7.2 Day 1

Friday, November 15, 2019 9:35 AM

Indefinite Integrals - U-sub

Definite Integral: net area between the curve and the x-axis

$\Rightarrow$ a finite number is the result.

Indefinite Integral: the family of all antiderivatives of a function f(x) is the indefinite integral of f w/ respect to x.

$$\int f(x)\,dx = F(x) + C$$

where C is arbitrary constant

$$\int f'(x)\,dx = f(x) + C$$

must have

Evaluate

Ex: $\int (x^2 - \sin x)\,dx = \frac{x^3}{3} - (-\cos x) + C$

$= \frac{x^3}{3} + \cos x + C$

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substitution

$\int \sin(3x)\,\boxed{dx}$

$\int \sin(u)\left(\frac{1}{3}du\right)$

Let $\boxed{3x} = u$

$\frac{du}{dx} = 3$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$\frac{1}{3}du = dx$

$\frac{1}{3}\int \sin u \cdot du$

$-\frac{1}{3}\cos u + C = -\frac{1}{3}\cos(3x) + C$

Ex $\int \boxed{\sin x} e^{\cos x} \cdot \boxed{dx}$

1. Pick the "inside" function and call it "u"

$u = \cos x$

2. Find the derivative of u w/ respect to x and write as $\frac{du}{dx}$

$\frac{du}{dx} = -\sin x$

3. Adjust the substitution so that the right-hand side looks like part of the integrand.

$du = -\boxed{\sin x\, dx}$

$-du = \sin x\, dx$

4. Substitute & integrate w/ respect to u.

$-\int e^{u} du$

$-e^{u} + C = \boxed{-e^{\cos x} + C}$

5. substitute the original function in for u.

Check $\frac{d}{dx}\left(-e^{\cos x} + C\right)$

$-e^{\cos x} \cdot (-\sin x) + 0$

$\sin x\, e^{\cos x}$:)

When can I substitute?

ex:

a. $\int 2x \sin(x^2)\, dx$:) $u = x^2$ $du = 2x\, dx$

ex:

a. $\int \underline{2x} \sin(x^2)\, \underline{dx}$ ☺

$\int \sin u\, du = -\cos u + C$

$= -\cos x^2 + C$

b. $\int \sin(x^2) \boxed{dx}$ ☹ $u = x^2$ $du = 2x\, dx$

c. $\int \boxed{x} \sin(x^2) \boxed{dx}$ $u = x^2$ $du = 2\boxed{x\, dx}$

$\frac{du}{dx} = 2x$ $\frac{1}{2} du = x\, dx$

$\frac{1}{2}\int \sin u\, du$

$= -\frac{1}{2}\cos u + C$

$= -\frac{1}{2}\cos(x^2) + C$

Ex: $\int u^{-1} du = \int \frac{1}{u} du = \ln|u| + C$

Evaluate:

$$\int \frac{5x}{\sqrt{x^2+1}}\, dx = \int \frac{1}{\sqrt{x^2+1}}\, \underline{5x \cdot dx}$$

$u = x^2 + 1$

$\frac{du}{dx} = 2x$

$du = 2\boxed{x\, dx}$

$\frac{1}{2} du = x\, dx$

$= 5\int \frac{1}{\sqrt{x^2+1}}\, x\, dx$

$= 5 \cdot \frac{1}{2} \int \frac{1}{}\, du$

$$= 5 \cdot \frac{1}{2} \int \frac{1}{\sqrt{u}}\, du$$

$$= \frac{5}{2} \int u^{-1/2}\, du = \frac{5}{2} \cdot 2u^{1/2} + C$$

$$= \boxed{5\sqrt{x^2+1} + C}$$

$$u^{-1/2+1} = \frac{u^{1/2}}{1/2}$$

$$2u^{1/2}$$

you try $\displaystyle\int \frac{e^{\frac{1}{x}}}{x^2}\, dx =$

$u = \frac{1}{x}$

$du = \frac{-1}{x^2}\, dx$

$-du = \frac{1}{x^2}\, dx$

$$\int e^{\frac{1}{x}} \cdot \boxed{\frac{1}{x^2}\, dx}$$

$$-\int e^u\, du = -e^u + C = -e^{\frac{1}{x}} + C$$

$$\int_{x=0}^{x=1} \boxed{6x^2} (x^3+5)^4\, \boxed{dx}$$

$u = x^3 + 5$

$u(1) = 1^3 + 5 = 6$

$u(0) = 0^3 + 5 = 5$

$du = 3x^2\, dx$

$\frac{1}{3} du = x^2\, dx$

$$\frac{1}{3} \cdot 6 \int_5^6 u^4\, du$$

$$2\left[\frac{u^5}{5}\right]_5^6 = \frac{2}{5} 6^5 - \frac{2}{5} 5^5$$

$$\int_0^{\frac{\pi}{3}} \sin x\, e^{\cos x}\, dx$$

$u = \cos x$

$-du = \sin x\, dx$

$u\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$u(0) = 1$

$\int_0 \sin x\, e^{\cos x}\, dx$ $\quad -du = \sin x\, dx \quad u(0) = 1$

$-\int_1^{1/2} e^u\, du = -e^u \Big|_1^{1/2} = -e^{1/2} - (-e^1)$