7.2 Day 2 Antidifferentiation by Substitution

Friday, February 3, 2017 8:29 AM

★ opener PP day 2

Find the indefinite integral:

a. $\int \frac{\ln x}{3x}\,dx =$

Allison

$u = \frac{1}{3}\ln x$

$du = \frac{1}{3}\cdot\frac{1}{x}\,dx$ ☆

$du = \frac{1}{3x}\,dx$

$3du = \frac{1}{x}\,dx$

$\int \frac{1}{3}\ln x \cdot \frac{1}{x}\,dx$

$\int \underbrace{\frac{1}{3}\ln x}_{u} \cdot \frac{1}{\cancel{x}}\,dx$

$3\int u\,du$

Logan

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\frac{1}{3}\int \ln x \cdot \frac{1}{x}\,dx$

$\frac{1}{3}\int u\,du$

$\frac{1}{3}\cdot\frac{u^2}{2} + C$

$\frac{(\ln^2 x)}{6} + C$

b. $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = \int \sin x \cdot \frac{1}{\cos x}\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$-1\,du = \sin x\,dx$

$= -\int \frac{1}{u}\,du = -\ln|u| + C$

$\boxed{= -\ln|\cos x| + C}$

c. $\int \tan^7\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right)dx$

$u = \tan\left(\frac{x}{2}\right)$

Fyi: $\tan^7 x = (\tan x)^7$ or $\sin^2 x = (\sin x)^2$

$\int \left(\tan\left(\frac{x}{2}\right)\right)^7 \cdot \left(\sec\left(\frac{x}{2}\right)\right)^2 dx$

$\frac{du}{dx} = \sec^2\left(\frac{x}{2}\right)\cdot\frac{1}{2}$

$du = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)dx$

$2du = \sec^2\left(\frac{x}{2}\right)dx$

$2\int u^7\,du = \frac{2u^8}{8} + C$

$$\boxed{= \frac{\left(\tan^8\left(\frac{x}{2}\right)\right)}{4} + C}$$

d. $\int \sin^3 x\,dx$ hint: $\sin^2 x = 1 - \cos^2 x$

$$\int \sin^2 x \cdot \sin x \, dx$$

$$\int (1-\cos^2 x)\sin x \, dx \qquad u=\cos x \qquad du=-\sin x \, dx$$

$$-du = \sin x \, dx$$

$$-\int (1-u^2)\, du \qquad -\left[u-\frac{u^3}{3}\right]+C$$

$$= -\cos x + \frac{\cos^3 x}{3} + C$$