7.3 Day 1 Integration By Parts $\Rightarrow$ Reverse of the Product Rule

Monday, December 2, 2019 8:56 AM

Recall the Product Rule!

$$\frac{d}{dx}\left[f(x)\cdot g(x)\right] = f'(x)\,g(x) + f(x)\cdot g'(x)$$

$$\underbrace{\int \frac{d}{dx}\left[f(x)\cdot g(x)\right]dx}_{} = \int\left[f'(x)g(x) + f(x)\cdot g'(x)\right]dx$$

$$f(x)\cdot g(x) = \int f'(x)\,g(x)\,dx + \boxed{\int f(x)\,g'(x)\,dx}$$

$$f(x)\cdot g(x) - \int f'(x)\,g(x)\,dx = \int f(x)\,g'(x)\,dx$$

$$\boxed{\int f(x)\,g'(x)\,dx = f(x)g(x) - \int f'(x)\,g(x)\,dx}$$

or.... where u & v are functions of x

$$\frac{d}{dx}u\cdot v = (u'\cdot v + v'\cdot u)$$

$$\int \frac{d}{dx}u\cdot v\,dx = \int v\cdot\frac{du}{\cancel{dx}}\cdot\cancel{dx} + \int u\cdot\frac{dv}{\cancel{dx}}\cdot\cancel{dx}$$

$$\frac{d}{dx}\int u\cdot v\,dx$$

$$u\cdot v = \int v\,du + \boxed{\int u\,dv}$$

$$\boxed{\int u\,dv = u\cdot v - \int v\,du}$$

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ex: $\int \overset{f(x)\,\cdot\,g'(x)}{x\cos x}\,dx$

$$\int f(x)\cdot g'(x)\,dx = f(x)\cdot g(x) - \underbrace{\int f'(x)\,g(x)\,dx}_{}$$

$u = f(x) = x \qquad v = g(x) = \sin x$

$du = f'(x) = 1 \cdot dx \qquad dv = g'(x) = \cos x \, dx$

$\int x \cos x \, dx = x \sin x - \int 1 \cdot \sin x \, dx$

$= x \sin x - (-\cos x) + C$

$\boxed{= x \sin x + \cos x + C}$

Ex: $\int x e^x \, dx$

$u = x \qquad v = e^x$

$du = 1 \, dx \qquad dv = e^x \, dx$

$= xe^x - \int e^x \cdot dx$

$\boxed{= xe^x - e^x + C}$

How do we choose u & dv?

u: something you can take a derivative

dv: something you can antidifferentiate

LIPET for choosing "u" effectively

Logarithms

Inverse trig.

3 2 1

Polynomial | Power (x^2-3, x, $\sqrt{x}$, $x-x^{-1}+1$)

Exponentials

Trig

Ex: $\int x^2 \ln x \, dx$

$u = \ln x$ $\qquad v = \frac{1}{3}x^3$

$du = \frac{1}{x}dx$ $\qquad dv = x^2 dx$

$= \frac{1}{3}x^3 \cdot \ln x - \int \frac{1}{3}x^3 \cdot \frac{1}{x}\, dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\cdot\frac{1}{3}x^3 + C$

$$\boxed{= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C}$$

you try... $\int x \sec^2 x \, dx$

$u = x$ $\qquad v = \tan x$

$du = dx$ $\qquad dv = \sec^2 x$

$= x\tan x - \int \tan x \, dx$

$= x\tan x - \int \frac{\sin x}{\cos x} dx$ $\qquad u = \cos x$, $du = -\sin x \, dx$

$= x\tan x + \int \frac{1}{u} du$

$= x\tan x + \ln|\cos x| + C$