6.3 Day 1

Thursday, January 17, 2019 7:47 AM

Rules for Definite Integrals

6.3

1. *Order of Integration:* $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$ A definition

2. *Zero:* $\int_a^a f(x)\,dx = 0$ Also a definition

3. *Constant Multiple:* $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$ Any number k

   $\int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx$ $k = -1$

4. *Sum and Difference:* $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

5. *Additivity:* $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

Example Suppose f & h are continuous functions and that:

$\int_{-1}^{1} f(x)\,dx = 5$ $\int_{1}^{4} f(x)\,dx = -2$ $\int_{-1}^{1} h(x)\,dx = 7$

Evaluate:

a. $\int_4^1 f(x)\,dx$

$-\int_1^4 f(x)\,dx$

$-(-2) = 2$

b. $\int_{-1}^4 f(x)\,dx$

$\int_{-1}^{1} f(x)\,dx + \int_{1}^{4} f(x)\,dx$

$5 + -2 = 3$

c. $\int_{-1}^1 [2f(x) + 3h(x)]\,dx$

$2\int_{-1}^1 f(x)\,dx + 3\int_{-1}^1 h(x)\,dx$

$2(5) + 3(7)$

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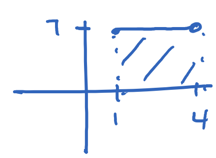
d. $\int_1^4 (f(x) + 7)\,dx$



$\int_1^4 f(x)\,dx + \int_1^4 7\,dx$

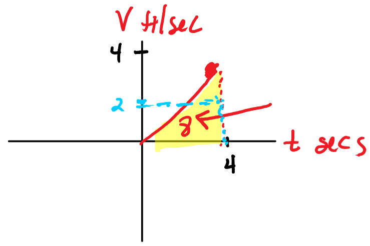
$-2 + 7(4-1)$

$-2 + 21 = 19$

Average value (Avg. output / Avg y)

$v(t) = t$ ft/sec

1. $\int_0^4 t\,dt = 8$ ft



2. Avg velocity [0,4] $\frac{\Delta s}{\Delta t} = \frac{\int_0^4 t\,dt}{4-0} = \frac{8}{4} = 2$ ft/sec

Average value

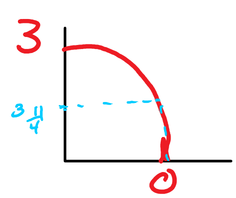
f(x) is a continuous function on the interval [a,b], then

$$\text{Avg. value } \frac{1}{b-a}\int_a^b f(x)\,dx$$

Ex: Find the avg. value of f(x):

a. w/o a calculator $f(x) = \sqrt{9-x^2}$ [0,3]

$$\frac{1}{3-0}\int_0^3 \sqrt{9-x^2}\,dx$$



$$\frac{1}{3}\cdot\frac{1}{4}\pi(3)^2 = \frac{3}{4}\pi$$

b. w/ calc. $f(x) = 4-x^2$ on [0,3]

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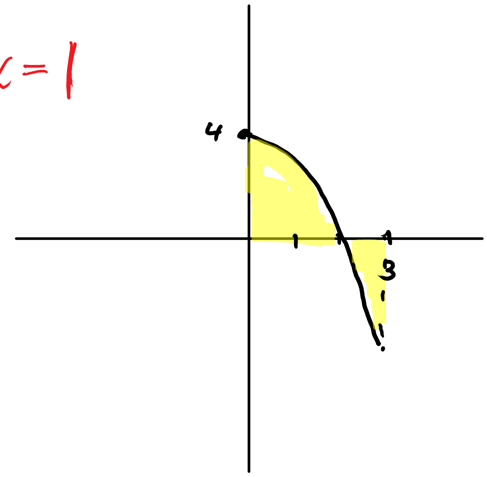
$$\text{Avg. value} = \frac{1}{3}\int_0^3 (4-x^2)\,dx = 1$$



Mean Value Theorem for Definite Integrals

If f is a continuous function on $[a,b]$,
then there is some c in $[a,b]$
where $f(c) = \frac{1}{b-a}\int f(x)\,dx$

w/ calculator

Example Find when $f(x)$ = the avg. value.

$f(x) = x^2 - 1$ $[0, \sqrt{3}]$

① Find the avg. value $\frac{1}{\sqrt{3}-0}\int_0^{\sqrt{3}} (x^2-1)\,dx = 0$

② $f(x) = \text{avg value}$

Avg value

$f(c) = c^2 - 1$ $c^2 - 1 = 0$ $c = \pm 1$ $\boxed{c = 1}$