

=6.1 packet

Monday, February 23, 2015 7:27 AM

1. $\frac{d}{dx}(\sin x) =$

2. $\frac{d}{dx}(\cos x) =$

3. $\frac{d}{dx}(\tan x) =$

4. $\frac{d}{dx}(\csc x) =$

5. $\frac{d}{dx}(\sec x) =$

6. $\frac{d}{dx}(\cot x) =$

7. $\frac{d}{dx}(4^x) =$

8. $\frac{d}{dx}(4^{x^2}) =$

9. $\frac{d}{dx}(e^x) =$

10. $\frac{d}{dx}(e^{x^2}) =$

11. $\frac{d}{dx}(\ln x) =$

12. $\frac{d}{dx}(\ln x^2) =$

$$13. \int \sin x dx =$$

$$14. \int \cos x dx =$$

$$15. \int \tan x dx =$$

$$16. \int \cot x dx =$$

$$17. \int e^x dx =$$

$$18. \int e^{2x} \cdot 2x^2 dx =$$

$$19. \int \frac{1}{t} dt =$$

$$20. \int \frac{2x}{(x^2+1)} dx =$$

Calculus – Initial Value Notes & Practice

1) If $f'(x) = 3x^2$ and $f(1) = 6$, find the particular solution to the differential equation.

$$f(x) = x^3 + C$$

$$6 = (1)^3 + C$$

$$C = 5$$

$$f(x) = x^3 + 5$$

Applications of Initial Value Problems

2) The velocity of a particle moving along a line is given by $v(t) = 4t^3 - 3t^2$ at time t . If initially the particle is at $x = 3$ on the line, find its position when $t = 2$.

$$s(t) = x \quad (t, x)$$

$$s(0) = 3 \quad s(2) = ?$$

$$s(t) = t^4 - t^3 + C$$

$$3 = 0^4 - 0^3 + C$$

$$C = 3$$

$$s(t) = t^4 - t^3 + 3$$

$$s(2) = 2^4 - 2^3 + 3$$

$$= 16 - 8 + 3$$

$$= 8 + 3$$

$$= 11$$

$$s(0) = 3 \quad s(2) = ?$$

$$\int_0^2 (4t^3 - 3t^2) dt = s(2) - s(0)$$

$$s(0) + \int_0^2 (4t^3 - 3t^2) dt = s(2)$$

$$3 + [t^4 - t^3]_0^2 = s(2) = 3 + [2^4 - 2^3] - [0]$$

$$2 + 8 = 11$$

3) Suppose that $a(t)$, the acceleration of a particle at time t , is given by $a(t) = 4t - 3$, that $v(1) = 6$, and that $s(2) = 5$, where $s(t)$ is the position function.

a) Find $v(t)$ and $s(t)$.

$$v(t) = 2t^2 - 3t + C$$

$$6 = 2 - 3 + C$$

$$C = -1 + 6$$

$$C = 5$$

$$v(t) = 2t^2 - 3t + 5$$

$$s(t) = \frac{2}{3}t^3 - \frac{3}{2}t^2 + 7t + C$$

$$5 = \frac{2}{3}(8) - \frac{3}{2}(4) + 7(2) + C$$

$$5 = \frac{16}{3} - 6 + 14 + C$$

$$C = -25/3$$

$$s(t) = \frac{2}{3}t^3 - \frac{3}{2}t^2 + 7t - 25/3$$

b) Find the position of the particle when $t = 1$.

$$s(1) = \frac{2}{3} - \frac{3}{2} + 7 - \frac{25}{3}$$

$$s(1) = -13/6$$

1) Decide which function from column 2 is a solution of the differential equation in column 1.

~~a) $\frac{dy}{dx} = 2y$~~

d I) $y = 2 \sin(x)$ $\frac{dy}{dx} = 2 \cos x$ $y'' = -2 \sin x = -y$

~~b) $y'' = -4y$~~

b II) $y = \sin(2x)$ $y = 2 \cos 2x$ $y'' = -4 \sin(2x)$
 $y'' = -4y$

~~c) $\frac{dy}{dx} = -2y$~~

a III) $y = e^{2x}$ $\frac{dy}{dx} = 2e^{2x} = 2y$ since $y = e^{2x}$

~~d) $\frac{d^2y}{dx^2} = -y$~~

c IV) $y = e^{-2x}$ $\frac{dy}{dx} = -2e^{-2x} = -2y$ since $y = e^{-2x}$

e) $y'' - 2y' + y = 0$

e V) $y = xe^x$ $y' = xe^x + e^x$ $y'' = xe^x + e^x + e^x$

$xe^x + 2e^x - 2(xe^x + e^x) + xe^x$
 $\cancel{xe^x} + 2e^x - 2\cancel{xe^x} - 2e^x + \cancel{xe^x} = 0$

2) A solution is given for each differential equation. Show that the function really is a solution of the differential equation and determine the particular solution that satisfies the given condition.

a. $\frac{dy}{dx} = \frac{1-2y}{x}$ $y = \frac{k}{x^2} + \frac{1}{2}$ $y(2) = 4$

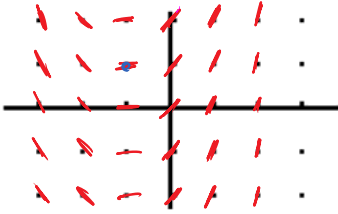
b. $\frac{dy}{dt} = \sqrt{y}$ $y = \frac{1}{4}(C+t)^2$ passes through (6,16)

SLOPE FIELDS

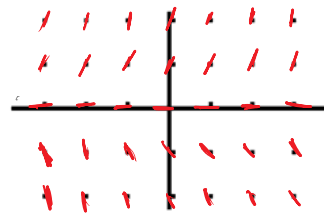
Draw a slope field for each of the following differential equations.

1. $\frac{dy}{dx} = x+1$

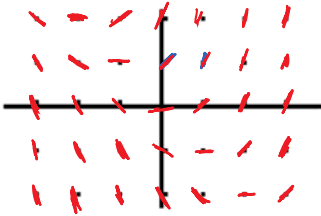
$y = \frac{x^2}{2} + x + C$



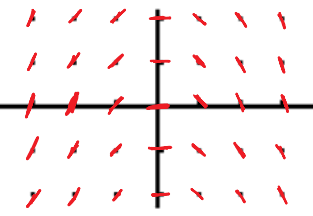
2. $\frac{dy}{dx} = 2y$



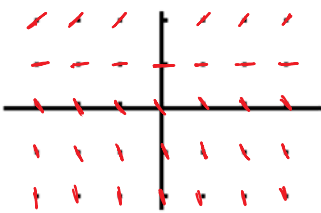
3. $\frac{dy}{dx} = x+y$



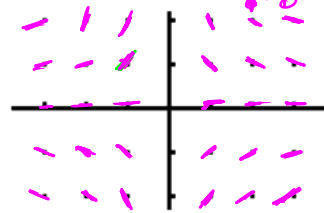
4. $\frac{dy}{dx} = 2x$



5. $\frac{dy}{dx} = y-1$



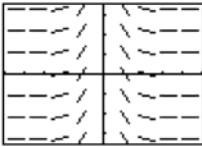
6. $\frac{dy}{dx} = -\frac{y}{x} = \frac{-1}{-1} = 1$



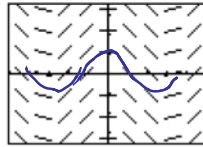
x	y	$\frac{dy}{dx}$
-1	-1	1
-1	2	2
-1	0	0
-1	-2	-2

Match each slope field with the equation that the slope field could represent.

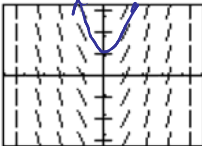
(A)



(B)



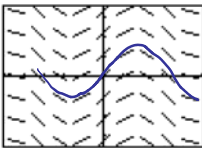
(C)



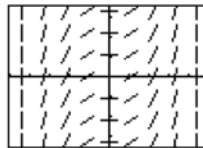
(D)



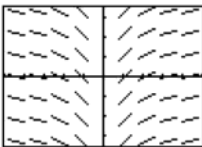
(E)



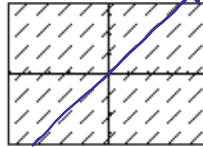
(F)



(G)



(H)



7. $y=1$ **D**

8. $y=x$ **H**

9. $y=x^2$ **C**

10. $y=\frac{1}{6}x^3$ **F**

11. $y=\frac{1}{x^2}$ **A**

12. $y=\sin x$ **E**

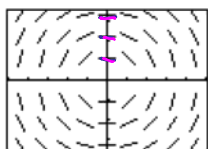
13. $y=\cos x$ **B**

14. $y=\ln|x|$ **G**

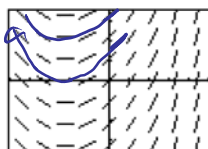
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Match the slope fields with their differential equations.

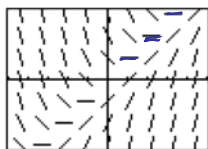
(A)



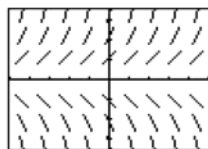
(B)



(C)



(D)



15. $\frac{dy}{dx} = \frac{1}{2}x + 1$ B

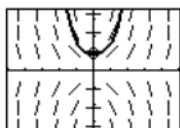
17. $\frac{dy}{dx} = x - y$ C

16. $\frac{dy}{dx} = y$ D

18. $\frac{dy}{dx} = -\frac{x}{y}$ A

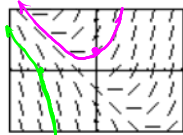
19. The calculator drawn slope field for the differential equation $\frac{dy}{dx} = xy$ is shown in the figure below. The solution curve passing through the point $(0, 1)$ is also shown.

- (a) Sketch the solution curve through the point $(0, 2)$.
- (b) Sketch the solution curve through the point $(0, -1)$.



20. The calculator drawn slope field for the differential equation $\frac{dy}{dx} = x + y$ is shown in the figure below.

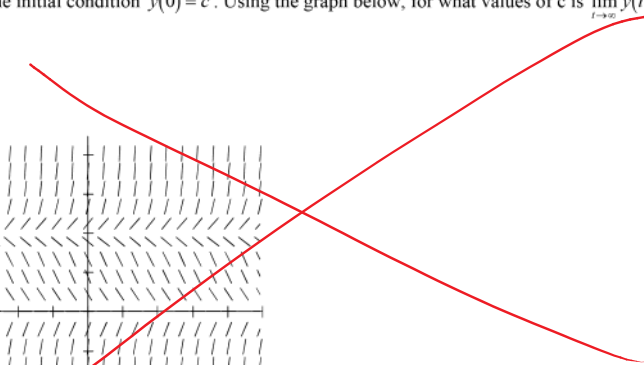
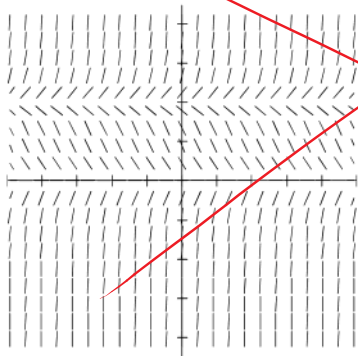
- (a) Sketch the solution curve through the point $(0, 1)$.
- (b) Sketch the solution curve through the point $(-3, 0)$.



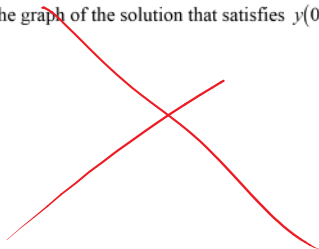
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Slope fields Practice

1. Suppose the initial condition $y(0) = c$. Using the graph below, for what values of c is $\lim_{t \rightarrow \infty} y(t)$ finite?



2. Using the above graph, sketch the graph of the solution that satisfies $y(0) = 1$.



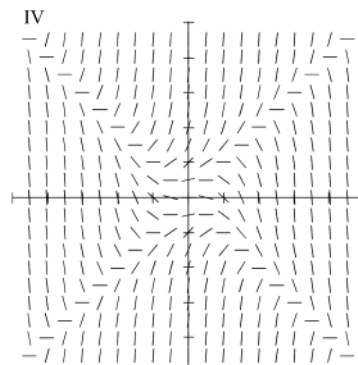
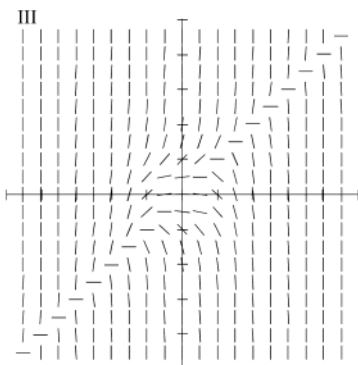
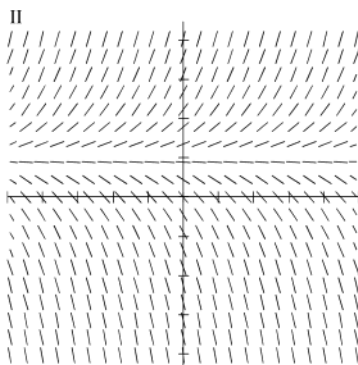
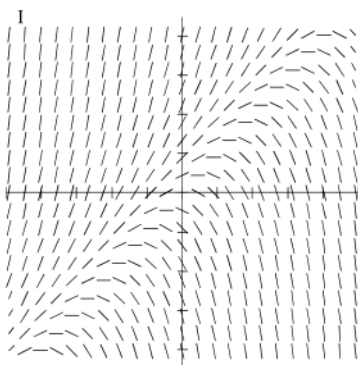
Match the differential equation with its slope field.

II 3. $\frac{dy}{dx} = y - 1$

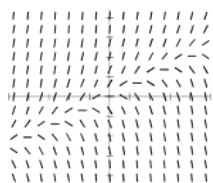
IV 4. $\frac{dy}{dx} = y^2 - x^2$

I 5. $\frac{dy}{dx} = y - x$

III 6. $\frac{dy}{dx} = y^3 - x^3$

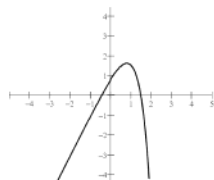


5.

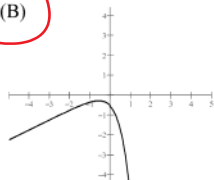


Which one of the following could be the graph of the solution of the differential equation whose slope field is shown above?

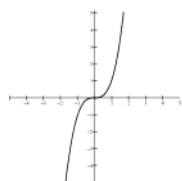
(A)



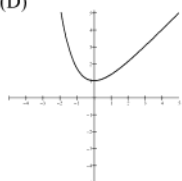
(B)



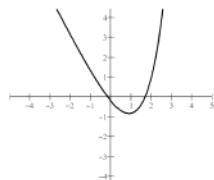
(C)



(D)



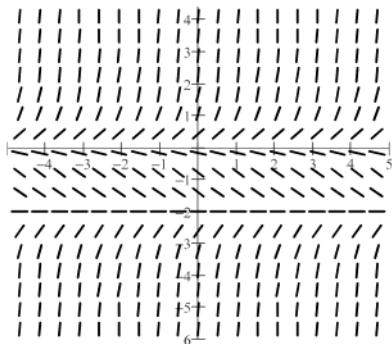
(E)



6. The slope field for the differential equation $\frac{dy}{dx} = \frac{x^2y + y^2}{4x + 2y}$ will have vertical segments when

- (A) $y = 2x$, only
- (B) $y = -2x$, only
- (C) $y = -x^2$, only
- (D) $y = 0$, only
- (E) $y = 0$ or $y = -x^2$

7.



Which statement is true about the solutions $y(x)$, of a differential equation whose slope field is shown above?

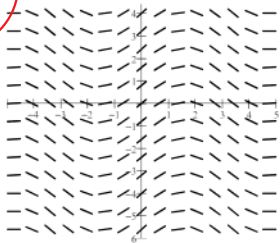
- I. If $y(0) > 0$ then $\lim_{x \rightarrow \infty} y(x) \approx 0$.
- II. If $-2 < y(0) < 0$ then $\lim_{x \rightarrow \infty} y(x) \approx -2$.
- III. If $y(0) < -2$ then $\lim_{x \rightarrow \infty} y(x) \approx -2$.

- (A) I only
- (B) II only
- (C) III only
- (D) II and III only
- (E) I, II, and III

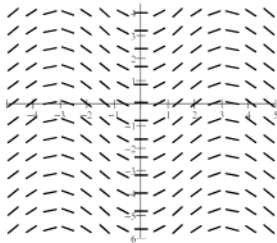
9. Which choice represents the slope field for $\frac{dy}{dx} = \cos x$?

$y = \sin x$

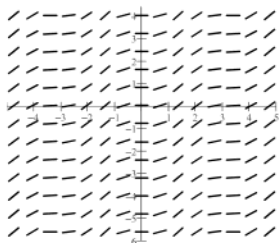
(A)



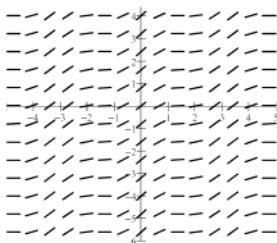
(B)



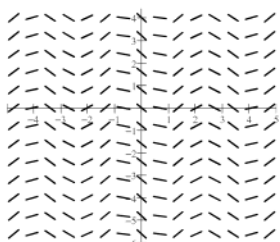
(C)



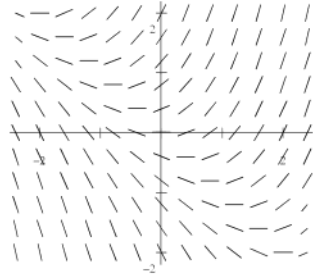
(D)



(E)



10.
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Shown above is the slope field for which of the following differential equations?

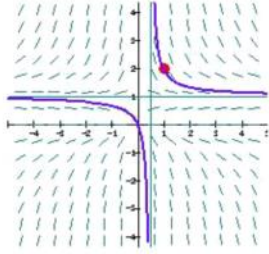
- (A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$ (D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

Answers:

4. D 5. B 6. B 7. D

8. (b) The particular solution is $y = \frac{x}{x - \frac{1}{2}}$, and the slope at $(0, 0)$ is $y'(0) = \frac{-\frac{1}{2}}{(0 - \frac{1}{2})^2} = -2$.

The asymptotes are $x = \frac{1}{2}$ and $y = 1$. (d) Graph below.



9. A

10. C