7.4 Day 1

Wednesday, February 15, 2017 8:43 AM

Separable Differential Equations

- Check in HW
- collect 7.1 packet
- Notes

online hw scale

90 - 8
85 - 7.5
80 - 7
75 - 6.5
70 - 6
65 - 5.5
60 - 5
55 - 4.5
50 - 4
45 - 3.5
40 - 3
35 - 2.5
30 - 2
25 - 1.5
20 - 1

A separable differential equation of the form $\frac{dy}{dx} = f(y) \cdot g(x)$ is called separable.

$$\frac{dy}{dx} = f(y) \cdot g(x)$$

$$\frac{1}{f(y)} \cdot dy = g(x) \cdot dx$$

Use separation of variables to solve the initial value problems. Indicate the domain over which the solution is valid.

ex 1: $\frac{dy}{dx} = x^2 y^2 \cdot \frac{1}{y^2}$ and $y=1$ when $x=1$

$$\cancel{dx} \; \frac{1}{y^2} \cdot \frac{dy}{\cancel{dx}} = x^2 \cdot dx$$

$$\int \frac{1}{y^2}\, dy = \int x^2\, dx$$

$$-y^{-1} = \frac{x^3}{3} + C$$

★ find C $\quad -(1)^{-1} = \frac{1^3}{3} + C$

$$-1 = \frac{1}{3} + C$$

$$C = -4/3$$

$$-y^{-1} = \frac{x^3 - 4}{3}$$

$$(y^{-1})^{-1} = \left(\frac{-x^3 + 4}{3}\right)^{-1}$$

$C = -4/3$

$-y^{-1} = \frac{x^3}{3} - \frac{4}{3}$

$(y) = \left(\frac{\quad}{3}\right)$

$y = \frac{3}{-x^3+4}$

Think about domain

$-x^3+4 \neq 0$

$-x^3 = -4$

$x \neq \sqrt[3]{4}$

D: $(-\infty, \sqrt[3]{4}) \cup \cancel{(\sqrt[3]{4}, \infty)}$

b/c $x=1$ falls in this interval

2. $\frac{dy}{dx} = \frac{x}{y}$ and $y=2$ when $x=1$

$\int y\,dy = \int x\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + C$

$y^2 = x^2 + C$

find C

$2^2 = 1^2 + C$

$4 = 1 + C$

$C = 3$

$y^2 = x^2 + 3$

$y = \pm\sqrt{x^2+3}$

Stop and think !!! output

$$\boxed{y = \sqrt{x^2+3} \qquad D: \mathbb{R}}$$

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3. $\frac{dy}{dx} = \sec^2 x\, e^{\tan x + y}$ $(0,0)$

$dx \cdot \frac{1}{e^y} \cdot \frac{dy}{dx} = \sec^2 x\, e^{\tan x} \cdot e^y \cdot \frac{1}{e^y}\, dx$

Hint:

$x^{a+b} = \underbrace{x^a \cdot x^b}$

$x^3 \cdot x^2 = x^{3+2}$

$\int \frac{1}{e^y}\, dy = \int \sec^2 x\, e^{\tan x}\, dx$

$u = \tan x \qquad du = \sec^2 x\, dx$

$u = -y$ $\int e^{-y}$ $\int \sec^2 \quad \tan x$

$u=-y$
$du=-dy$

$u=\tan x \quad du=\sec^2 x\,dx$

$$\int e^{-y}\,dy = \int \sec^2 x\, e^{\tan x}\,dx$$

$$-e^{-y} = \int e^{u}\,du$$

$$-e^{-y} = e^{u} + C$$

$$= -e^{-y} = e^{\tan x} + C$$

$$= -e^{-0} = e^{\tan 0} + C$$

$$= -1 = e^{0} + C$$

$$= -1 = 1 + C$$

$$C = -2$$

$$= -e^{-y} = e^{\tan x} - 2$$

$$= e^{-y} = -e^{\tan x} + 2$$

$$= \ln e^{-y} = \ln\left|-e^{\tan x} + 2\right|$$

$$= -y = \ln\left|-e^{\tan x} + 2\right|$$

$$= \boxed{y = -\ln\left|-e^{\tan x} + 2\right|}$$

Domain $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$