \bullet \ln x}{x}$ $\quad$ if $\quad y(e) = 1$

$\frac{1}{4\sqrt{y}}\,dy = \frac{\ln x}{x}\,dx$

$\int \frac{1}{4} y^{-1/2}\,dy = \int \ln x \cdot \frac{1}{x}\,dx$ $\qquad u = \ln x \quad du = \frac{1}{x}\,dx$

$\frac{1}{2} y^{1/2} = \int u\,du$

$\frac{1}{2} y^{1/2} = \frac{u^2}{2} + C$

$\frac{1}{2} y^{1/2} = \frac{\ln^2 x}{2} + C$

$y^{1/2} = \ln^2 x + C$

$1^{1/2} = \ln^2 e + C$

$1 = 1 + C$

$C = 0$

$y^{1/2} = \ln^2 x$

$y = \pm\sqrt{\ln^2 x}$ since $y(e) > 0$

$y = \sqrt{\ln^2 x} = |\ln x|$ $\qquad$ Fyi $\sqrt{x^2} = |x|$

$y = |\ln x|$