AP Calculus AB **Notes 7.2 Day 4 – U-Substitution & Algebra**

Tips for Integration

Evaluate the following integrals.

1. $\int \frac{1}{x^2}dx = \int x^{-2}dx = -x^{-1} + C$

2. $\int \frac{1}{x}dx = \ln|x| + C$

3. $\int \frac{1}{x^2+1}dx = \tan^{-1}x + C$

4. $\int \frac{x}{x^2+1}dx$

$u = x^2 + 1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\frac{1}{2}\int \frac{1}{u}du$

$= \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|x^2+1| + C$

Does #4 above confuse you? Here is tip #1.

I. Look for "The Hidden ln( )" in the integrand

When you have a rational function, look to see if the numerator is the derivative of the denominator. If so, substitute "u" for that denominator function, solve for "du" and then complete the integration.

Examples

$\int \frac{x}{x^2+1}dx = \frac{1}{2}\ln|x^2+1| + C$

$\int \tan(x)dx = \int \frac{\sin x}{\cos x}dx$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$-\int \frac{1}{u}du$

$= -\ln|\cos x| + C$

II. Try to determine if you just need to do algebra to simplify before integrating, or if u-substitution will be required. Particularly if the degree in the numerator is bigger than the degree in the denominator, you will have to use synthetic division to simplify.

Now use these facts to evaluate the following.

$$\int \frac{x^2 - 4x + 1}{x - 1}\,dx$$ $\neq$ synthetic division is necessary

$$\begin{array}{r|rrr} 1 & 1 & -4 & 1 \\ & & 1 & -3 \\ \hline & 1 & -3 & -2 \end{array}$$

$$\int \left(x - 3 - \frac{1}{x-1}\right) dx = \frac{x^2}{2} - 3x - 2\ln|x-1| + C$$

$u = x - 1$

$du = 1\,dx$

$$\int \frac{\boxed{x - 1}}{2x^2 - 4x + 4}\boxed{dx}$$ =

$u = 2x^2 - 4x + 4$

$du = 4x - 4\,dx = 4(x-1)\,dx$

$\frac{1}{4}\,du = x - 1\,du$

$$\frac{1}{4}\int \frac{1}{u}\,du = \frac{1}{4}\ln|u| + C$$

$$\boxed{= \frac{1}{4}\ln|2x^2 - 4x + 4| + C}$$

$$\int \frac{x^3 - 3x^2 - 4}{x}\,dx$$

$$\int \left(\frac{x^3}{x} - \frac{3x^2}{x} - \frac{4}{x}\right) dx = \int x^2 - 3x - \frac{4}{x}\,dx = \boxed{\frac{x^3}{3} - \frac{3}{2}x^2 - 4\ln|x| + C}$$

III. When making "u" substitutions in definite integrals, remember to change your bounds to "u" bounds!

$$\int_0^{\sqrt{3}} x\sqrt{x^2 + 1}\,dx$$

$u = x^2 + 1$ $\quad du = 2x\,dx$

$u(\sqrt{3}) = 4$ $\quad \frac{1}{2}du = x\,dx$

$u(0) = 1$

$$\frac{1}{2}\int_1^4 \sqrt{u}\,du$$

$$\frac{1}{2}\cdot\frac{2}{3}u^{3/2}\Big|_1^4$$

$$\frac{1}{3}u^{3/2}\Big|_1^4 = \frac{1}{3}(4)^{3/2} - \frac{1}{3}(1)^{3/2}$$

$$= 8/3 - 1/3 = \boxed{7/3}$$

$$\int_0^1 x^3\left(x^4 + 1\right)^5 dx$$

$u = x^4 + 1$ $\quad du = 4x^3\,dx$

$u(1) = 2$ $\quad \frac{1}{4}du = x^3\,dx$

$u(0) = 1$

$$\frac{1}{4}\int_1^2 u^5\,du = \frac{1}{4}\cdot\frac{1}{6}u^6\Big|_1^2 =$$

$$\frac{1}{24}(64) - \frac{1}{24}(1)$$

$$\frac{8}{3} - \frac{1}{24} = 21/8$$