6.6 Day 2

Wednesday, May 15, 2019 9:03 AM

$z = \frac{1}{2} + \frac{\sqrt{3}}{2} i$ write z^2, z^3, z^4, z^5, and z^6 in trig form (polar) then standard form

$r = 1$ $\theta = \frac{\pi}{3}$

recall: $z^n = r^n \text{cis}(n\theta)$ (polar form)

	Trig form	Standard form
$z^2 =$	$1^2 \text{cis}(2 \cdot \frac{\pi}{3}) = \text{cis} \frac{2\pi}{3}$ $\cos\frac{2\pi}{3} + i \sin\frac{2\pi}{3}$	$-\frac{1}{2} + \frac{i\sqrt{3}}{2}$
$z^3 =$	$1^3 \text{cis}\, 3 \cdot \frac{\pi}{3} = \text{cis}\, \pi$	$-1 + 0i = -1$
$z^4 =$	$1^4 \text{cis}(4 \cdot \frac{\pi}{3}) = \text{cis} \frac{4\pi}{3}$	$-\frac{1}{2} - \frac{i\sqrt{3}}{2}$
$z^5 =$	$1^5 \text{cis}(5 \cdot \frac{\pi}{3}) = \text{cis} \frac{5\pi}{3}$	$\frac{1}{2} - \frac{i\sqrt{3}}{2}$
$z^6 =$	$1^6 \text{cis}(6 \cdot \frac{\pi}{3}) = \text{cis}(2\pi)$	$1 + 0i = 1$

Root of a Complex Number

A complex number $v = a + bi$ is the n^{th} root of z if $v^n = z$

* if $z = 1$, then v is an n^{th} root of unity

** Finding n^{th} roots of a complex number

If $z = \underbrace{r(\cos\theta + i\sin\theta)}_{r\,\text{cis}\,\theta}$, then the n distinct complex numbers $= \sqrt[n]{r}\left(\cos\frac{\theta + 2\pi k}{n} + i\sin\frac{\theta + 2\pi k}{n}\right)$

or $\sqrt[n]{r}\, \text{cis}\left(\frac{\theta + 2\pi k}{n}\right)$

or $\sqrt[n]{r}\ \text{cis}\left(\frac{\theta+2\pi k}{n}\right)$

Ex: Find the cube root of $8i$

$z^3 = 8i$ solve the equation:

Assume the cube roots of z are in the form $z = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$

$$\underline{z^3} = \underline{8i} \Rightarrow 0 + 8i$$

$$r^3\,\text{cis}(3\theta) = 8\,\text{cis}\,\frac{\pi}{2}$$

$$r^3 = 8 \qquad \boxed{r = \sqrt[3]{8}} \Rightarrow \boxed{r = 2}$$

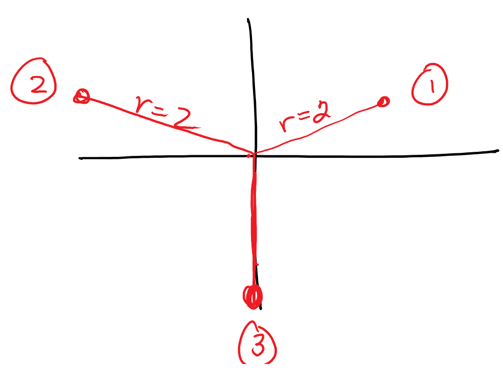
$$\frac{3\theta}{3} = \frac{\frac{\pi}{2}}{3} + \frac{2\pi k}{3}$$

$$\theta = \frac{\pi}{6} + \frac{2\pi k}{3}$$

Solution 1
$k=0$ $\quad 2\,\text{cis}\left(\frac{\pi}{6} + \frac{2\pi(0)}{3}\right) = 2\,\text{cis}\,\frac{\pi}{6} = 2\left(\frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)\right) = \sqrt{3} + i$

solution 2
$k=1$ $\quad 2\,\text{cis}\left(\frac{\pi}{6} + \frac{2\pi}{3}\cdot 1\right) = 2\,\text{cis}\,\frac{5\pi}{6} \qquad 2\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = -\sqrt{3} + i$

solution 3
$k=2$ $\quad 2\,\text{cis}\left(\frac{\pi}{6} + \frac{2\pi}{3}(2)\right) = 2\,\text{cis}\,\frac{3\pi}{2} \qquad 2(0 + -1i) = -2i$



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Ex. Find the fourth roots of $z = 5\,\text{cis}\,\frac{\pi}{3}$

$r^4 = 5$

$r = \sqrt[4]{5}$

$4\theta = \frac{\pi}{3} + 2\pi k$

$\theta = \frac{\pi}{12} + \frac{\pi}{2}k$

sol. 1 $k=0$ $\sqrt[4]{5}\,\text{cis}\left(\frac{\pi}{12} + \frac{\pi}{2}(0)\right) = \sqrt[4]{5}\,\text{cis}\left(\frac{\pi}{12}\right)$

sol. 2 $k=1$ $\sqrt[4]{5}\,\text{cis}\left(\frac{\pi}{12} + \frac{\pi}{2}(1)\right) = \sqrt[4]{5}\,\text{cis}\,\frac{7\pi}{12}$

sol. 3 $k=2$ $\sqrt[4]{5}\,\text{cis}\left(\frac{\pi}{12} + \frac{\pi}{2}(2)\right) = \sqrt[4]{5}\,\text{cis}\,\frac{13\pi}{12}$

sol. 4 $k=3$ $\sqrt[4]{5}\,\text{cis}\left(\frac{\pi}{12} + \frac{\pi}{2}(3)\right) = \sqrt[4]{5}\,\text{cis}\,\frac{19\pi}{12}$

Find the 8^{th} roots of unity

$z = 1$ $1 + 0i$

$r^8 = 1$

$\sqrt[8]{1} = 1$

$8\theta = 0 + 2\pi k$

$\theta = \frac{\pi}{4}k$

sol. 1 $k=0$ $\sqrt[8]{1}\,\text{cis}\,\frac{\pi}{4}(0) = \text{cis}\,0$ $1 + 0i$

sol. 2 $k=1$ $\sqrt[8]{1}\,\text{cis}\,\frac{\pi}{4}(1) = \text{cis}\,\frac{\pi}{4}$ $\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

sol. 3 $k=2$ $\sqrt[8]{1}\cdot\text{cis}\,\frac{\pi}{4}(2) = \text{cis}\,\frac{\pi}{2}$ $0 + 1i$

$\vdots$

sol. 8 $\quad K=7 \quad \sqrt[8]{1}\,\mathrm{cis}\left(\frac{\pi}{4}(7)\right) = \mathrm{cis}\,\frac{7\pi}{4} \qquad \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$

Example: simplify $\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)^8$ $\qquad$ Hint: (root or power?)

$r^n\,\mathrm{cis}(n\theta)$

$r = ? \qquad \theta = ?$

$r = 1 \qquad \theta = \frac{3\pi}{4}$

$r^n\,\mathrm{c\cancel{o}is}(n\theta)$

$1^8\,\mathrm{c\cancel{o}is}\left(8\left(\frac{3\pi}{4}\right)\right) \Rightarrow \mathrm{c\cancel{o}is}(6\pi) = 1 + 0i = 1$